# Gas Laws and Wealth Laws

FRANK W. PRESTON

*The author, a native of Leicester, England, has a Ph.D. from London University (1925). He came to the United States in 1921 and has been owner and director of the Preston Laboratories, near Butler, Pennsylvania, since 1928.*

EIGHTY years ago, or thereabouts, the law relating to the distribution of velocities among the molecules of a gas was worked out and, from this, the law governing the partition of energy among those molecules. No molecule can have less than zero energy, and very few approach this figure. Many molecules have a modest energy, and a few have very high ones. Since the matter is highly impersonal, no one was greatly offended at this.

The analytical expression is

$$dN = N \frac{2}{\sqrt{\pi} \cdot (kT)^{3/2}} \cdot e^{\frac{\epsilon}{kT}} \sqrt{\epsilon} \cdot d\epsilon,$$

where $dN$ is the number of molecules, out of total of $N$, that have energies between $\epsilon$ and $\epsilon + d\epsilon$, when $T$ is absolute temperature, and $k$ is the Boltzmann constant. The graphical expression is given in Figure 1.

The problem was originally one of physics, but essentially the same problem can be posed as a matter of mathematics, with little reference to gas molecules. It is then spoken of as "statistical mechanics," and of course by any other name it would produce the same results. We then have a "canonical ensemble." (R. C. Tolman, *Statistical Mechanics* [ACS Monographs]. New York: Chemical Catalog Co., 1927.)

If it were considered unjust that a few molecules should enjoy exceptional privileges in the form of phenomenal speeds or energies, we might set a Maxwell demon to catch them and take their energies from them. If this energy were redistributed among the less fortunate, slower molecules, it would not stay so redistributed but would promptly revert to the canonical form. This would keep the temperature at its old value. If, on the other hand, the demon confiscated the energy, or a percentage of it, from the faster molecules, or annihilated them, the distribution of the energy among all the other molecules would change. The truncated distribution left after the demon had operated is not canonical and cannot persist. A new distribution, of the canonical form, promptly comes about, but since some energy has been extracted, the temperature falls. The over-all potential of the populations is less than before, and it can do less work, but the distribution of energy among the individuals is as uneven as before.

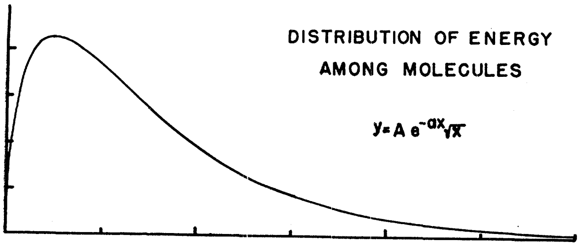


FIGURE 1.

If the demon takes to himself seven others, each worse than himself, and works overtime, singling out the more energetic molecules for the ax, he may ultimately get the energy equally distributed. This will occur when the temperature reaches absolute zero, and no molecule has any energy whatever. It will not occur before.

Around 1895, some time after the Maxwell and Boltzmann laws dealing with gas molecules had been discovered, it occurred to Pareto that wealth tends to distribute itself according to a recognizable pattern. Excluding the very wealthy and the very poor, he found that in all times and all places, the distribution could be very well represented by the equation $N = Ax^{-a}$ where $N$ is the number of people having an income of $x$ units or higher, $A$ is a constant depending on the size of the community (so that $N/A$ is a fraction) and $a$ is an index with a value near 1.5.

In Figure 2 I have taken data not available to Pareto—namely, the incomes of individuals in the United States in the year 1926, as reported in *World Almanac* for 1930. It will be seen that Pareto's law holds very well indeed over a long interval. This law, being much less impersonal

