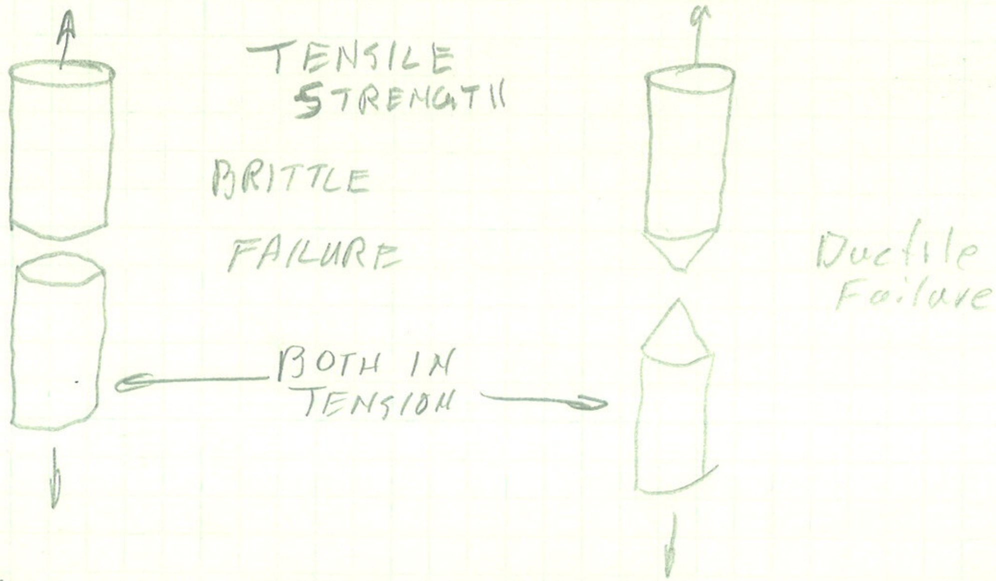
STATIC FAILURE THEORY - BRITTLE MATERIALS

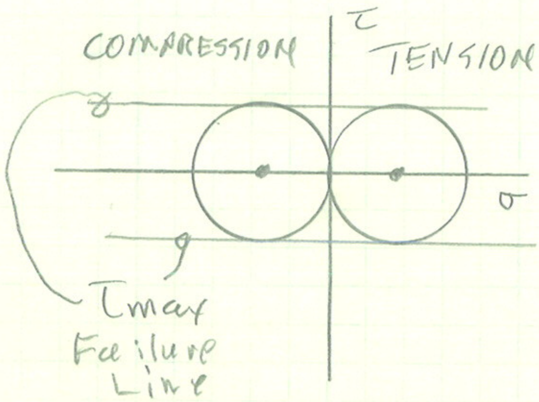
For brittle materials it is a little more complex, Most ductile materials and some brittle materials, the strength is the same for both compression and tension.

For the brittle materials, they fracture instead of yielding. In general, when a brittle material fails, when loaded in tension it will fail in tension. This is different from the ductile material that fails in shear



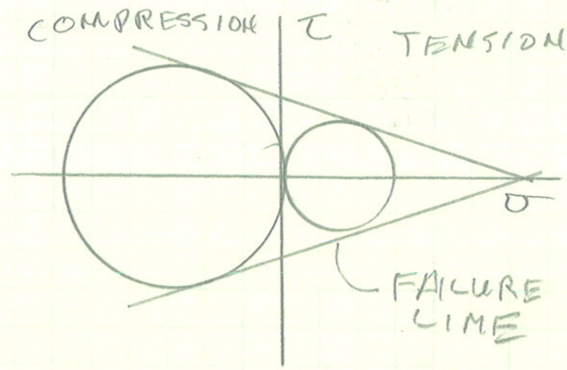
EVEN AND UNEVEN MATERIALS

EVEN MATERIALS



USE MAXIMUM SHEAR STRESS THEORY

UNEVEN MATERIALS



COMPRESSION STRENGTH IS GREATER THAN THE TENSILE STRENGTH

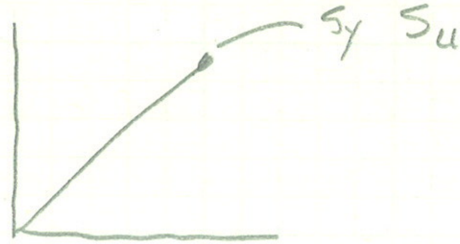
UNEAVEN MATERIALS

For uneven materials, we have:

S_{ut} = the ultimate tensil strength

S_{uc} = the ultimate compressive strength

If the uneven material is brittle, which is often the case, there is no S_y.



