Sensitivity Analysis of Nonequilibrium Adaptation Parameters for Modeling Mining-Pit Migration

Dong Chen, M.ASCE[1]; Kumud Acharya[2]; and Mark Stone, A.M.ASCE[3]

Abstract: The nonequilibrium adaptation parameters of a depth-averaged two-dimensional hydrodynamic and sediment transport model were examined in the study. Calculated results were compared to data measured in two sets of published laboratory experiments that investigated mining-pit migration under well-controlled boundary conditions including steady flow and uniform rectangular cross sections along the flume except in the vicinity of the experimental mining area. The two sets of experiments were chosen as representatives of bed-load-dominated and suspended-load-dominated cases, respectively. A sensitivity analysis was conducted to estimate the influence of the nonequilibrium adaptation parameters on mining-pit migration simulation. Calculated results indicate that appropriate selection of the adaptation parameters is critical in order to close the nonequilibrium sediment transport formulas when modeling mining-pit migration.

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Introduction

Sand and gravel on riverbeds have been considered as an attractive (high quality and low cost) source of building material for centuries (Kondolf 1994; Rinaldi et al. 2005). After excavation of sediment over time, scouring and deposition occur on the channel bed, which results in migration and deformation of the mining pit. The migration of mining pit is a complex morphodynamic process resulting from the interaction between streamflow, sediment, and movable boundaries. As water flows over a mining pit, the dividing streamlines separate and converge at the upstream and downstream ends of the pit, respectively. Streamline separation causes eddy rollers and headcut erosion at the upstream end, while streamline convergence causes bed degradation at the downstream end of the pit. Concurrently, incoming sediment from upstream is trapped in the upstream portion of the pit. The overall effect is downstream migration of the gravel pit as deposition occurs at the upstream front while the tail end degrades from local scour. Such patterns have been observed in both natural streams (Chang 1987; Kondolf 1997; Neyshabouri et al. 2002; Rinaldi et al. 2005) and laboratory experiments (Fredsøe 1978; Kornis and Laczay 1988; Lee et al. 1993; Lee and Chen 1996; Neyshabouri et al. 2002).

In contrast with the rapidly varied streamflow over the mining pits, the sediment transport process cannot rapidly reach new equilibrium states. Noticeable temporal and spatial lags exist between sediment transport and streamflow variations. Wu (2008) stated that the assumption of local equilibrium transport is usually unrealistic and may have significant errors in the case of strong erosion and deposition. Bell and Sutherland (1983) conducted a series of experiments and concluded that the predictions of mathematical models are poor in the local scour region if an equilibrium transport formulation is used. Since strong nonuniform flow and nonequilibrium sediment transport phenomena exist around mining-pit areas, it is necessary to introduce nonequilibrium sediment transport schemes when modeling mining-pit migration (Yue and Anderson 1990; Guo and Jin 1999; Wu 2008).

Theory and Objective

Streambed deformation is calculated using various forms of the sediment continuity equation. For only suspended-load transport, the bed change is attributed to the net suspended sediment flux and thus determined by

$$(1-P)\frac{\partial Z_c}{\partial t}=\alpha\omega_s(C-C_*) \tag{1}$$

Similarly, we can calculate the bed change when only bed-load transport exists

$$(1-P)\frac{\partial Z_c}{\partial t}=\frac{1}{L}(Q_s-Q_{s*}) \tag{2}$$

where Z_c=calculated bed elevation (m); P=porosity of bed material; ω_s=settling velocity of suspended sediment (m s^{-1}); C=depth-averaged volumetric suspended-load concentration; C_*=equilibrium depth-averaged volumetric suspended-load concentration; Q_s=volumetric bed-load transport flux per unit width (m^2 s^{-1}); Q_{s*}=equilibrium volumetric bed-load transport flux per unit width (m^2 s^{-1}); α=adaptation coefficient for suspended load; and L=adaptation length for bed-load (m).

[1]Assistant Research Professor, Desert Research Institute, 755 East Flamingo Rd., Las Vegas, NV 89119 (corresponding author). E-mail: dong.chen@dri.edu

[2]Associate Research Professor, Desert Research Institute, 755 East Flamingo Rd., Las Vegas, NV 89119.

[3]Assistant Professor, Dept. of Civil Engineering, Univ. of New Mexico, MSC01 1070, Albuquerque, NM 87131.

