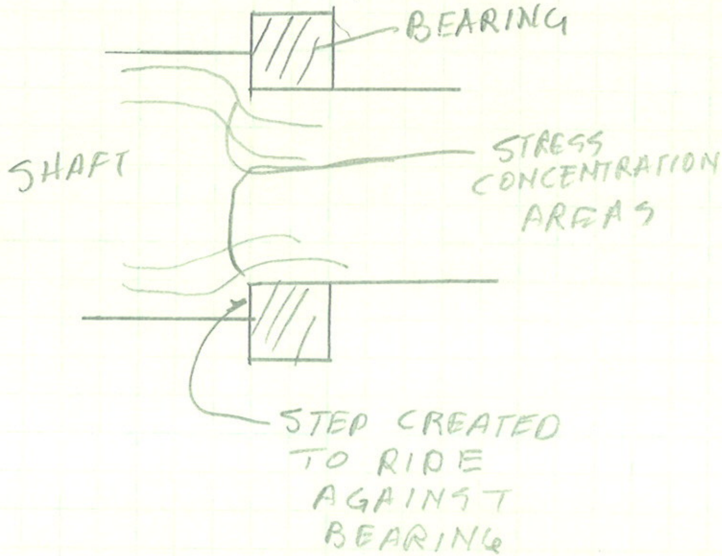
FATIGUE FAILURE

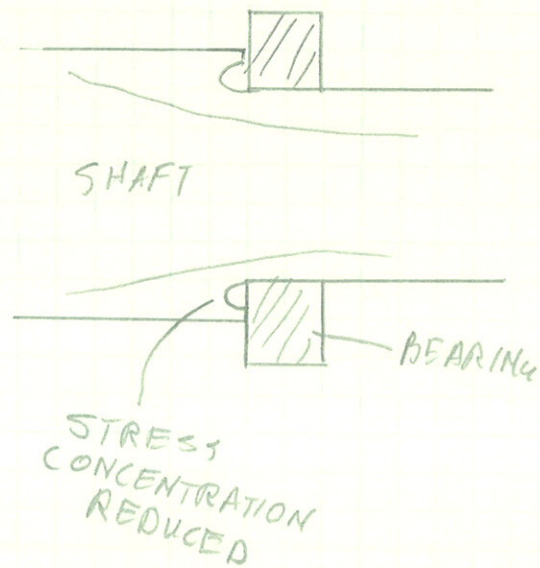
1) Most machinery failures are due to fatigue failures rather than static failures.

2) Fatigue failures usually start at a notch or other stress concentration.

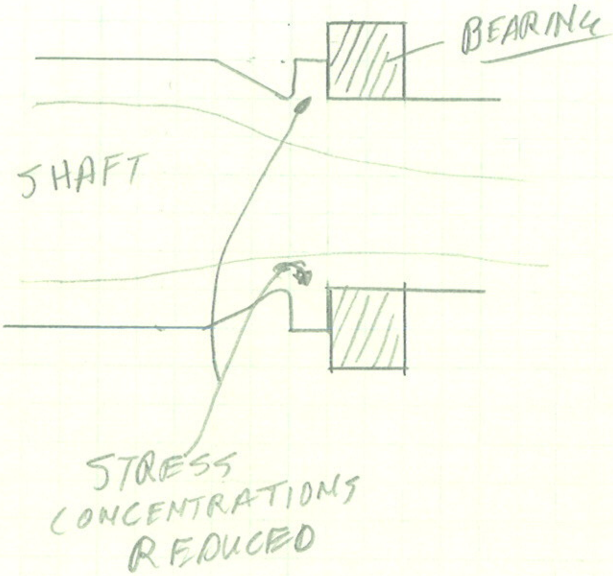
It is critical that dynamically loaded parts be designed to minimize stress concentrations.

