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Chapter 9

MATCHED PAIRS OR DEPENDENT *t*- test

MATCHED PAIRS OR DEPENDENT *t* - test

WHEN IS THE MATCHED-PAIRS OR DEPENDENT *t*-TEST USED?

1. When pairs of subjects are **matched** on pre-specified variables (e.g., age, gender, education, economic status) in order to reduce random error variance

2. All subjects experience **each level** of treatment
   - Known as a **repeated measures** or **within-subjects** design

3. The population SD is unknown and

4. When the data are of at least interval or ratio scale

HOW DOES THE MATCHED-PAIRS OR DEPENDENT *t*-TEST WORK?

1. Subjects are matched on one or more variables believed to be correlated with the dependent variable
2. One member of each pair is assigned to treatment or placebo (by default, the other member is placed into the other group)
3. The difference between subjects' scores is measured
4. The average or mean difference across all pairs of subjects is obtained and compared to the value expected by the null hypothesis

WHAT DOES THE NULL HYPOTHESIS EXPECT?

1. The null hypothesis expects the average or mean difference across all pairs of subjects to be zero
   - In other words, the average effect of the IV across all pairs of subjects is expected to be zero

2. If the obtained average or mean difference across all pairs of subjects is some value other than zero, the null hypothesis assumes it is solely due to random error

WHAT IS THE AVERAGE OR MEAN DIFFERENCE?

Lecture	Lecture+Activity	Difference
82	88	-6
73	72	1
77	84	-7
71	74	-3
80	93	-13
	SUM	-28
	N	= 5
	MEAN DIFFERENCE	-5.6

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FORMULA:

Average or mean difference

Expected value from null hypothesis

$$t_{\overline{D}} = \frac{\overline{D} - \mu}{SE_{\overline{D}}}$$

Estimated standard error of the mean difference

(Excel will do this for you)

DEGREES OF FREEDOM FOR MATCHED PAIRS *t* - TEST:

$$df = n - 1$$

N is the number of pairs of subjects

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EXAMPLE 1:

An educational researcher wanted to know if in-class activities significantly improved students' learning compared to traditional lecture only teaching methods. Ten students matched on GPA, Year in school, and academic major were randomly selected from the current UNM student population. One student from each pair was randomly assigned to either the Lecture Only class or Lecture plus Group Activities class. At the end of the semester, students' final exam scores were recorded and teaching methods were compared.

EXAMPLE 1:

❶ Null hypothesis

- We expect the average difference between final exam scores for students who either attended a Lecture only class or Lecture+Activities class to be zero. Any non-zero average difference observed is assumed to be solely due to random error.

EXAMPLE 1:

1. Average or mean difference final exam score is -5.6
2. The sample SD for the difference scores is 5.18
3. Sample size is N = 5

$$t_{\overline{D}} = \frac{-5.6 - 0}{\frac{5.18}{\sqrt{5}}}$$

EXAMPLE 1:

$$t_{\overline{D}} = \frac{-5.6}{2.34}$$

$$t_{\overline{D}} = -2.39$$

EXAMPLE 1:

❶ Degrees of Freedom (DF) is 5 – 1 or 4

$$t_{.025} = -2.776$$

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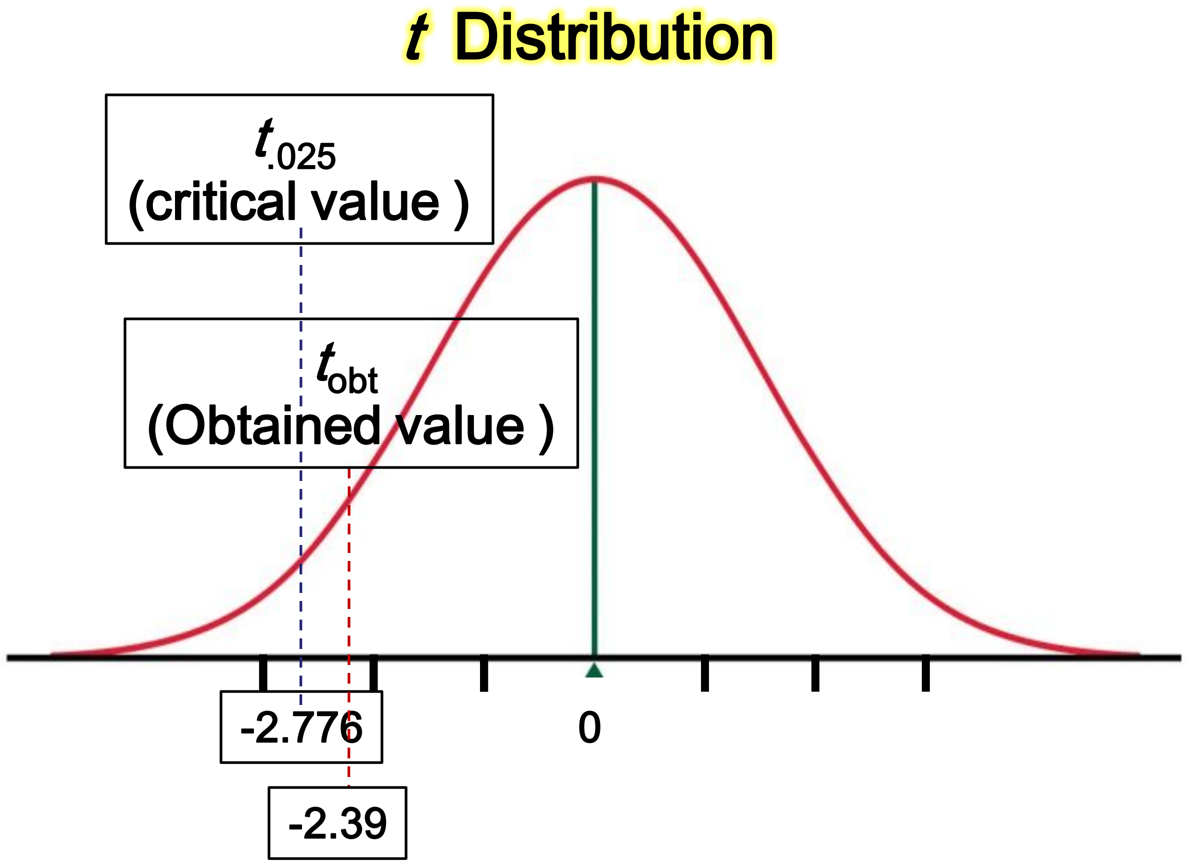
EXAMPLE 1:



❶ **We fail to reject the null hypothesis since the obtained value is less extreme than the critical value (p = .05)**

EXAMPLE 1:

❶ Statistical conclusion:

- "Since t (4) = -2.39, $p>.05$; Fail to reject the null hypothesis."

❷ Interpretation:

- "The average difference between final exam scores comparing students' performance in Lecture only and Lecture+Activities was not significant (p = .05). The observed average difference of -5.6 points may only be due to random error."

COMPARING TESTS OF SIGNIFICANCE TO THE 95% CONFIDENCE INTERVAL

1. When the null hypothesis is rejected, the confidence interval SHOULD NOT contain the expected value from the null hypothesis
   - The sample mean is NOT representative of the population mean expected by the null hypothesis
2. When we fail to reject the null hypothesis, the confidence interval SHOULD contain the expected value from the null hypothesis
   - This indicates that the sample mean IS representative of the population mean

FORMULA FOR 95% CONFIDENCE INTERVAL FOR MEAN DIFFERENCE

$$\overline{D} \pm SE_{\overline{D}}(t_{.025})$$

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MATCHED PAIRS OR DEPENDENT *t* - test

EXAMPLE 1:

1. Average or mean difference final exam score is -5.6
2. The sample SD for the difference scores is 5.18
3. Sample size is N = 5

$$-5.6 \pm 2.34(2.776)$$

$$-5.6 \pm 6.50$$

EXAMPLE 1:

1. Lower Limit → -12.1
2. Upper Limit → .90
3. Interpretation:
   - "We are 95% confident that the true average mean difference between final exam scores for students in the Lecture only class and students in the Lecture+Activities class is in the range of -12.1 points to .90 points."