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L characterizes the distance for bed load to adjust from a nonequilibrium state to an equilibrium state, while α represents, theoretically, the ratio between the near-bed and depth-averaged suspended sediment concentrations. An equivalent adaptation length, L_s, for suspended load may be defined in terms of α as (Wu 2001)

$$L_s = \frac{q}{\alpha \omega_s} \tag{3}$$

where q=flow rate per unit width ($m^2\ s^{-1}$).

Both L and α are related not only to the streamflow, sediment size and nonuniformity, but also to the "degree of nonequilibrium" (i.e., the difference between sediment load and the sediment transport capacity of flow). Researchers have reported a wide range of values for L and α. Bell and Sutherland (1983) investigated nonequilibrium sediment transport by discontinuing sediment supply at the upstream end of their flume. They found that the length for bed-load sediment to adjust from a nonequilibrium state to an equilibrium state was about the length of the first occurrence of a sand dune or scour hole, although the sand dune or scour hole extended and migrated progressively downstream throughout the experiment. Soni (1981) also found that L was related to flow condition and it changed with time in an experimental case of bed aggradation. Galappatti and Vreugdenhil (1986) found that the adaptation length for which the mean concentration approaches the mean equilibrium concentration is depended on sediment size and Chézy coefficient. Armanini and Di Silvio (1988) also indicated that L should vary with sediment size and flow characteristics (flow depth, Chézy coefficient, etc.). In the experiments conducted by Wang (1999), the adaptation length was determined by the so-called "bed inertia," which represents the difference between sediment load and the sediment transport capacity of flow. In modeling practice, Phillips and Sutherland (1989) and Wu et al. (2000) adopted the nonequilibrium adaptation length as the averaged saltation step length of bed material particles approximated as a hundred times d_{50} for bed load. Rahuel et al. (1989) gave much larger values by estimating L as two times the numerical grid length when dealing with natural channels. As for the parameter α, Han (1980) and Wu and Li (1992) suggested α is 1 for strong scour, 0.25 for strong deposition, and 0.5 for weak scour and deposition.

The objective of this research is to inform the selection of adaptation parameters for nonequilibrium sediment transport conditions. This was accomplished through an extensive sensitivity analysis to investigate the influence of both the nonequilibrium adaptation length, L, and the adaptation coefficient α to the process of mining-pit migration.

Experimental Data Preparation and Comparison

Simulation results using a range of nonequilibrium parameters were tested against data from two sets of published laboratory experiments on mining-pit migration: (1) a set of experiments by Lee et al. (1993) and (2) a set of experiments by Delft Hydraulics Laboratory (DHL) (Galappatti and Vreugdenhil 1986; van Rijn 1986; Guo and Jin 1999). Both experiments were conducted with steady flow and uniform rectangular cross sections except near the artificial mining areas. The two sets of experiments were chosen as representatives of bed-load-dominated and suspended-load-dominated cases, respectively. Details of the experiments are given below.

Table 1. Hydraulic Conditions for the Laboratory Experiments of Lee et al. (1993) and the Present Numerical Model

	Q (m^3/s)	h (cm)	U (m/s)	F	H_p (cm)	L_p (cm)	d_{50} (mm)
Experiment	0.031	10.6	0.501	0.500	4	54	1.4
Model	0.031	10.0	0.519	0.518	4	54	1.4

Experiments by Lee et al.

Experiments of Lee et al. (1993) investigated the migration behavior of several rectangular pits of different sizes composed of uniform bed material. The authors presented the resulting channel geometries which were used in this study to compare with numerical results using various adaptation parameters. The experiments were conducted using a 17-m-long by 0.6-m-wide recirculation flume. The bed sediment was uniform sand with a median particle size, d_{50}, of 1.4 mm. The rectangular pit was 54-cm long and 4-cm deep, with the upstream end located about 9.5 m from the flume entrance. The width of the pit was equal to the width of the flume. No sediment was supplied from upstream. Most of the sediment movement were in the bed-load transport mode and no significant bed forms were observed. All tests were conducted under a subcritical flow regime. Hydraulic conditions reported for flume study and used in the present numerical modeling study are summarized in Table 1. Q is flow rate; h is flow depth; U is velocity; F_r is Froude number; and H_p and L_p are the depth and length of the mining pit, respectively.

