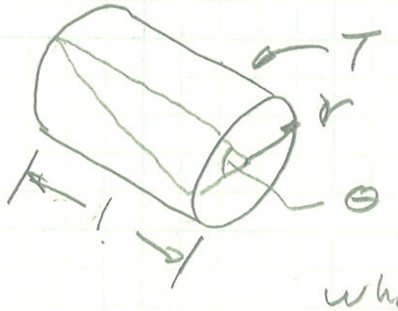
TORSION CIRCULAR SECTIONS



The shear at any point in the cylinder can be computed with.

$$\tau = \frac{T\rho}{J}$$

where T = the torque

ρ = the distance from the center

J = the polar area moment of inertia

The maximum shear

$$\tau_{max} = \frac{Tr}{J}$$ RADIUS of cylinder

The angular deflection is

$$\theta = \frac{Tl}{JG}$$

$J = \frac{\pi d^4}{32}$ solid

$J = \frac{\pi (d_o^4 - d_i^4)}{32}$ Hollow

where T = Torque

l = Length of the cylinder

J = polar area moment of inertia

G = The shear modulus

$$G = \frac{E}{2(1+\nu)}$$

E — Young's Modulus

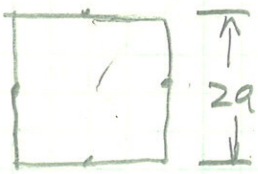
ν — poisson's ratio

NON-CIRCULAR SECTIONS

$$\tau_{max} = \frac{T}{Q}$$ determined by geometry

$$\theta = \frac{Tl}{KG}$$ determined by geometry

SOLID SQUARE



$$K = 2.25\,a^4 \qquad Q = \frac{a^3}{0.6}$$

HOLLOW SQUARE

$$K = \frac{2t^2(a-t)^4}{2at - 2t^2}$$

$$Q = 2t(a-t)^2$$

EXAMPLE

Given a square hollow tube

Steel $a = 25$ mm $\quad t = 1$ mm $\quad G = 80.8$ GPa

Length = 1 meter $\quad T = 10$ N-m

The tube sizes are expressed in mm but we would like them in meters

$a = 25mm = .025m$

$t = 1mm = .001m$

$$K = \frac{2t^2(a-t)^4}{2at-2t^2} = \frac{2(.001)(.025-.001)^4}{2(.025)(.001)-2\cdot .001^2}$$

$K = 1.382E-8 \ m^4$

COMPUTING THE AMOUNT OF TWIST

$$\theta = \frac{TL}{GK} = \frac{10(1)}{(8.08E10)(1.382E-8)} = 0.00895 \text{ rad}$$

CONVERTING TO DEGREES

$\theta = 0.51°$

COMPUTING the SHEAR

$Q = 2t(a-t)^2 = 2(.001)(.025-.001)^2$

$Q = 1.152E-6 \ m^3$

$$\tau = \frac{T}{Q} = \frac{10}{1.152E-6} = 8.7 MPa$$

or 1259 psi

TRANSVERSE SHEAR

In a previous section, we looked at bending moments in beams. Loads on beams produce moments and shear forces. If the beam is long the stresses caused by the moment $\frac{Mc}{I}$ dominate and we can exclude the shear forces in our calculations of the principal stresses.

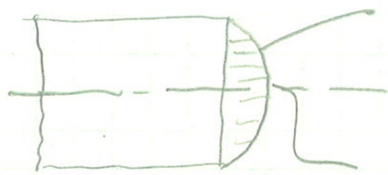
- If the beam is relatively short
  Length < 10 · Height

Then the shear should be considered

<u>SHEAR PROFILE</u>

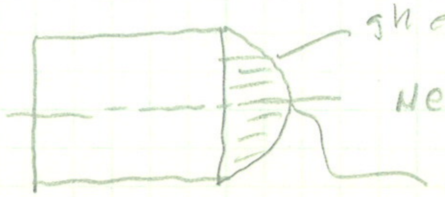
The shear profile is not constant throughout the beam. It varies, usually reaching maximum somewhere the neutral axis. The actual value depends upon the shape of the beam.