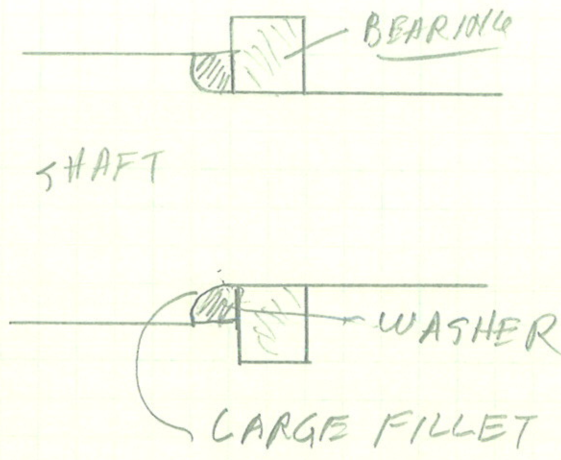


POOR DESIGN



BETTER DESIGN





METHODS TO REDUCE STRESS CONCENTRATIONS

3) Crack growth and fatigue failures are due to tensile stresses.

FATIGUE MODELS

There are three

1) SN stress Life
2) εN strain Life
3) LEFM Linear elastic fracture mechanism

Two different fatigue regimes

1) LCF Low cycle fatigue $N \leq 1000$ cycles
2) HCF High cycle fatigue $N \geq 1000$ cycles

STRESS LIFE

This is a stress based model which is used to compute a maximum stress which is low enough that failure does not occur.

The design goal is that stresses and strains everywhere remain in the elastic region and no local yielding occurs to initiate a crack.

This method is most applicable for designing parts with indefinite life.

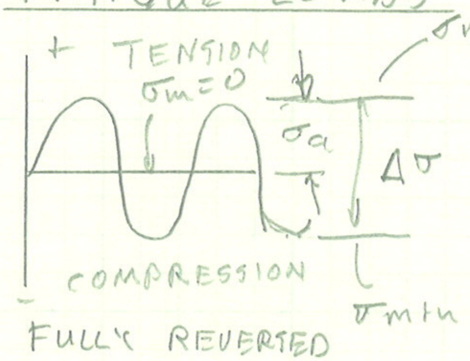
STRAIN LIFE

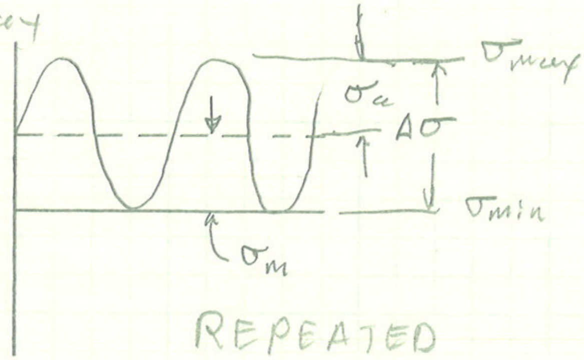
This method is most often applied to LCF, finite life problems where cyclic stresses are high enough to cause local yielding.

LEFM APPROACH

Usually applied where stresses are high enough to cause cracks. Most usefule in predicting the remaining life of parts in service.