Remember, we classify brittle materials as those whose strain is less than 5%

$$\varepsilon_u < 5\%$$

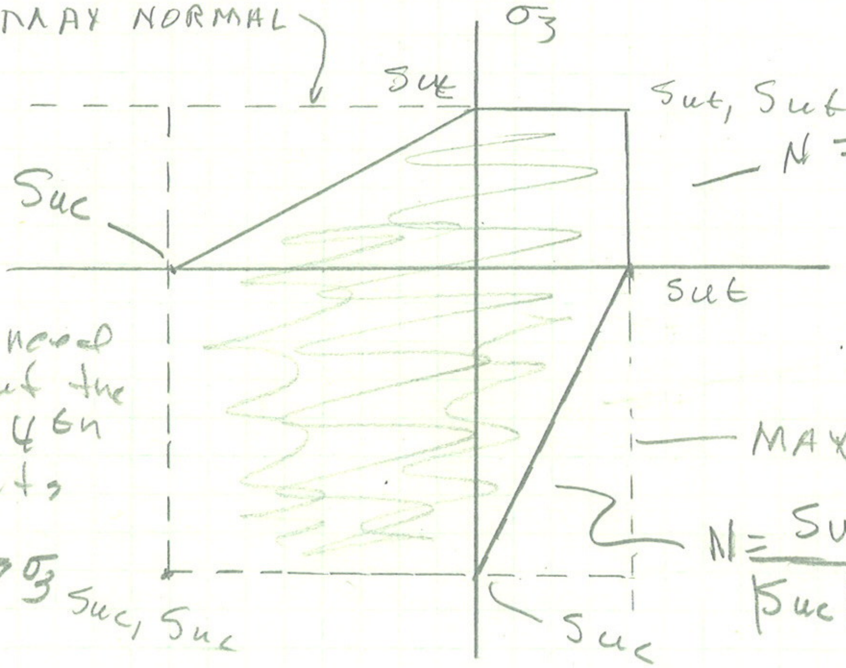
For tension of an unequal material brittle fracture is due to the normal stress alone. In this case, we can use normal stress theory

$$N = \frac{S_{ut}}{\sigma_1}$$

This formula is not reliable for ductile materials but seems to work well for brittle materials in tension.

In compression, fracture is due to some combination of normal compressive stress and shear and requires a different theory of failure.

COULOMB-MOHR THEORY

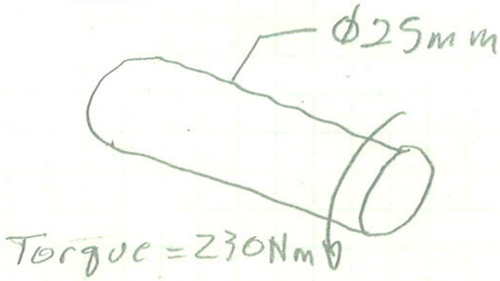


$N = \dfrac{S_{ut}}{\sigma_1}$

We only need to look at the 1st and 4th quadrants if $\sigma_1 > \sigma_2 > \sigma_3$