Experiments by DHL

The migration of mining pits was also investigated in the flume experiments carried out at DHL (Galappatti and Vreugdenhil 1986; van Rijn 1986; Guo and Jin 1999). The experiments were conducted under steady and uniform flow conditions using a mining pit with 1:10 side slopes in a 30-m-long, 0.5-m-wide, and 0.7-m-deep flume. The trench was 0.16 m deep initially. The width of the mining pit was set equal to the width of the flume. The mean flow velocity and the flow depth were 0.51 m s^{-1} and 0.39 m, respectively. The bed consisted of fine sand (d_{50} =0.16 mm). Unlike the deficient sediment input in the experiments by Lee et al. (1993), the sediment concentration profile was fully developed before the flow reached the mining pit. Suspended sediment transport was the dominant mode.

Table 2 shows the hydraulic conditions reported for the DHL flume study (Galappatti and Vreugdenhil 1986) and the conditions used for the present numerical modeling study. Both top and bottom lengths of the pit are shown (6.2 and 3.0 m, respectively) in Table 2 because the pit has a side slope of 1:10.

Evaluating Goodness-of-Fit

The goodness-of-fit between computed and measured bed elevations was evaluated by three statistical parameters including the

Table 2. Hydraulic Conditions in the DHL Flume Experiments of Galappatti and Vreugdenhil (1986) and the Present Numerical Model

	Q (m^3/s)	h (cm)	U (m/s)	F	H_p (m)	L_p (m) (T/B)	d_{50} (mm)
Experiment	0.09945	39.0	0.51	0.26	0.16	6.2/3	0.16
Model	0.09945	38.4	0.62	0.32	0.16	6.2/3	0.16

bias, the average geometric deviation (AGD), and the root-mean-square (RMS). Each parameter provides a measure of the goodness-of-fit between the computed and measured bed elevations from a slightly different perspective. They are described as follows.

1. Bias

$$\text{bias} = \sum_{j=1}^{J} (Z_{cj} - Z_{mj})/J \tag{4}$$

where Z_c and Z_m=computed and measured bed elevations, respectively, and j=data set number. The *bias* with a unit of *centimeter* in the study represents the arithmetic mean of the difference between computed and experimental bed elevations. A positive value of bias is produced when the calculated bed elevations are generally higher than the observed conditions.

2. AGD

$$\text{AGD} = \left(\prod_{j=1}^{J} RR_j\right)^{1/J}, \quad RR_j = \begin{cases} Z_{cj}/Z_{mj} & \text{for } Z_{cj} \geqslant Z_{mj} \\ Z_{mj}/Z_{cj} & \text{for } Z_{cj} < Z_{mj} \end{cases} \tag{5}$$

The dimensionless parameter AGD represents the geometrical mean of the special discrepancy ratio, RR_j.

3. RMS

$$\text{RMS} = \left[\sum_{j=1}^{J} (Z_{cj} - Z_{mj})^2/J\right]^{1/2} \tag{6}$$

The root-mean-square represents the quadratic mean of the difference between the computed bed elevations and the measured values. RMS is especially useful when deviations are both positive and negative such as overestimation and underestimation of bed deformation in the current calculation. RMS has a unit of centimeter in this study.

These three statistical parameters (bias, AGD, and RMS) provide a comprehensive evaluation of the goodness-of-fit between the computed and measured bed elevation.

Sensitivity Analysis

Nonequilibrium Adaptive Length L

The bed-load-dominated flume experiments of Lee et al. (1993) were simulated by applying CCHE2D, a depth-averaged two-dimensional (2D) hydrodynamic and sediment transport model, which implements a nonequilibrium transport model for bed-material load including both bed load and suspended load (Jia and Wang 2001; Wu 2001). Computational time step and grid size of the 2D mesh have been carefully tested and chosen as 10 s and 0.02 m, respectively.