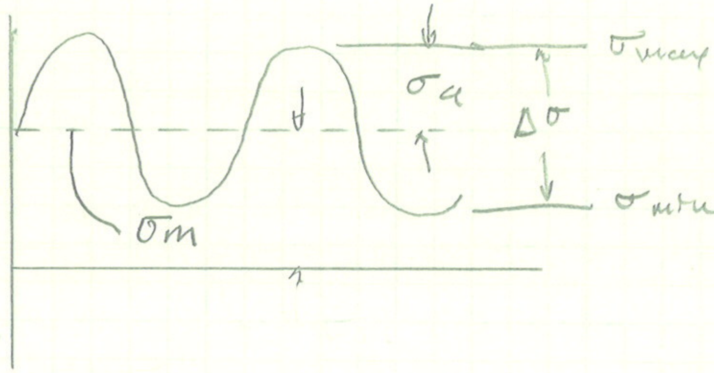
FATIGUE LOADS





$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

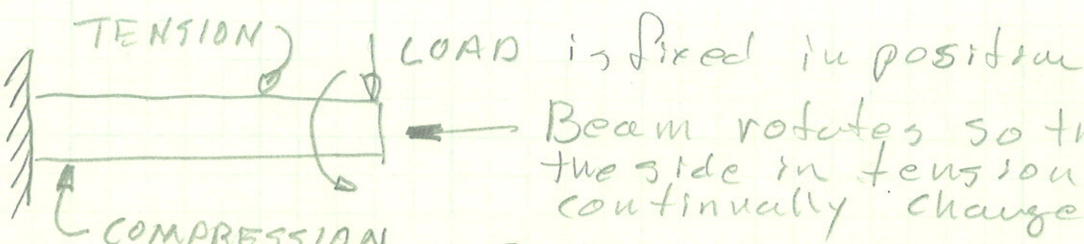
$$A = \frac{\sigma_a}{\sigma_m}$$



FLUCTUATING

The exact wave shape does not have much influence on the fatigue life so a sinusoidal wave shape is usually used

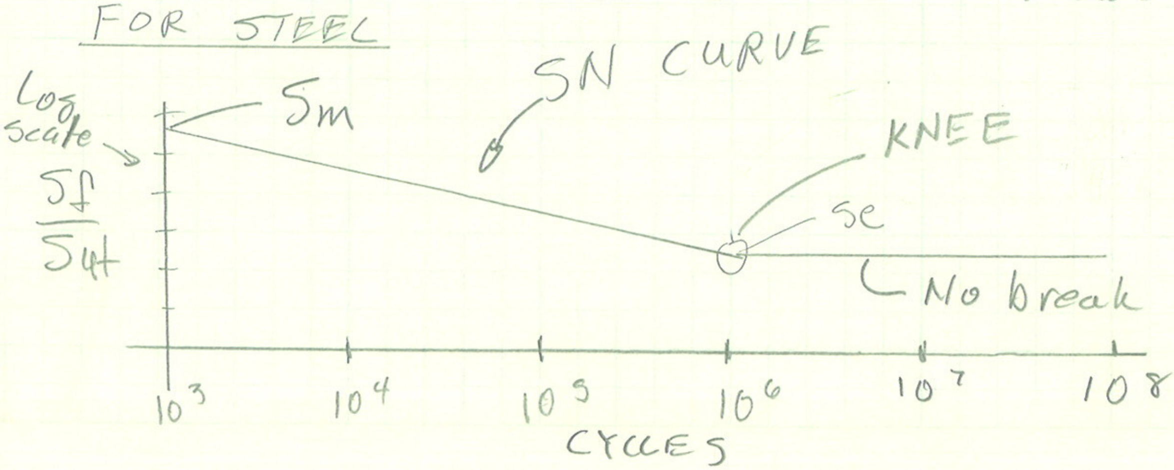
ROTATING BEAM TEST



Beam rotates so that the side in tension continually changes

Bending $S_m = 0.9\, S_{ut}$

FOR STEEL



FOR MILD STEELS

$$S'_{UT} \leq 1{,}400\,MPa \rightarrow S_e \cong 0.5\,S_{UT}$$

FOR HIGH STRENGTH STEELS

$$S'_{UT} > 1{,}400\,MPa \quad S_e \cong 700\,MPa$$

ALUMINUM

Aluminum does not have an endurance limit like steel and will always fail.