$N = \dfrac{S_{ut}|S_{uc}|}{|S_{uc}|\sigma_1 - S_{ut}\sigma_3}$

EXAMPLE

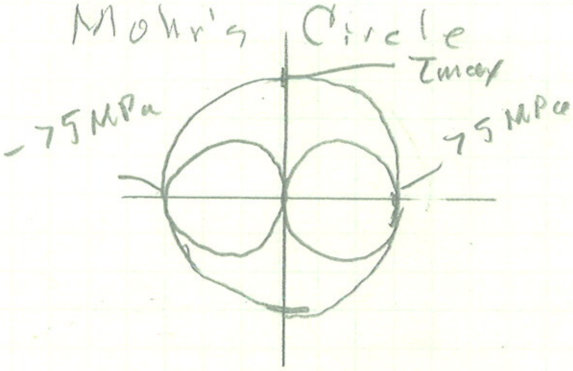


Cast Aluminum 195-T6

$S_{uc} = 170$ MPa $\quad S_{ut} = 160$ MPa

Pure shear

Mohr's Circle



$$\tau = \frac{Tr}{J}$$

$$J = \frac{\pi D^4}{32} = \frac{\pi 25^4}{32} = 38{,}349$$

$$\tau = \frac{230(1000)(12.5)}{38{,}349}$$

$$\tau = = 74.97 = \underline{75 \text{ MPa}}$$

We are in the 4th quadrant because $\sigma_1 > 0$ and $\sigma_3 < 0$

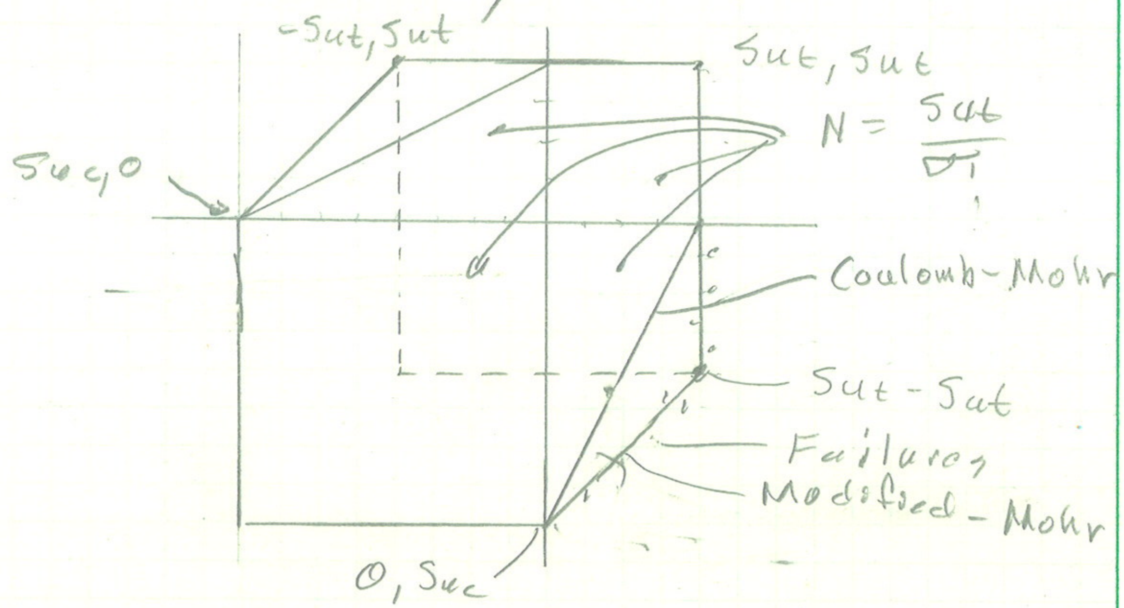
The safety factor becomes

$$N = \frac{S_{ut}|S_{uc}|}{|S_{uc}|\sigma_1 - S_{ut}\sigma_3}$$

$$N = \frac{160 \cdot 170}{170(75) - 160(-75)} = \underline{1.1}$$

Modified - Mohr Theory

Researchers working with cast iron and other brittle materials noticed that in the 4th quadrant, breaking took place outside the boundary.



Modified Mohr

$$N = \frac{S_{ut}|S_{uc}|}{|S_{uc}|\sigma_1 - S_{ut}(\sigma_1 + \sigma_3)}$$

When applying these equations, compute with the equation above and with

$$N = \frac{S_{ut}}{\sigma_1}$$

and use the smaller value.