Lee and his collaborators (Lee and Chen 1996; Lee et al. 1993) divided the migration process of mining pits into two periods, namely, the convection period and the diffusion period. The convection period extends from the beginning of pit deformation to the moment when the upstream boundary of the mining pit moves to the original downstream end of the pit. The diffusion period extends from the conclusion of the convection period to the end of the experiment or observation period. Fig. 1 compares between the calculated results and the measurement at t=2 h which belongs to the "convection period." Four different adaptation lengths (L=1, 2, 6, and 8 cm) were applied in the same computational domain. All the other parameters, including flow parameters, sediment parameters, and computational mesh, were kept the same. As shown in the figure, the smallest adaptation length (L=1 cm) induced excessive bed aggradation at the downstream of the pit, indicating that the equilibrium or nearly equilibrium sediment transport scheme is not suitable for the scenario. As water flows past the eddy-roller region at the downstream end of the mining pit, its velocity decreases dramatically along the channel. The nearest to equilibrium scheme (L=1 cm) will force sediment to settle on the bed immediately, which is unlike to occur in the real situation. Fig. 1 also shows that larger L resulted in more sediment deposition in the pit as well as a gentler frontal surface slope of the upstream end of the pit. However, the bed deformation in upstream and downstream reaches of the pit did not vary greatly for $L > 2$ cm, which indicates that L is only sensitive in the pit area during the convection period.

Fig. 2 shows the comparison between the numerical model and

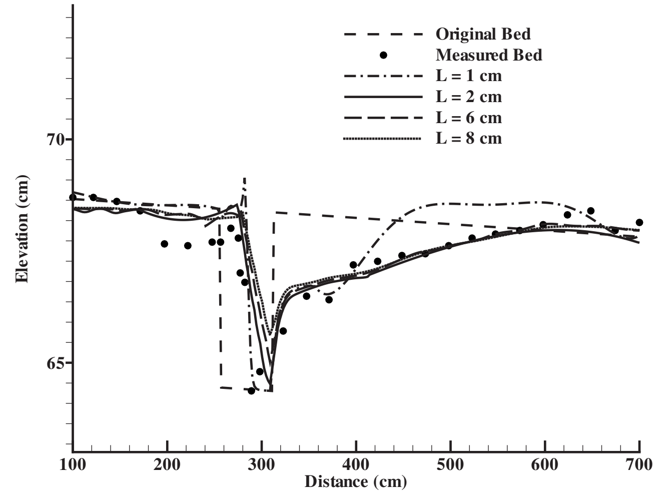

Fig. 1. Sensitivity analysis of nonequilibrium adaptation length of bed-load transport in convection period [flow and original morphology data from Lee et al. (1993)]

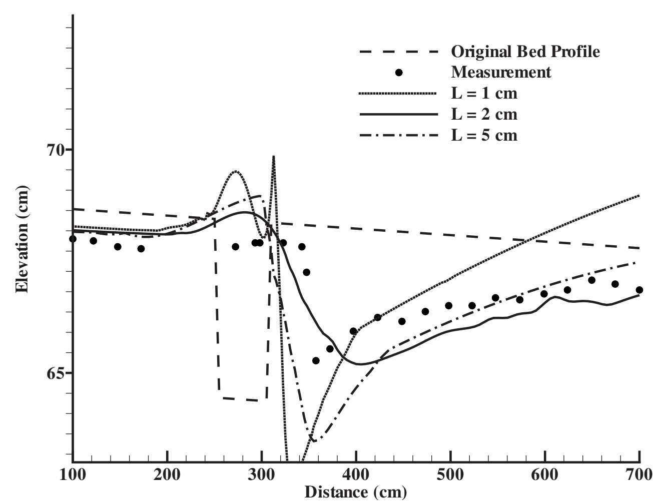

Fig. 2. Sensitivity analysis of nonequilibrium adaptation length of bed-load transport in diffusion period [flow and original morphology data from Lee et al. (1993)]

Table 3. Comparison of Goodness-of-Fit for Computed and Measured Results of Lee et al. (1993) Experiments

Experiments	Lee et al. (1993)						
Time	2 h				5 h		
L (cm)	1	2	6	8	1	2	6
Minimum difference (cm)	−0.41	−0.45	−0.26	−0.23	−4.83	−1.13	−3.56
Maximum difference (cm)	2.28	1.17	1.40	1.75	2.11	0.86	1.05
Bias (cm)	0.51	0.16	0.25	0.27	0.09	−0.17	−0.42
AGD	1.08	1.05	1.05	1.05	1.22	1.08	1.16
RMS (cm)	0.75	0.43	0.52	0.57	1.72	0.56	1.28
Number of points	29	29	29	29	25	25	25