For LOWER STRENGTH ALUMINUMS

$$S'_f \text{ @ } 5 \times 10^8 \text{ cycles} \cong 0.4\,S_{ut}$$

where $S_{ut} < 330\,MPa$

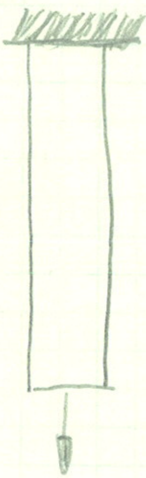
SHOW CURVE

For STRONGER Aluminum

$$S'_f \text{ @ } 5 \times 10^8 \text{ CYCLES} \cong 130\,MPa$$

Where $S_{ut} > 330\,MPa$

AXIAL FATIGUE TEST



Repeated Axial Loading

TEST SHOW S_f for tensile stress in 10%-30% Less than the rotating beam

$$\underset{\text{Rotating beam}}{0.7\,S'_f} \leq S'_{f\,tensile} \leq 0.9\,S'_{f\,\text{rotating beam}}$$

$$S_m = 0.75\,S_{ut}$$

ESTIMATING FATIGUE FAILURE CRITERIA

Designers need information about fatigue failure in the materials and devices they design. There are 3 approaches we can use

1) BUILD AND TEST — Done with aircraft and some other high cost items. It is very expensive and time consuming
2) Test the materials that will be used in building the product
3) Test materials that are representative of material to be used
4) Estimate the fatigue resistance of the material.

We will look at (4). It is the least reliable but frequently it is the best we can do. We can use the table below for a reasonable estimate

STEEL	$S_e' = 0.5 S_{ut}$	$S_{ut} < 1{,}400$ MPa
	$S_e' = 700$ MPa	$S_{ut} > 1{,}400$ MPa
IRON	$S_e' = 0.4 S_{ut}$	$S_{ut} < 400$ MPa
	$S_e' = 160$ MPa	$S_{ut} > 400$ MPa
ALUMINUM	$S_{f @ 5E8}' = 0.4 S_{ut}$	$S_{ut} < 330$ MPa
	$S_{f @ 5E8}' = 130$ MPa	$S_{ut} > 330$ MPa
COPPER ALLOYS	$S_{f @ 5E8}' = 0.4 S_{ut}$	$S_{ut} < 280$ MPa
	$S_{f @ 5E8}' = 100$ MPa	$S_{ut} > 280$ MPa

S_e represents the endurance limit at the knee somewhere around 10^6 cycles

S_f represents the fatigue strength @ 5×10^8 cycles.

CORRECTIONS

Several correction factors must be applied to those values,

$$S_e = C_{LOAD}\, C_{size}\, C_{SURF}\, C_{TEMP}\, C_{RELIB}\, S_e'$$

$$S_f = C_{LOAD}\, C_{size}\, C_{SURF}\, C_{TEMP}\, C_{RELIB}\, S_f'$$

LOADING C_{LOAD} or K_c — SHIGLEY

$$K_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Torsion} \end{cases}$$

If we use von Mises stresses it combines torsion with the other stresses – Remember

$$S_{sy} = 0.577\, S_y$$

from distorsion theory so the two are very similar.

SIZE BENDING K_b — Shigley

ROTATING (Cylinderical Shape)

IMPERIAL

$C_{size} = 0.879\, d^{-.107}$ $\qquad 0.11 \le d \le 2$ inches

$C_{size} = 0.91\, d^{-.157}$ $\qquad 2 \le d \le 10$ inches

METRIC

$C_{size} = 1.24\, d^{-.107}$ $\qquad 2.79 \le d \le 51$ mm

$C_{size} = 1.51\, d^{-.157}$ $\qquad 51 < d \le 254$ mm

Above 10" or 254 mm use

$$C_{size} = .6$$

NON - ROTATING

For non-rotating parts we use what is called A_{95} or the area beyond the 95% point

For a cylinder

$$A_{95} = \frac{\pi}{4}\left(d^2 - (0.95d)^2\right) = .0766d^2$$

solving for d

$$d_{equiv} = \sqrt{\frac{A_{95}}{.0766}}$$

Shigley presents A_{95} for various shapes. The equivalent d can be computed with the above formula.

Once d_{equiv} is computed, substitute it into the previous equations for C_{size} to get the actual correction factor.

FOR AXIAL LOADS

$C_{size} = 1$

Since the entire cross section is under the same load.

