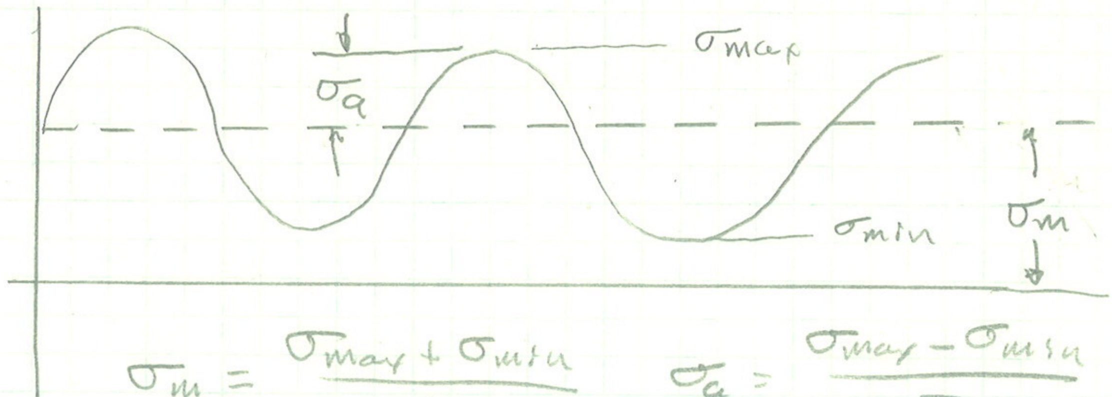
High Cycle Fatigue

If we have simple loading and the stresses are fully reversed, an S-N curve is sufficient for determining the fatigue life of the material.

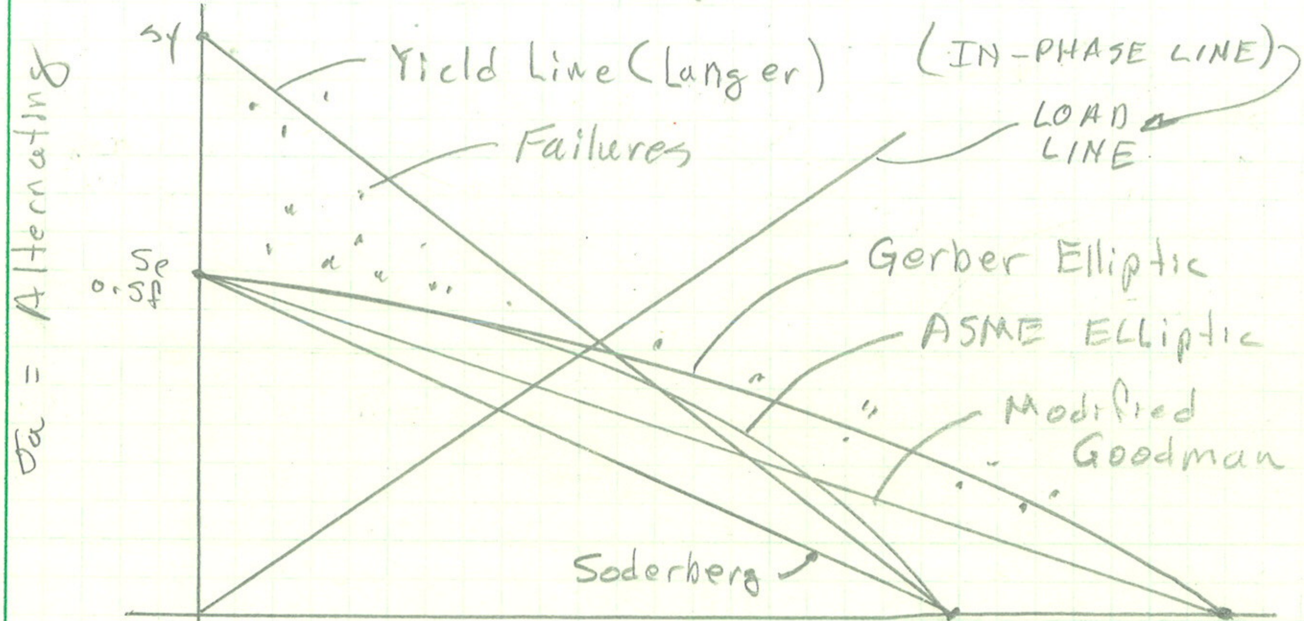
Frequently though, we have a more complex situation and a simple S-N curve will not work.