Rectangular beam



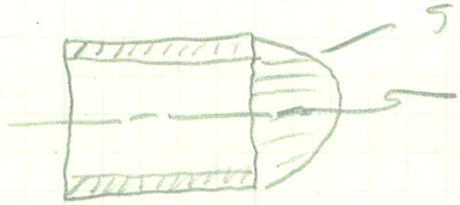
$\tau_{max} = \frac{3}{2}\frac{V}{A}$ — V: shear, A: area

Cylinderical shaft

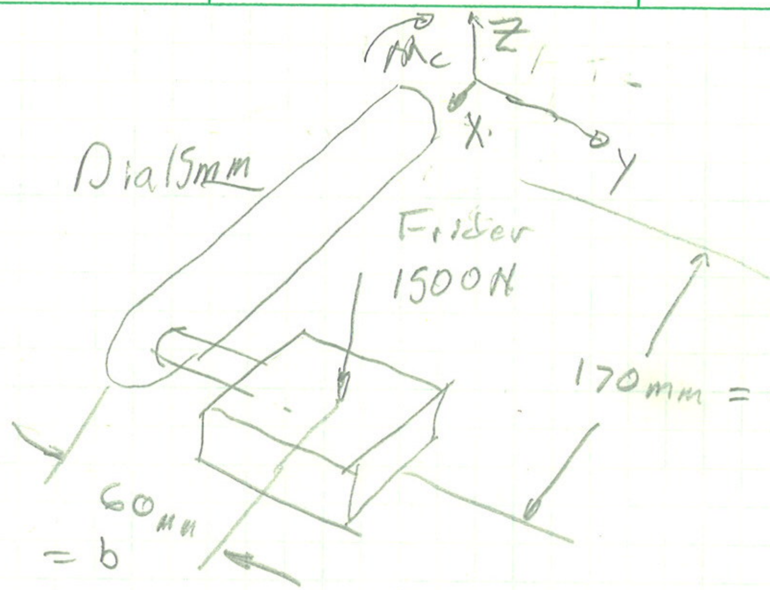


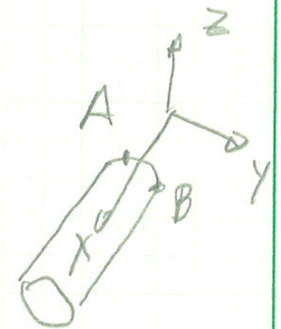
$\tau_{max} = \frac{4}{3}\frac{V}{A}$

hollow cylinderical shaft



$\tau_{max} = \frac{2V}{A}$





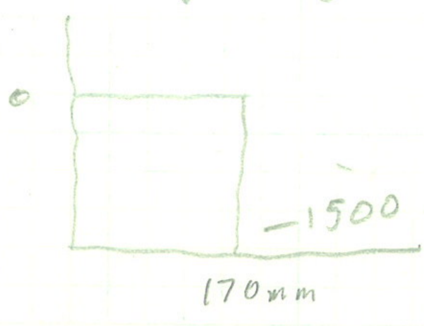
$$M = F_{rider} \cdot a = (1500)(.170) = 255 \text{ Nm}$$