SURFACE EFFECTS

CAST IRON $C_{SURF} = 1.0$

OTHER MATERIALS

$$C_{SURF} = A\left(S_{ut}\right)^{b}$$ IN MPa

IF

$C_{SURF} > 1.0$ then $C_{SURF} = 1.0$

FINISH	A (MPa)	b
Ground	1.58	−0.085
COLD-ROLLED	4.51	−0.265
HOT ROLLED	57.7	−0.718
FORGED	272	−0.995

TEMPERATURE

For Steel Only

$T \leq 450°C$ $C_{TEMP} = 1$

$450°C \leq T \leq 550°C$ $C_{TEMP} = 1 - 0.0058(T - 450)$

Do NOT use above 550°C

RELIABILITY

Choose from a table

Reliability	C_{RELIAB}
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620

TOPS. 35500

ESTIMATING S-N DIAGRAMS

FOR 10^3 CYCLES

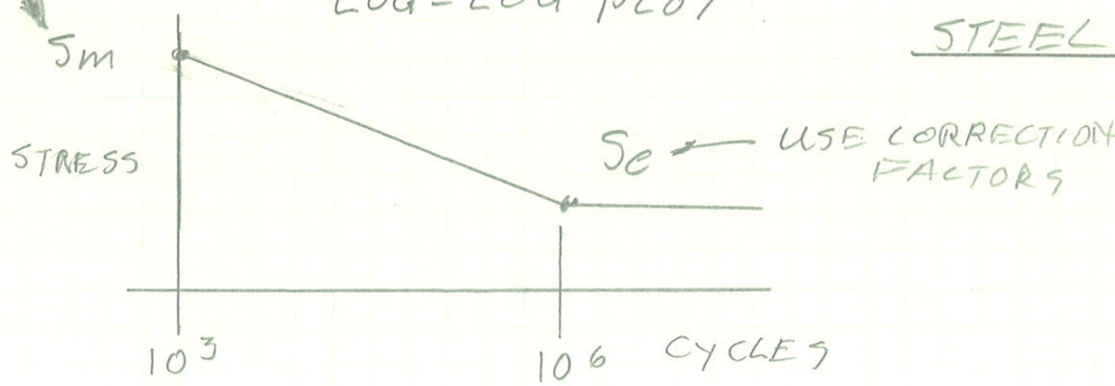
Bending $S_m = 0.9 S_{ut}$

Axial Loading $S_m = 0.75 S_{ut}$

The previously discussed correction factors ARE NOT applied to S_m.