experimental results of Lee and others at $t=5$ h which belongs to the "diffusion period." Three different values of L (1, 2, and 5 cm) were applied in the same computational domain. In the diffusion period, the upstream boundary of the mining pit has migrated past the original downstream end of the pit. The eddy-roller erosion at the original downstream end of the pit ceased and the channel bed tended to "recover" from the lowered bed to the original bed elevation. Three conclusions can be drawn from Fig. 2. First, the calculation with the smallest adaptation length ($L=1$ cm) generated the deepest pit at the end of convection period since the nearest to equilibrium scheme induced unrealistic bed scouring at the original downstream end of the pit. Second, the simulated results using a large adaptation length ($L=5$ cm) underestimated the "recovering" rate of the lowered bed in the diffusion period since this "supernonequilibrium" scheme has over-retarded the settlement of particles from the decelerating sediment-laden flow. Third, the calculated channel bed elevations with $L=2$ and $L=5$ did not vary greatly from each other in the upstream reach (before 250 cm) and downstream reach (after 450 cm) which indicates that L is influential only in the vicinity of the pit area (250～450 cm).

The overall goodness-of-fit descriptions between the flume experiments and numerical model results are summarized in Table 3. The lowest values of bias, AGD, and RMS in Table 3 indicate that 2 cm was the most suitable value for the parameter L in both convection and diffusion periods.

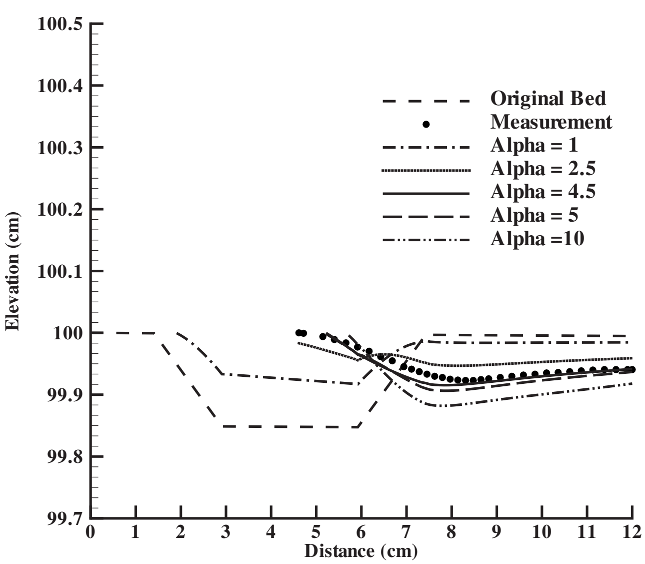

Fig. 3. Sensitivity analysis of nonequilibrium adaptation coefficient of suspended transport [flow and original morphology data from Galappatti and Vreugdenhil (1986)]

Nonequilibrium Adaptive Coefficient α

The suspended-sediment-dominated flume experiments of DHL (Galappatti and Vreugdenhil 1986) were simulated by applying the same sediment transport model with a range of α values (1, 2.5, 4.5, 5, and 10). A value of 4.5 was obtained from the equation of Armanini and Di Silvio (1988)

$$\frac{1}{\alpha} = \frac{a}{h} + \left(1 - \frac{a}{h}\right)\exp\left[-1.5\left(\frac{a}{h}\right)^{-1/6}\frac{\omega_s}{u_*}\right] \tag{7}$$

where h=flow depth (m); a=thickness of bed-load layer (m); ω_s=particle settling velocity (m s^{-1}); and u_*=upstream bed-shear velocity (m s^{-1}). In this study, $h=0.55$ m, $a=2d_{50}=0.32$ mm, ω_s and u_* are calculated as 0.0118 and 0.0405 m/s, respectively. All other model parameters were kept the same. Computational time step and grid size of the 2D mesh have been chosen as 10 s and 0.125 m, respectively. The simulated bed elevations were compared with the experimental results at $t=15$ hours, which belongs to the diffusion period as shown in Fig. 3. Data in the convection period are not available for DHL experiments. Based on the results, we can conclude that: (1) simulation with the scheme $\alpha=1$ seriously underestimated the mining-pit migration rate which is not suitable herein; (2) larger α resulted in over-steep frontal surface slope of the upstream end of the pit which is due to the limitation of equilibrium computational schemes; and (3) variations in simulated bed elevations due to different α values diminished downstream from the pit area as the sediment transport rate approached the equilibrium condition.