EXAMPLE 1:

❶ The 95% confidence interval and the statistical conclusion are consistent. We failed to reject the null hypothesis (p = .05) which means the true average difference between the final exams for the two class formats may be zero. The 95% confidence interval contains zero which may be the true mean difference for the population.

INDEPENDENT GROUPS *t* - Test

THE INDEPENDENT GROUPS *t* - TEST IS USED WHEN:

❶ Comparing two independent sample means

- Subjects in each treatment group are independent from each other

❷ The difference between the two sample means is compared to the expected difference between two population means

❸ When σ is unknown and when the data are of at least interval or ratio scale

❹ Assumes the two samples have equal variance (the ratio of the largest to the smallest should not exceed 4 to 1)

FORMULA:

$$t_{\overline{X}_1-\overline{X}_2} = \frac{\left(\overline{X}_1 - \overline{X}_2\right) - \mu}{\sqrt{\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}}}$$

- Where SD^2 is the sample standard deviation squared

EXAMPLE 2:

A study on facial expression and anxiety was conducted to determine if anxious people exhibit more or fewer facial expressions compared to non-anxious people. The number of facial expressions exhibited in response to 25 emotional cues for 25 anxious subjects and 25 non-anxious subjects are shown on the next slide.

EXAMPLE 2:

Anxious			Non-Anxious	
2	11		4	9
12	23		17	9
17	5		12	18
15	3		13	16
27	26		15	15
18	11		8	18
28	16		15	14
16	26		16	14
16	18		13	15
19	26		12	9
11	20		10	17
25	19		18	17
13			14	

EXAMPLE 2:

$$t_{\overline{X}_1-\overline{X}_2}=\frac{(16.92-13.52)-0}{\sqrt{\dfrac{54.33}{25}+\dfrac{13.26}{25}}}$$

- Where SD² is the sample standard deviation squared

EXAMPLE 2:

$$t_{\overline{X}_1 - \overline{X}_2} = 2.07$$

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1. The obtained *t* value is equal to +.2.07. We now need to compare this value to the critical value at *p* = .025
2. The critical value is found in the *t* table using:

$$df = N_1 + N_2 - 2$$

- Where N_1 and N_2 are the samples sizes for each group

- For the example:

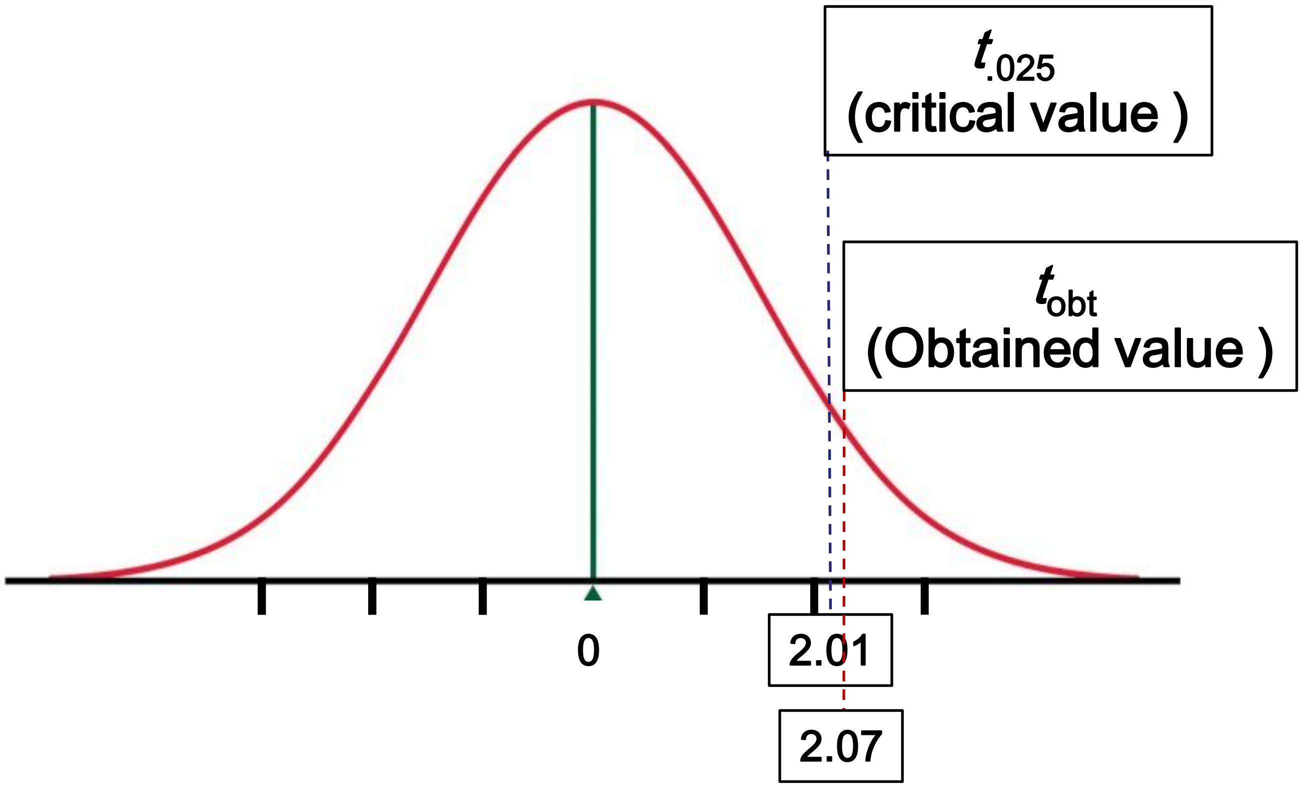
$$df = 25 + 25 - 2 = 48$$

- The critical value of t is found in the table for 48 degrees of freedom

$$t_{.025} = \pm 2.01$$

EXAMPLE 2:

t Distribution



❶ We reject the null hypothesis since the obtained value is more extreme than the critical value (p = .05)

INDEPENDENT GROUPS *t* - Test

EXAMPLE 2:

❶ Statistical conclusion:

- "Since t (48) = 2.07, $p < .05$; Reject the null hypothesis."

❷ Interpretation:

- **"It appears, on the average, anxious people exhibit significantly more facial expressions compared to non-anxious people when exposed to emotional cues (p = .05). The difference between group means does not appear to be solely due to random error, but suggests that increased anxiety may increase the number of facial expressions a person exhibits."**

COMPARING THE CONFIDENCE INTERVAL TO THE TEST OF SIGNIFICANCE:

❶ The following formula will give us the 95% confidence interval for the difference between two sample means

$$\overline{X}_1 - \overline{X}_2 \pm \sqrt{\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}}\left(t_{.025}\right)$$

COMPARING THE CONFIDENCE INTERVAL TO THE TEST OF SIGNIFICANCE:

$$16.92 - 13.52 \pm \sqrt{\frac{54.33}{25} + \frac{13.26}{25}}\,(2.01)$$

$$3.40 \pm 3.305$$

COMPARING THE CONFIDENCE INTERVAL TO THE TEST OF SIGNIFICANCE:

❶ Lower Limit → .095 facial expressions

❷ Upper Limit → 6.705 facial expressions

❸ Interpretation:

- "We are 95% confident that the true difference between the average number of facial expressions of anxious and non-anxious adults is in the range of .095 and 6.705."

❶ The 95% confidence interval and the statistical conclusion are consistent. We rejected the null hypothesis ($p = .05$) and the value expected by the null hypothesis (i.e., zero) was not captured in the range of values for the confidence interval obtained. It appears that increased anxiety may increase the number of facial expressions a person exhibits.

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THAT'S IT FOR CHAPTER 9