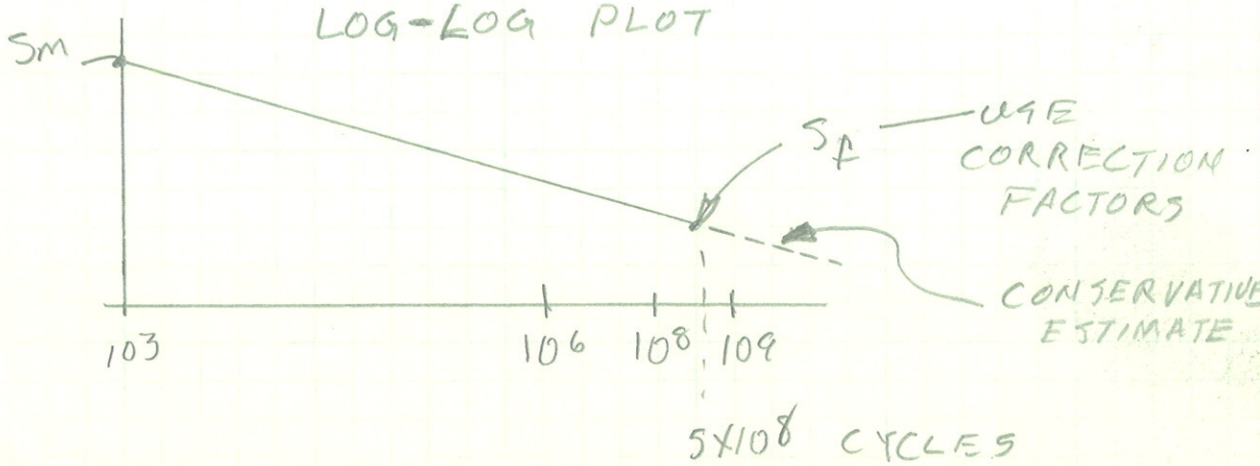
LOG-LOG PLOT

STEEL



OTHER MATERIALS

LOG-LOG PLOT



EXAMPLE

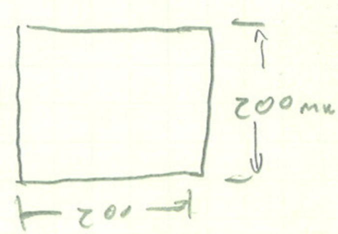
Draw the SN DIAGRAM FOR STEEL

$S_{ut} = 700\,MPa$

Cold rolled

Temp = 500°C

99% Reliability



FULL REVERSE BENDING

① $S_m = 0.9\ S_{ut} = 0.9(700) = 630$ MPa

② $S_{e'} = 0.5\ S_{ut} = 0.5(700) = 350$ MPa

③ $\underline{C_{LOAD}} = 1.0$ Bending

④ FOR SQUARE SHAPE

$A_{95} = 0.05bh = 0.05(200)(200) = 2000$

Diameter equivalent to $d_{equiv} = \sqrt{\dfrac{A_{95}}{0.0766}} = \sqrt{\dfrac{2000}{.0766}} = 161.6$

