Shaft Fatigue Problem

The shaft shown on a following page is machined from ANSI 1040 CD steel. It has an ultimate tensile strength, S_{ut} of 400 MPa and the yield strength, S_y is 250 MPa. A gear and a sheave are mounted on the shaft and an average of 2000 watts of power is transferred from the gear to the sheave. The gear is powered by a piston engine and the power variance is 50% of the average. The shaft changes diameters several times to hold the placement of the shaft relative to the bearings, gear, and sheave. The diameter must change by a minimum of 2 mm at each step to effectively hold these positions.

Write a MATLAB script that computes the diameters of the shaft at d0, d1, d2, and d3 as needed for infinite life and a safety factor no less than 2.5 when computed with the Modified Goodman method. For reference also report the final safety factor as computed using the Gerber and ASME methods. Bearings, gears, and sheaves come in limited sizes and the shaft diameters must coincide with these diameters. The usable diameters are shown in the table below.

Allowable Shaft Diameters (mm)					
10	12	15	17	20	25
30	35	40	45	50	55

The fillet at each step is 0.5 mm. Compute the stress concentrations caused by these steps and check for local yielding.

Both bearings must have the same shaft diameter.

The sheave transfers the power from the shaft to a belt. The belt is under tension on both sides of the sheave as shown in the diagram. The tension on the drive side F1 five times the tension on the slack side F2.

The force turning the gear is applied 20° from tangential as shown in the diagram. This produces both a tangential and normal force on the gear.

S_{ut} = 400 MPa	S_y = 250 MPa	Reliability = 99%
Temperature = Ambient	Fillet = 0.5 mm	Finish = Cold Drawn, machined
Shaft Speed = 1000 RPM	Safety Factor minimum = 2.5	Power Average = 2000 watts
Power Variance = ±50%		

Suggested Procedure

1. Define problem parameters including a table of allowable shaft sizes.
2. Compute the midrange and alternating torques.
3. Compute the midrange and alternating forces on the sheave.
4. Compute the midrange and alternating normal and tangential forces on the gear.
5. Compute the midrange and alternating moments on the shaft. Combine the horizontal and vertical moments vectorially at each location to fine the maximum moment.

6. Compute the midrange and alternating bending stresses at each location along the shaft.
7. Compute the midrange and alternating shears due to the power being transmitted along the shaft.
8. Compute the endurance stress at each location.
9. Compute the static stress concentration factors for both bending and shear.
10. Reduce the bending and shear stress concentration factors due to fatigue.
11. Combine the alternating and midrange bending and shear stresses along the shaft. The von Mises formulation is used to combine these.
12. Compute Langer and modified Goodman safety factors at each location along the shaft. If the safety factor is too low, increase the size of the shaft and repeat the process by going back to step 8. Continue looping until no size change is necessary. Note: an increase in size at one location may cause other areas of the shaft to increase in size to maintain the 2 mm minimum difference in shaft diameter at adjacent locations along the shaft.
13. Compute the Gerber and ASME safety factors at each location along the shaft.
14. Print the results.