The overall goodness-of-fit descriptions between the measured and simulated bed elevations are summarized in Table 4. The

Table 4. Comparison of Goodness-of-Fit for Computed and Measured Results of DHL Experiments

Experiments	DHL				
Time	15 h				
α	1	2.5	4.5	5	10
Minimum difference (cm)	−7.51	−2.17	−1.68	−2.37	−4.84
Maximum difference (cm)	6.09	2.43	2.74	2.74	2.74
Bias (cm)	2.41	1.09	−0.48	−0.98	−2.62
AGD	1.06	1.02	1.01	1.01	1.03
RMS (cm)	5.33	1.89	1.10	1.50	3.28
Number of points	34	34	34	34	34

lowest values of bias (−0.48 cm), AGD (1.01), and RMS (1.10 cm) occurred for α value of 4.5, which was calculated by Eq. (7).

Summarizing the results of "Nonequilibrium Adaptive Length L" and "Nonequilibrium Adaptive Coefficient α" sections, we can conclude that the "near-equilibrium" sediment transport schemes with small L or large α were not suitable for the nonequilibrium scenarios since (1) the near-equilibrium sediment transport calculation caused an over-steep frontal surface slope of the upstream end of the pit in both convection and diffusion periods; (2) the near-equilibrium sediment transport calculation resulted in a over-deep pit at the end of convection period; (3) the near-equilibrium sediment transport calculation brought excessive bed aggradation in the reach behind the eddy-roller region at the downstream end of the mining pit; and (4) the nonequilibrium effects were found to be more remarkable in the vicinity near the mining pit and a short distance downstream from the pit.

Conclusions

This paper examines the nonequilibrium adaptation parameters of a depth-averaged 2D hydrodynamic and sediment transport model through comparison with data measured in two sets of published laboratory experiments on the topic of mining-pit migration. Based on a sensitivity analysis, the following conclusions can be drawn:

1. The equilibrium or "near-equilibrium" sediment transport schemes are not suitable for the calculation of mining-pit migration.
2. After evaluating the overall goodness-of-fit between measured and computed bed elevations using a range of adaptation parameters, $L=2$ cm most accurately reproduced the results similar to Lee et al. (1993), while the α value calculated by Armanini and Di Silvio (1988) equation best fit the DHL experimental data.
3. The sensitivity analysis indicated that the supernonequilibrium scheme (large L) resulted in overestimation of sediment deposition in the mining pit during the convection period with gentler frontal surface slopes at the pit's upstream end.
4. Numerical simulations using the supernonequilibrium scheme (large L or small α) tend to underestimate the bed recovering rate in the diffusion period. However, the near-equilibrium scheme usually results in an over-deep pit at the end of convection period.
5. The error introduced by improper selection of L and α is most noticeable in the vicinity of the mining pit where sediment transport condition is furthest from the equilibrium condition.

However, the above conclusions suffer from limitations. For instance, L or L_s essentially characterizes the distance for sediment-laden flow to adjust from a nonequilibrium state to an equilibrium state and this distance may vary from a few centimeters in laboratorial flumes to a thousand meters in real rivers (personal discussion with CCHE2D developers). The current case study is not capable to give an empirical express of L or α. Besides, both L and α were assumed unchangeable during all the calculation although their values should vary as the bed deformed.

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Notation

The following symbols are used in this technical note:

a = thickness of bed-load layer (m);
C = depth-averaged volumetric suspended-load concentration;
C_* = equilibrium depth-averaged volumetric suspended-load concentration;
d_{50} = the median size which 50% of the particles are finer;
F = Froude number;
H_p = depth of a pit (m);
h = flow depth (m);
L = adaptation length for bed-load (m);
L_p = length of a pit (m);
L_s = equivalent adaptation length of suspended load;
P = porosity of bed material;
Q = flow rate ($m^3\ s^{-1}$);
Q_s = volumetric bed-load transport flux per unit width ($m^2\ s^{-1}$);
Q_{s*} = equilibrium volumetric bed-load transport flux per unit width ($m^2\ s^{-1}$);
q = flow rate per unit width ($m^2\ s^{-1}$);
U = flow velocity ($m\ s^{-1}$);
U_* = shear velocity ($m\ s^{-1}$);
Z_c = calculated bed elevation (m);
Z_m = measured bed elevation (m);
α = adaptation coefficient for suspended-load; and
ω_s = settling velocity of suspended sediment ($m\ s^{-1}$).

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