See diagrams pg 360

$\underline{C_{SIZE}} = 1.189(161.6)^{-0.097} = .726$

⑤ Temperature

From table 6-4 Page 291

$C_{TEMP} = \dfrac{S_T}{S_{RT}} = .768$

⑥ SURFACE FACTOR

$C_{SURF} = A\ (S_{ut})^b$

COLD ROLLED $A = 4.51$ $b = -0.265$

$C_{SURF} = A(S_{ut})^b = 4.51(700)^{-0.265}$

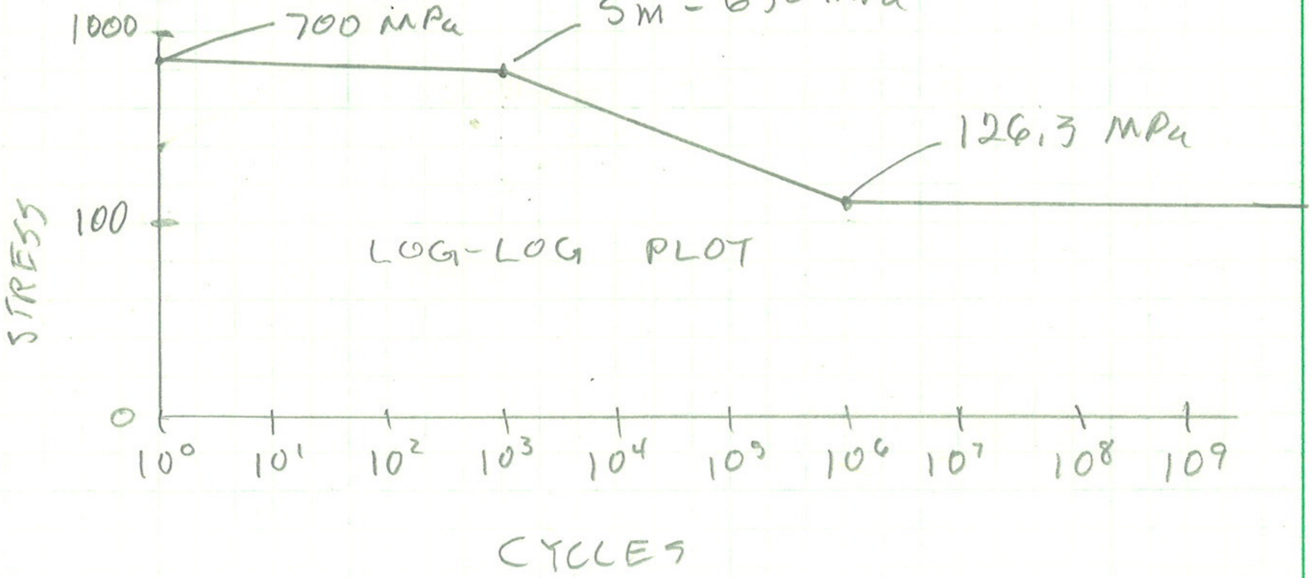
$\underline{C_{SURF}} = 0.795$

⑦ $C_{RELIB} = 0.814$

$S_e = C_{LOAD}\, C_{size}\, C_{SURF}\, C_{TEMP}\, C_{RELIP}\, S_e'$

$S_e = (1.0)(.726)(.795)(.768)(.814)\,350$

$S_e = 126.3$



EXAMPLE

If the part above in cyclically stressed at 200MPa what will be the expected life in cycles

$S(N) = aN^b$ for linear Log-Log Plots

$$b = \frac{1}{z} \log\left(\frac{S_m}{S_e}\right)$$

where

$$z = \log(N_1) - \log(N_2)$$ use $\log_{10}$

$N_1 = 1000$ cycles

$N_2 = 10^6$ cycles

$z = \log(1000) - \log(10^6) = 3 - 6 = -3$

$$b = -\frac{1}{3} \log\left(\frac{630}{126.3}\right) = -.23265$$

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At 5m $S(N) = 630$ $N = 1000$

$$S(N) = aN^b$$

solving for a

$$a = \frac{S(N)}{N^b} = \frac{630}{1000^{-.23265}} = 3142.61$$

$$a = 3.1821\ E-4$$

Now solving for N

$$S(N) = aN^b$$

$$Log(S(N)) = Log(a) + b\,Log(N)$$

$$Log(200) = Log(3142.61) - .23265\,Log(N)$$

$$Log(N) = \frac{LOG(200) - LOG(3142.61)}{-.23265}$$

$$LOG(N) = 5.142$$

$N = 10^{5.142} = \underline{138,675}$ cycles

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Determining the S-N Diagram for Non-Ferrous Materials

We have a 6061-T6 has been tested.

$S_{ut} = 310$ MPa Round Bar dia = 38mm

Loading is fully reversed torsion (Room Temperature)

A) Draw the estimated S-N Curve

B) What is the corrected fatigue strength at 2×10^7 cycles?

Use reliability at 99%

The uncorrected Fatigue strength is taken at 5×10^8 cycles.

1) We do not have fatigue strength so we estimate it.

$$S_f' \cong 0.4 S_{ut}$$

$$\underline{S_f' = (0.4)(310) = 124 \text{ MPa}}$$

This value is at $N = 5 \times 10^8$ cycles

2) The loading is in pure torsion. So the load factor is

$$\underline{C_{LOAD} = 1.0}$$

Because the torsional stress will be converted to von Mises stress.

3) We can compute the size factor with the formula

$$C_{size} = 1.189\, d^{-0.097}$$

$$C_{size} = (1.189)(38)^{-0.097} = \underline{.835}$$

This formula was developed for steel and may not be as accurate for aluminum.

(4) The surface factor is found from eqn 6.7 page 361 using data in Table 6-3.