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Shaft Design

Ultimate tensile stress =    400.0 MPa
Yield stress =    250.0 MPa
Fillet Radius =    0.500 mm
Minimum Safety Factor =     2.50
Rotational velocity =  1000.00
Average Power Transmitted =  2000.00
Power variance =    0.500
Gear Diameter =   150.00
Sheave Diameter =   125.00
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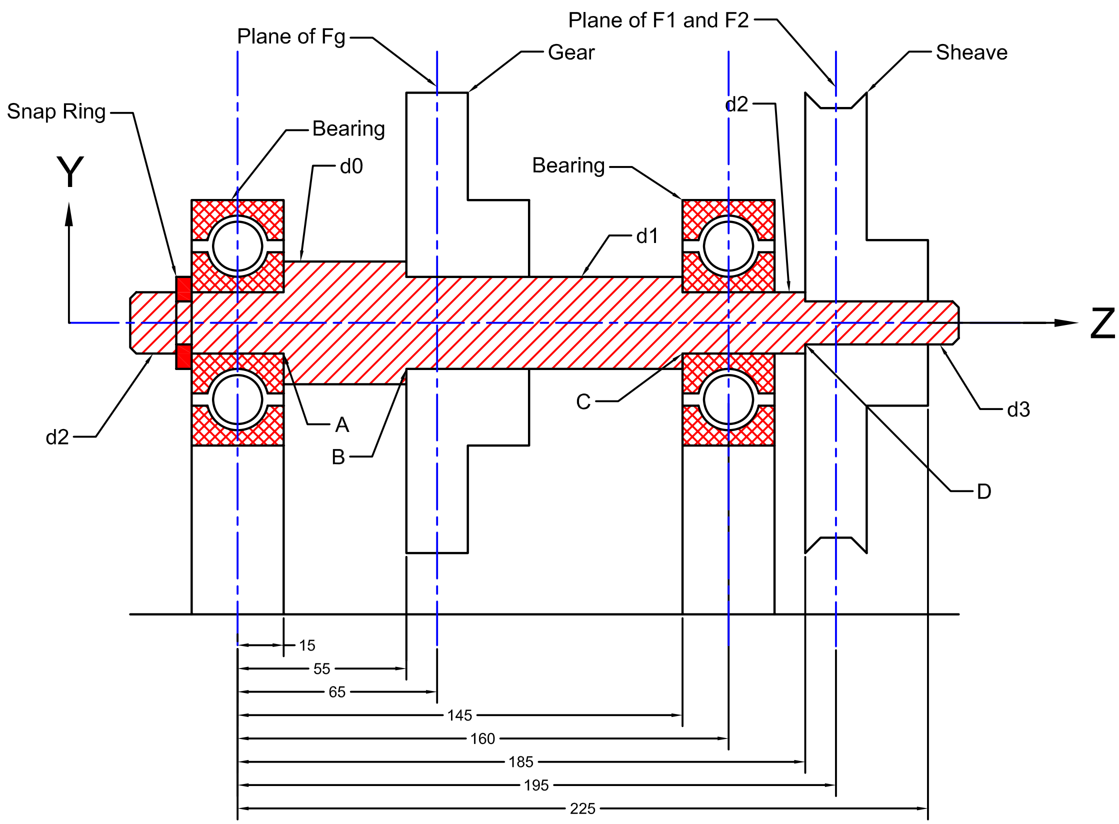
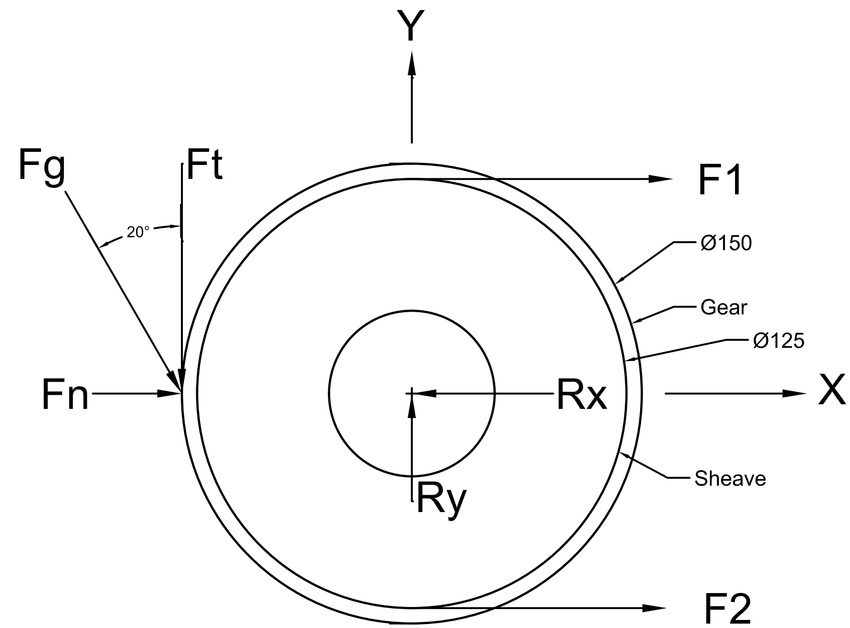
	A	B	C	D
Mean Moments (N-mm)	1983	7403	13949	4584
Alt Moments (N-mm)	992	3702	6974	2292
Alt Stress (MPa)	2.1	4.7	14.5	6.9
Mean Stress (MPa	4.1	9.4	28.9	13.8
Alt Shear (MPa)	0.0	0.0	9.9	14.4
Mean Shear (MPa)	0.0	0.0	19.8	28.8
Se (MPa)	137	135	137	139
Kt	2.165	2.285	2.118	2.102
Kts	1.884	1.476	1.703	1.521
Kf	1.645	1.712	1.620	1.610
Kfs	1.552	1.297	1.438	1.325
VM alt (MPa)	3.4	8.1	29.0	27.3
VM mean (MPa)	6.8	16.1	68.5	69.8
Langer N	24.637	10.328	2.563	2.574
Goodman N	24.081	10.095	2.614	2.678
Gerber N	30.099	12.489	3.259	3.352
ASME N	33.484	13.872	3.677	3.808
Shaft Dia (mm)	17.0	20.0	17.0	15.0



Snap Ring
Y
Bearing
d0
Plane of Fg
Gear
Plane of F1 and F2
Sheave
d2
Bearing
d1
Z
A
B
C
D
d2
d3
15
55
65
145
160
185
195
225



Fg
Ft
20°
Y
F1
Ø150
Gear
Ø125
Fn
Rx
X
Ry
Sheave
F2