EXAMPLE

Reworking our previous example with modified Mohr equation

$\sigma_{ut} = 160$ MPa $\quad \sigma_{uc} = 170$ MPa

$\sigma_1 = 75$ MPa $\quad \sigma_3 = -75$ MPa

$$N = \frac{S_{ut}}{\sigma_1} = \frac{160}{75} = \underline{2.13}$$

— Normal stress theory

Also

$$N = \frac{S_{ut}|S_{uc}|}{|S_{uc}|\sigma_1 - S_{ut}(\sigma_1 + \sigma_2)}$$

$$N = \frac{160(170)}{170(75) - 160(75-75)}$$

$N = \underline{2.31}$ The same

Note that modified Mohr is less conservative than the Coulomb-Mohr theory.

DOWLING FACTORS

N. E. Dowling, a researcher who did much of the experimental work on uneven materials came up with a method that is computationally easier because you do not have to look for the quadrant.

FOR UNEAVEN MATERIALS

We have

S_{ut} = ultimate tensile strength

S_{uc} = ultimate compressive strength

compute

$$C_1 = \frac{1}{2}\left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_1 + \sigma_2)\right]$$

$$C_2 = \frac{1}{2}\left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_2 + \sigma_3)\right]$$

$$C_3 = \frac{1}{2}\left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_3 + \sigma_1)\right]$$

Next we compute

$$\tilde{\sigma} = MAX(C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3)$$

Max picks the largest value. If all of the values are negative

$$\tilde{\sigma} = 0 \quad \text{if } MAX < 0$$

The Safety can be computed with

$$N = \frac{S_{ut}}{\tilde{\sigma}}$$

EXAMPLE

We look at the same example again

$\sigma_{ut} = 160 MPa$ $\sigma_{uc} = 170 MPa$

$\sigma_1 = 75 MPa$ $\sigma_3 = -75 MPa$

We will use the Dowling method

$$C_1 = \frac{1}{2}\left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_1 + \sigma_2)\right]$$

$\sigma_2 = 0$ in this case so

$$C_1 = \frac{1}{2}\left[\sigma_1 + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_1)\right]$$

$$C_1 = \frac{1}{2}\left[75 + \frac{2(160) - 170}{-170}(75)\right] = 4.413$$

$$C_2 = \frac{1}{2}\left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-S_{uc}}(\sigma_2 + \sigma_3)\right]$$

$$C_2 = \frac{1}{2}\left[75 + \frac{2(160) - 170}{-170}(-75)\right] = 70.59$$

$$C_3 = \frac{1}{2}\left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_3 + \sigma_1)\right]$$

$$C_3 = \frac{1}{2}\left[150 + \frac{2 \cdot 160 - 170}{-170}(0)\right] = 75$$

$\tilde{\sigma} = MAX(4.413, 70.59, 75, 75, 0, -75)$

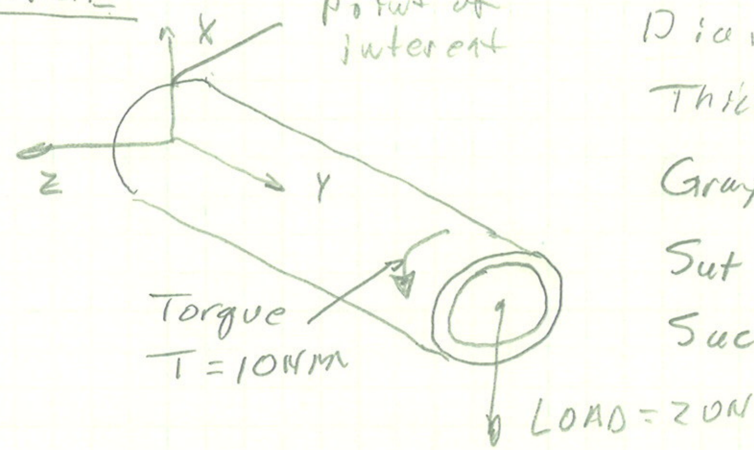
$\tilde{\sigma} = 75$

so our safety factor becomes

$$N = \frac{S_{ut}}{\tilde{\sigma}} = \frac{160}{75} = \underline{\underline{2.13}}$$

this is the same value we got from the modified Mohr's method.