$$T = F_{rider} \cdot b = (1500)(.06) = 90 \text{ Nm}$$

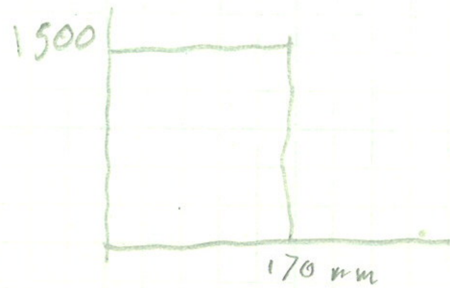
Let

$$F = F_{rider} = 1500 \text{ N}$$

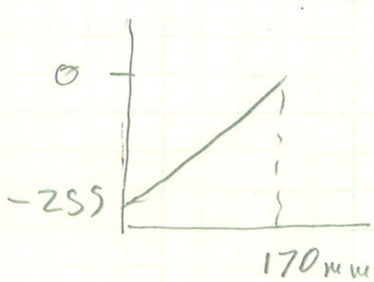
Loading Diagram



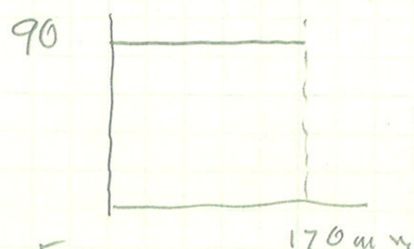
SHEAR DIAGRAM

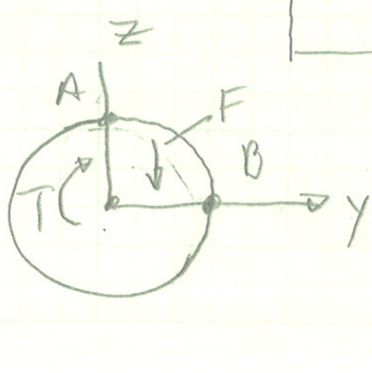


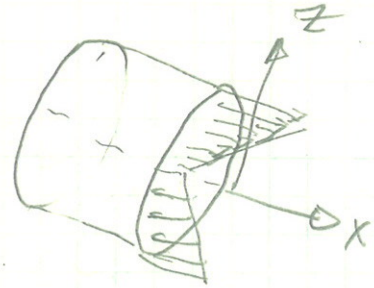
Moment Diagram



Torque Diagram







BENDING NORMAL STRESSES

$$\sigma_x = \frac{Mc}{I} \qquad I = \frac{\pi d^4}{64}$$

TORSION SHEAR

$$\tau_{xy} = \frac{Tr}{J} \qquad J = \frac{\pi d^4}{32}$$

TRANSVERSE SHEAR

$$\tau_{max} = \frac{4}{3}\frac{V}{A}$$

AT POINT A

$$I_a = \frac{\pi d^4}{64} = \frac{\pi(.015)^4}{64} = 2.485 \times 10^{-9}\ M^4$$

Bending Stress X direction

$$\sigma_x = \frac{MC}{I} = \frac{255(.0075)}{2.485 \times 10^{-9}} = 770\ MPa$$

$$\sigma_y = 0$$

Shear Stress due to torque

$$\tau_{xy} = \frac{Tr}{J} = \frac{90(.0075)}{2(2.485E-9)} = 136\ MPa$$

Principal Stress at A

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

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$$\sigma_1 = \frac{770+0}{2} + \sqrt{\left(\frac{770-0}{2}\right)^2 + 136^2}$$

$$\underline{\sigma_1 = 793 \text{ MPa}}$$

$$\underline{\sigma_2 = 0} \text{ MPa}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_3 = \frac{770}{2} - \sqrt{\left(\frac{770}{2}\right)^2 + 136^2}$$

$$\underline{\sigma_3 = -23 \text{ MPa}}$$

Principal Stress at B

No bending stresses at B since it is on the neutral axis

Transverse Shear

$$\tau_{xz} = \frac{4}{3}\frac{F}{A} \rightarrow \text{Transverse shear}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(.015)^2}{4} = 1.77E-4 \text{ m}^2$$

$$\tau_{xz} = \frac{4(1500)}{3(1.77E-4)} = 11.3E6$$

Torsion shear

$$\tau_{xz} = 136 E6$$

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Total shear = Transverse + Torsion

$$\tau_{xz} = 11.3E6 + 136E6 = 147.3E6$$

Normal stresses

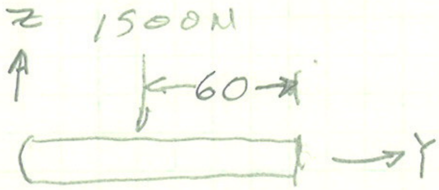
$\sigma_x = 0$ $\sigma_z = 0$

$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = 147.3 \text{ MPa}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = -147.3 \text{ MPa}$$

STRESS ON PEDAL SCREW



$M = 1500 \cdot (.060) = 90 NM$

DISTANCE TO NEUTRAL AXIS

$$C = \frac{d}{2} = \frac{.012}{2} = .006 \text{ m}$$

MOMENT OF INERTIA

$$I = \frac{\pi d^4}{64} = \frac{\pi (.012)^4}{64} = 1.018 \times 10^{-9} \text{ m}^4$$

<u>Bending Stress Y Direction</u>

$$\sigma_Y = \frac{MC}{I} = \frac{90(.006)}{1.018 \times 10^{-9}} = 530.5 \text{ MPa}$$

$\sigma_Z = 0$ MPa

$\tau_{XY} = 0$

since these are zero

$\sigma_1 = \sigma_Y$

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