From the table, $A = 272$ $b = -0.995$

$$C_{surf} = A\, S_{ut}^{\,b} = 272(310)^{-0.995} = .903$$

Expressed in MPa

5) $C_{temp} = 1.0$ because this is room temperature

6) Reliability

$C_{reliab} = 0.814$ — From Table

* (7) The corrected fatigue strength at $N = 5 \times 10^8$ cycles is

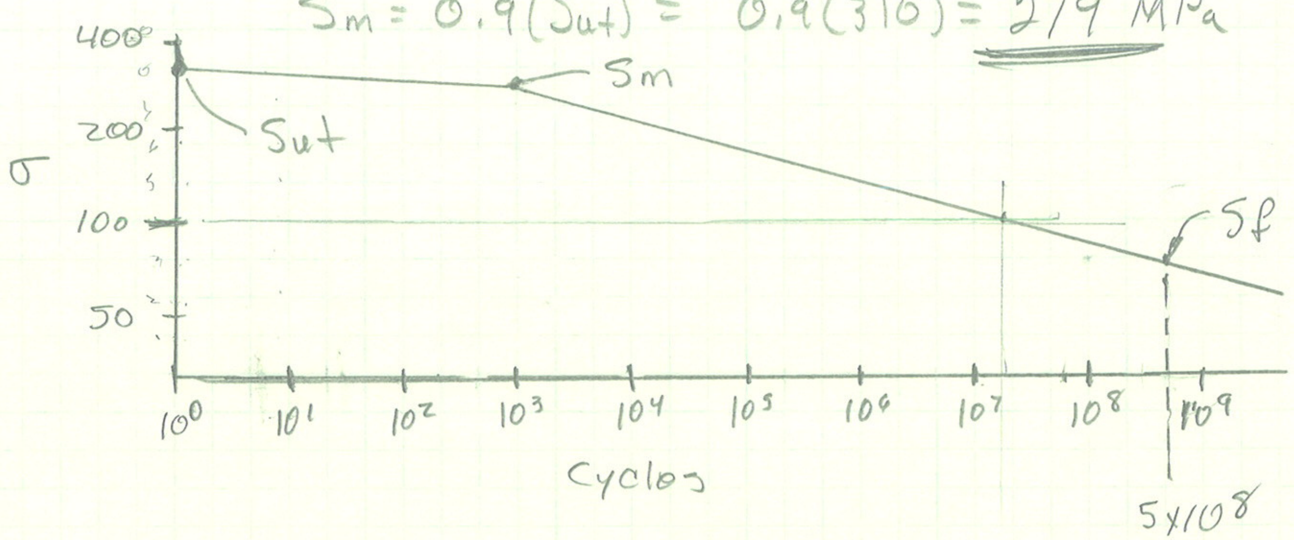
$$S_f = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_f'$$

$$S_f = (1.0)(.835)(.903)(1.0)(.814)(124)$$

$$S_f = 76.1 \text{ MPa}$$

(8) We can compute Sm for bending stress

$$S_m = 0.9(S_{ut}) = 0.9(310) = 279 \text{ MPa}$$



Now we compute the fatigue strength at 2×10^7 cycles

$$S(N) = aN^b$$

We can solve this with the equations

$$b = \frac{1}{z} \log\left(\frac{S_m}{S_f}\right)$$

$$z = \log(N_1) - \log(N_2)$$

at S_m at S_f USE $\log_{10}$

$$z = \log(10^3) - \log(5 \times 10^8)$$

$$z = -5.70$$

Solving for b

$$b = \frac{1}{z} \log\left(\frac{S_m}{S_f}\right) = \frac{1}{-5.70} \log\left(\frac{279}{76.1}\right)$$

$$b = -.09898$$

at $S_m = 279$ $N = 1000$

$$S_m = 279 = a(1000)^{-.1}$$

$$a = \frac{279}{1000^{-.09898}} = 552.77$$

Check at $S_f = 76.1$

$$S(N) = aN^b = 552.77(5 \times 10^8)^{-.09898} = 76.1$$

IT CHECKS

The fatigue strength at 2×10^7 cycles can be computed

$$S(N) = a N^b$$

$$S(N) = 552.77 (2 \times 10^7)^{-.09898}$$

$$S(N) = 104.7 \text{ MPa}$$