EXAMPLE



Diameter 25 mm

Thickness = 1 mm

Gray Cast Iron

$S_{ut} = 362$ MPA

$S_{uc} = -1130$ MPA

SHEAR STRESS DUE TO TORQUE T

$$\tau_{yz} = T/Q$$

Meters

$$Q = \frac{\pi(d_o^4 - d_i^4)}{32\, r_o} = \frac{\pi(.025^4 - .023^4)}{32(.0125)}$$

$$Q = 8.7 \times 10^{-7}$$

$$\tau_{xz} = 10/8.7\times10^{-7} = 11.5 \text{ MPa}$$

NORMAL STRESS DUE TO LOAD

$$\sigma_x = \frac{Mc}{I}$$ where $C = 12.5\text{mm} = .0125\text{m}$

$M = 20 \times 0.25 = 5$

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi}{64}\left(.025^4 - .023^4\right)$$

$$I = 5.44 \times 10^{-9}$$

$$\sigma_x = \frac{5(.0125)}{5.44\times10^{-9}} = 11.5 \text{ MPa}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \sqrt{\left(\frac{11.5 - 0}{2}\right)^2 + 11.5^2}$$

$$\tau_{max} = 12.85 \text{ MPa}$$

$$\sigma_1 = \frac{\sigma_y + \sigma_z}{2} + \tau_{max} = \frac{11.5}{2} + 12.85 = 18.6 \text{ MPa}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \tau_{max} = \frac{11.5}{2} - 12.85 = -7.1 \text{ MPA}$$

USING DOWLING FACTORS

$$C_1 = \frac{1}{2}\left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_1 + \sigma_2)\right]$$

$$= \frac{1}{2}\left[18.6 + \frac{2(362) - 1130}{-1130}(18.6)\right]$$

$$C_1 = 12.64$$

$$C_2 = \frac{1}{2}\left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_2 + \sigma_3)\right]$$

$$C_2 = \frac{1}{2}\left[7.1 + \frac{2(362) - 1130}{-1130}(-7.1)\right]$$

$$C_2 = 2.29$$

$$C_3 = \frac{1}{2}\left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|}(\sigma_3 + \sigma_1)\right]$$

$$C_3 = \frac{1}{2}\left[|-7.1 - 18.6| + \frac{2(362) - 1130}{-1130}(-7.1 + 18.6)\right]$$

25.7 11.5

$C_3 = 14.85$

Finding The MAX

$$\tilde{\sigma} = \text{MAX} \quad \sigma_1 \quad \sigma_2 \quad \sigma_3 \quad C_1 \quad C_2 \quad C_3$$