We talked about fluctuating stresses at the beginning of our discussion

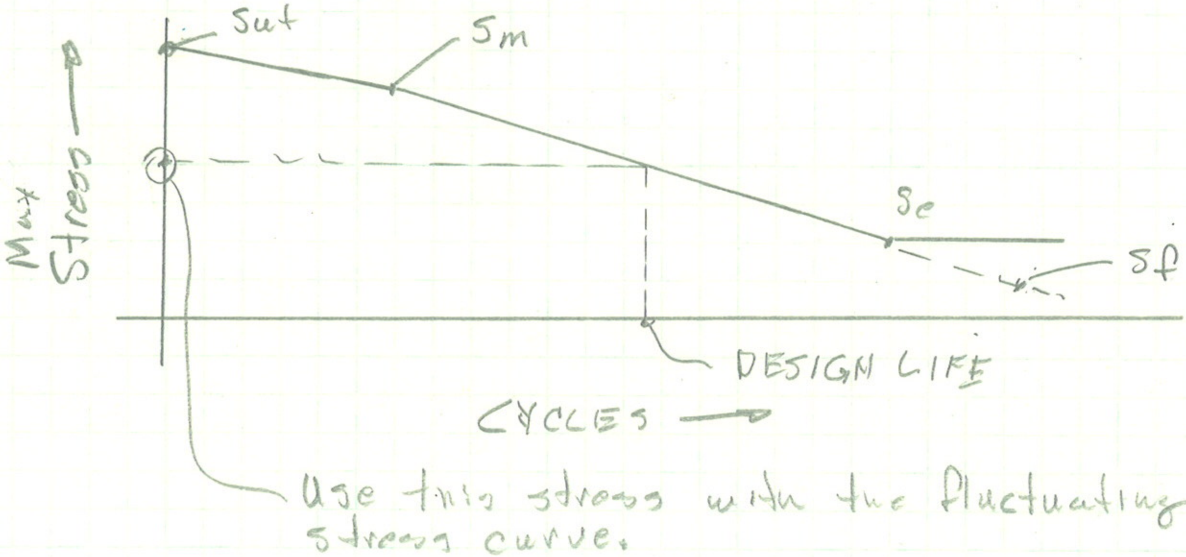


$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Several different investigators have looked at this and developed slightly different methods for handling it



Se and Sf are the strength at the required number of cycles. Se is used for materials with an infinite life knee and Sf for other materials



Usually we use Se or Sf from the Morin corrections.

You can tell from the curves that some of these curves are more conservative than others. Our textbook stresses the Gerber and ASME Elliptic while other authors stress the Modified Goodman.

Much of the work in subsequent chapters in our book will use the modified Goodman curve. It is convenient since it is linear.

SAFETY FACTORS

Langer static yield

$$n = \frac{S_y}{\sigma_a + \sigma_m}$$

Used in test for first cycle failure

Soderberg

$$n_f = \frac{S_e S_y}{S_y \sigma_a + S_e \sigma_m}$$

Not used much. Far too conservative

Modified - Goodman

$$n_f = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e}$$

A little conservative but widely used

Gerber

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$\sigma_m > 0$$

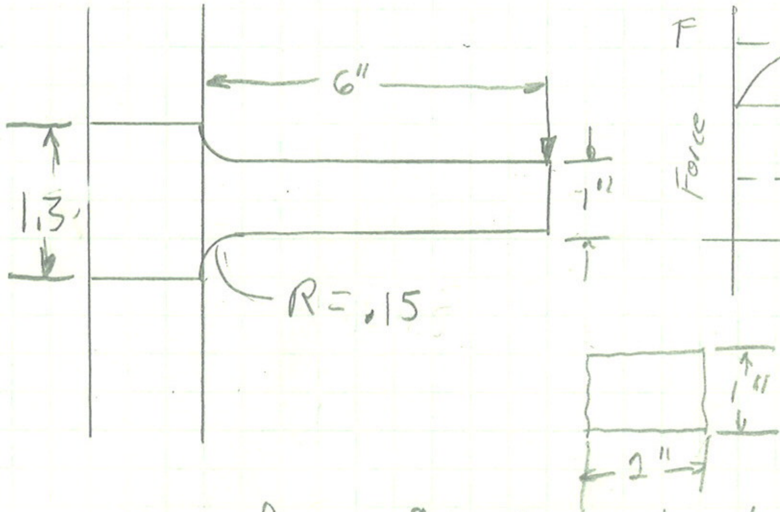
ASME

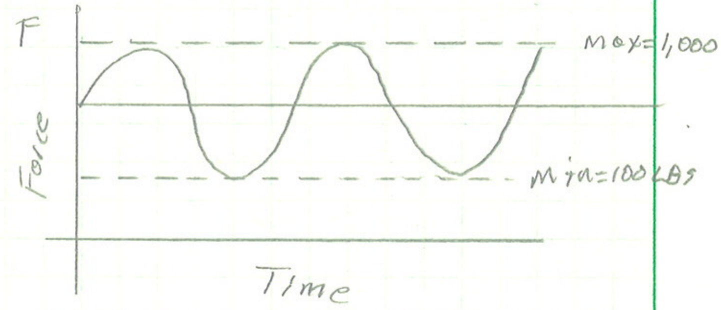
$$n_f = \sqrt{\frac{1}{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2}}$$

We compute each of these and compare with the Langer (Static Failure) safety factor. Choose the smaller. If Langer is smaller, the part will fail due to static stresses. If the other way around, the part will fail due to fatigue.