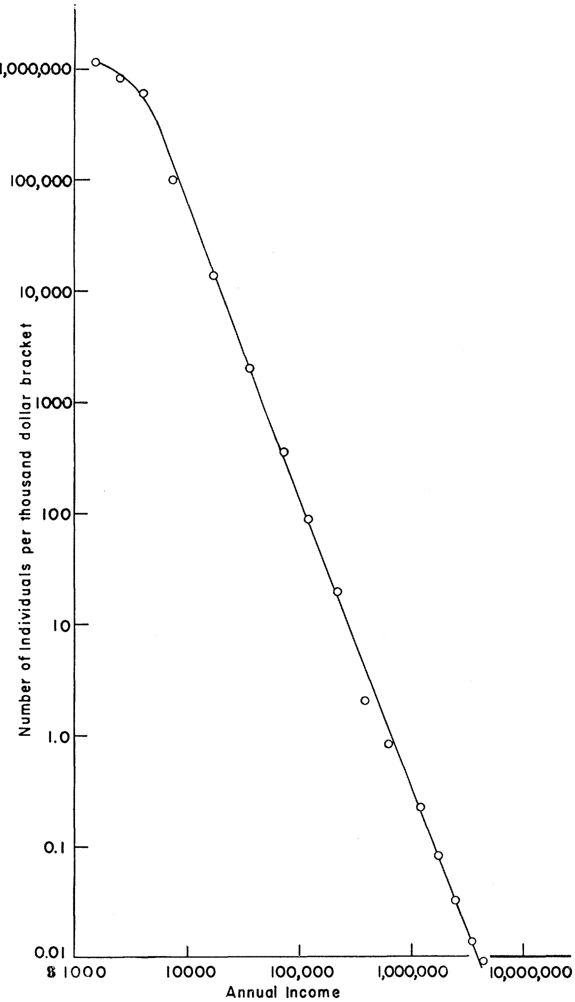


FIGURE 2.*

than one relating to gas molecules, has often been assailed, not on the grounds that it is untrue, but on the grounds that it is unjust. Income, many think, ought to be distributed uniformly, or at any rate *more* uniformly, and many demons have been hired to try to make it so. There is a great difference of opinion as to how much difference of income is tolerable, or perhaps desirable. But it is inherent in the natural distribution that there should be more people with less than the average income than with more. Few people really desire to have less than the average, and so more than half the people can be found in most communities

* The method of plotting is slightly different from that implicit in the formula given, since what I have plotted as the ordinate is the number of individuals in each income bracket of $1,000, not the integrated number of individuals having incomes as great as this *or greater*. Therefore, the slope, or exponent, for the straight part, is about –2.5 rather than –1.5. Actually, it is 2.621.

to vote against Pareto's law. One remedy is a graduated income tax, which, pushed to its logical conclusion, will establish equality when everyone has no income at all and the community reaches absolute economic zero.

It has been long known that Pareto's law does not hold accurately for the very rich or very poor (our own curve shows this), and it has been known for some time that when the poor are taken into account the distribution of income can be better represented by a "log-normal" curve (Fig. 3). That is, instead of finding as we go to lower and lower incomes that there are more and more individuals with such incomes, there is a peak in the curve, and then fewer and fewer people have smaller and smaller incomes below this level. This is more or less self-evident, and produces a total national income of approximately the correct amount, whereas Pareto's law, if carried to its logical conclusion, would call for a much larger total income than is the case, by reason of the vast numbers drawing comparatively small incomes.

Pareto's law in its original form does not look much like a law of physics, but a good many things do seem to conform in practice to log-normal distributions. The "normal," or "Gaussian," curve is a very common approximation to the facts in a great deal of experimentation, both in physics and in biology. And where the normal curve fails to fit, there, with a skew distribution, it will often be found that a log-normal curve does fit. The implication is that in trying to fit the normal curve we have chosen the wrong concept to be plotted as abscissa. What ought to be plotted is a ratio, a relation between two things, a geometric series rather than an arithmetical one. This is equivalent to plotting the abscissa on a logarithmic scale; the ordinate is plotted on a "natural," or arithmetic, scale. The curve then loses it skewness and shows up a symmetrical bell-shaped one of familiar appearance.

Attention has been called to this fact by several writers, and in very different fields. I myself have shown (Commonness, and Rarity, of Species. *Ecology,* 1948, **29,** [3]) that in a random collection of insects or birds or other biological material, the

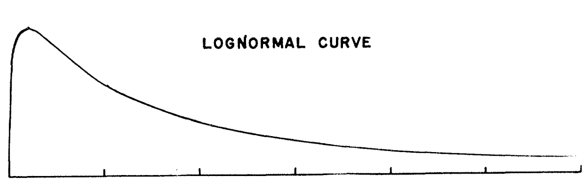


FIGURE 3.

commonness or rarity of the various species tends to plot to a log-normal curve, and a good deal can be explained on this basis.

The richness of a species in individuals bears a certain analogy to the richness of an individual in possessions. Possessions have a way of multiplying just as individuals do. If wealth takes the form of cattle, this is obvious; if it takes the form of invested money, it is scarcely less so. Thus it may not be too farfetched to argue that we might expect wealth, or income, to distribute itself to individuals according to a log-normal curve; and perhaps, if we find that it does, we ought not to be indignant about it.

Perhaps, indeed, we ought not to try to change it. It is not always wise to try to change the laws of nature. Success does not usually attend such attempts, and if success is only to be expected when all life has ceased and nothing moves any longer in the economic system, we should hardly welcome the success. We might do far better to try to understand the law, and the causes that bring it about, than to try to change it out of indignation and without understanding.

## SMO ON THE AIR

| Station | Sponsor | Time |
|---|---|---|
| | **Monday** | |
| KVOR, Colorado Springs | University of Denver (Science Reporter) | 9:45 P.M. |
| | **Wednesday** | |
| KFKA, Greeley | University of Denver (Science Reporter) | 8:15 P.M. |
| | **Friday** | |
| WEVD, New York City | Wendell W. Rázim (Science for the People) | 8:45 P.M. |
| | **Saturday** | |
| KGHF, Pueblo | University of Denver (Science Reporter) | 11:45 A.M. |
| KLZ, Denver | University of Denver (Science Reporter) | 3:15 P.M. |
| KFXJ, Grand Junction | University of Denver (Science Reporter) | 5:30 P.M. |
| KUBC, Montrose | University of Denver (Science Reporter) | 5:30 P.M. |

The Editor of THE SCIENTIFIC MONTHLY will be glad to cooperate with university or other educational stations interested in securing scientific material suitable for broadcasting.