18.6	0	−7.5	12.64	2.29	14.85

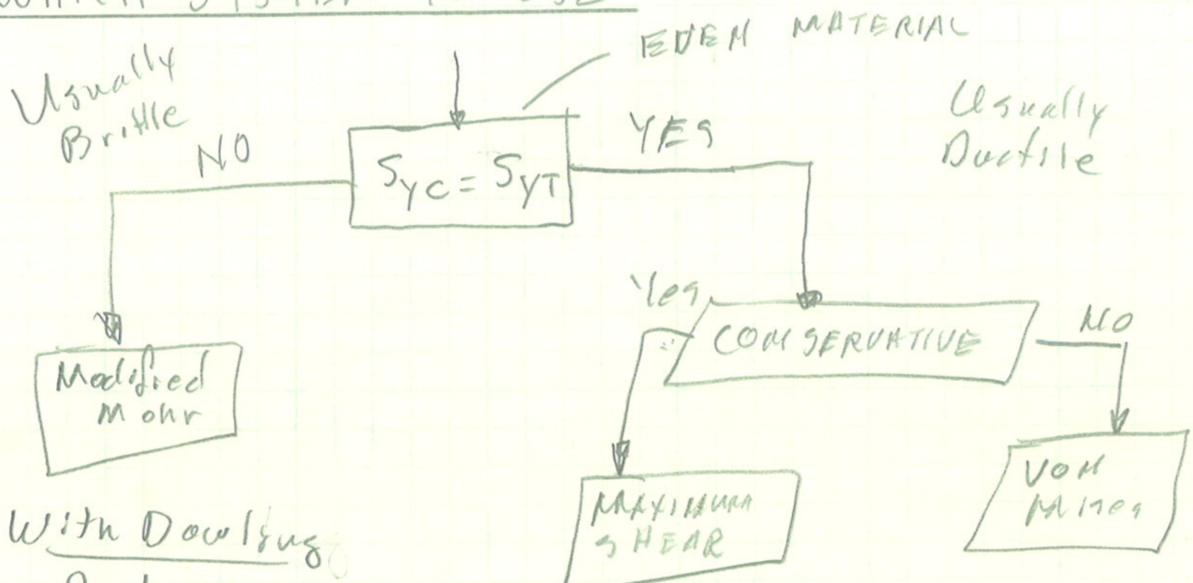
$$\tilde{\sigma}_1 = 18.6$$

SAFETY FACTOR

$$N = \frac{S_{ut}}{\tilde{\sigma}} = \frac{362}{18.6} = \underline{19.46}$$

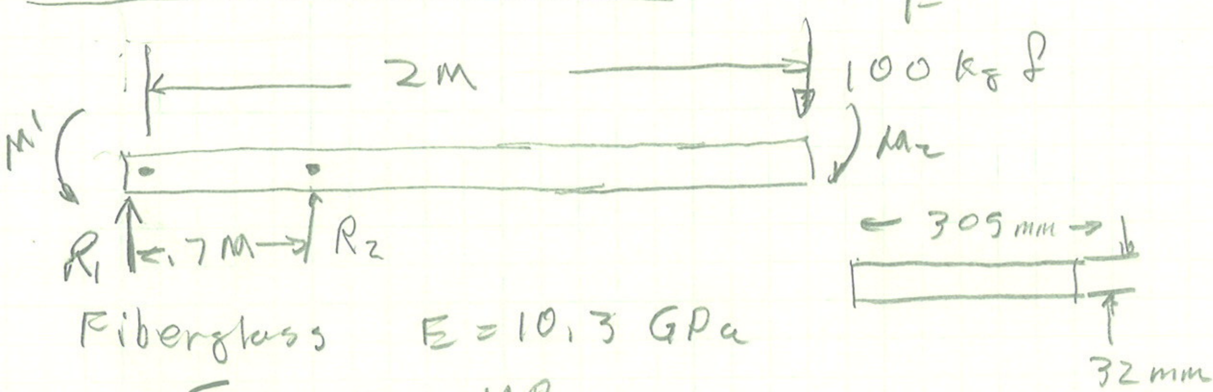
MODIFIED MOHR

WHICH SYSTEM TO USE



With Dowling Factors

5-10 DIVING BOARD



Fiberglass $E = 10.3$ GPa

$S_{ut} = 130$ MPa

$F = -100$ kg $= -981$

The maximum moment occurs at R_2

so $M_{max} = 981 \cdot (2 - 0.7) = 1275$

Distance to center fiber $= 32/2 = .016$ m

Moment of Inertia

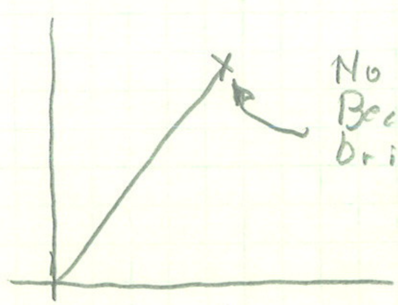
$$I = \frac{wt^3}{12} = \frac{.305 \cdot .032^3}{12} = 8.33 \times 10^{-7}$$

$$\sigma_1 = \sigma_x = \frac{M_{max} \cdot C}{I} = \frac{1275(0.016)}{8.33 \times 10^{-7}}$$

$$\underline{\sigma_1 = \sigma_x = 24.5 \text{ MPa}}$$

$\sigma_2 = 0$ $\sigma_3 = 0$

Not S_y



No Yield Point Because it is brittle

$$N = \frac{S_{ut}}{\sigma_1}$$

$$N = \frac{130}{24.5}$$

$$\underline{N = 5.3}$$

Simplified Case if we have more than just σ_1 then we use the C values.