EXAMPLE

EXAMPLE Cantilever Bracket for Fluctuating Bending





Reliability 99.9%
Machined Finish

Design for 10^9 cycles without Failure

SAE 1040 normalized carbon steel $S_{ut} = 80$ kpsi
$S_y = 60$ kpsi

(1) COMPUTE FORCES

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{1100 + 100}{2} = 600 \text{ lbs}$$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{1100 - 100}{2} = 500 \text{ Lbs}$$

2) COMPUTE MOMENTS

$$M_m = F_m(6") = 600(6) = 3600 \text{ in-lbs}$$

$$M_a = F_a(6") = 500(6) = 3000 \text{ in lbs}$$

3 Compute Stresses

$$I = \frac{bd^3}{12} = \frac{2(1)^3}{12} = .1667 \text{ in}^4$$

$$c = \frac{d}{2} = \frac{1.0}{2} = 0.5 \text{ inches}$$

$$\sigma_{a_{nom}} = \frac{M_a c}{I} = \frac{3000(.5)}{.1667} = 8998 \text{ psi}$$

$$\sigma_{m_{nom}} = \frac{M_m c}{I} = \frac{3600(.5)}{.1667} = 10,798$$

4) Compute the stress concentration factor

$$K_t = A\left(\frac{r}{d}\right)^b \qquad \frac{D}{d} = \frac{1.3}{1} = 1.3$$

From Table

$$A = .95880 \qquad b = -.27269$$

$$K_t = .95880\left(\frac{.15}{1}\right)^{-.27269} = 1.608$$

5) Compute Neuber's constant

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \qquad K_f = 1 + q(K_t - 1)$$

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

$$\sqrt{a} = 0.246 - 3.08(10^{-3})(80) + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

$$\sqrt{a} = .0826$$

$$q = \frac{1}{1 + \frac{.0826}{\sqrt{.15}}} = .824$$

$$K_f = 1 + 0.824(1.608 - 1) = 1.50$$ Reduced from K_t

6) Check for yielding

$$\sigma_{max} = K_f(\sigma_{a\,nom} + \sigma_{m\,nom})$$

$$\sigma_{max} = 1.50(8998 + 10{,}798) = 29{,}694 \text{ psi}$$

$$\sigma_{max} < S_y \quad \therefore \quad 29{,}694 < 60{,}000$$

so

$$K_{fm} = K_f = 1.5$$

7) Find the mean and alternating notch stresses

$$\sigma_a = K_f \sigma_{a\,nom} = 1.5(8998) = 13{,}497 \cong 13.5\text{ kpsi}$$

$$\sigma_m = K_{fm} \sigma_{m\,nom} = 1.5(10{,}798) = 16{,}197 \cong 16.2\text{ kpsi}$$

8) The uncorrected endurance limit

$$S_e' = 0.5 S_{ut} = 0.5(80{,}000) = 40{,}000\text{ psi}$$

9) The size factor

The size is rectangular so we must compute an equivalent diameter. From table page 290

$$d_e = 0.808\sqrt{hb} = 0.808\sqrt{2} = 1.143$$

$$C_{size} = 0.879(d_e)^{-.107} = .879(1.143)^{-.107} = .8665$$

10) Compute surface factor

$$C_{SURF} = a\, S_{ut}^{b}$$

From table page 288

$a = 2.70 \qquad b = -.265$

$$C_{SURF} = 2.70(80)^{-.268} = .834$$

11) Other factors

$C_{LOAD} = 1.0$ Bending

$C_{relib} = .753$ Table page 293

$C_{TEMP} = 1.0$ Room Temperature

12) Corrected Endurance Limit

$$S_e = C_{SURF}\, C_{size}\, C_{LOAD}\, C_{TEMP}\, C_{RELIB}\, S_e'$$

TOPS 35500

$$S_e = (.834)(.8665)(1)(1)(.753)(40{,}000) = 21{,}767 \text{ psi}$$

13) Static Yield safety factor

$$n = \frac{S_y}{\sigma_a + \sigma_m} = \frac{60}{13.5 + 16.2} = 2.02$$

14) Modified Goodman safety factor

$$n_f = \frac{S_e S_{ut}}{\sigma_a S_{ut} + \sigma_m S_e} = \frac{(21.8)(80)}{13.5(80) + 16.2(21.8)}$$

$$n_f = 1.22$$

15) Gerber

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$n_f = \frac{1}{2}\left(\frac{80}{16.2}\right)^2 \frac{13.5}{21.8}\left[-1 + \sqrt{1 + \left(\frac{2(16.2)(21.8)}{80(13.5)}\right)^2}\right]$$

$$n_f = 1.47$$

16) ASME

$$n_f = \sqrt{\frac{1}{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2}} = \sqrt{\frac{1}{\left(\frac{13.5}{21.8}\right)^2 + \left(\frac{16.2}{60}\right)^2}}$$

$$n_f = 1.48$$

Summation:
1) The bar will fail due to fatigue
2) Modified Goodman is the most conservative
3) Gerber & ASME are similar
4) All are too small for comfort.