Algebra in Atomic America
Exponential Growth and Decay

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Academic Setting

Cibola High School is a middle-class urban school in the most rapidly growing section of Albuquerque. The student body numbers about 2,300. Cibola's student body is quite diverse. The majority of students are Anglo, with a substantial Hispanic minority, as well as African-American, Asian and Native American students. Cibola HS will begin a freshman academy in the 2000-2001 school year. The same core of teachers will teach all of the entering freshmen. The teachers in the freshman academy will coordinate their curriculums with each of the other academic departments. One of the goals of the freshman academy is to de-compartmentalize the learning process and to demonstrate that the individual subjects do not stand alone, but are relevant to each other.

This curriculum unit is designed for a freshman algebra class. It will be presented about the middle of the second semester in the teaching of logarithms and exponential functions. The lesson will be presented over a period of five class periods. The goals of the unit will be for the students to understand and apply the concepts of exponential growth, exponential decay, and the mechanics of manipulating exponential expressions. Supplies needed for this unit will include graphing calculators and graph paper.

Introduction

Why do I have to learn this? This is one of the most common questions I hear from my Algebra I students. The response "because I said so" doesn't really provide suitable motivation for any but the most docile student, nor does it encourage critical thinking. Except for over-achievers and students who want business, scientific, or engineering careers, most students have no desire to learn any more math than it takes to balance a check book. There are the rare high school students who love the perfection, elegance and logic of math for its own sake (and what a treat it is to teach those people). There is the more common group who see math simply as a tool needed for other disciplines. And finally, there are those who take only what they must because they are compelled. It has been my experience that the third group is the majority in any Algebra I class.

What to do? I don't think I am persuasive enough to convince the average 15-year-old that algebra is actually useful, no matter how many relevant real world examples I demonstrate. Perhaps putting the math in a broader historical and social context will provide some additional stimulation to the less motivated math student. As a teenager, I was excited by things that go boom. For a child of the Cold War that meant the atomic bomb. Can I relate a mathematical equation suitable for Algebra I to the bomb? Of course I can. A bomb is a device that releases energy very rapidly, and in the case of a nuclear device, that energy release can be modeled by an exponential growth function. The dangers of the radioactive contamination left over can be modeled by an exponential decay function. This gives a modern historical context with plenty of local relevance to teach an algebra unit on logarithms and exponential functions.

Approach

In order to introduce the concept of exponential growth, we will first need to provide some background on the idea of criticality and what constitutes a self-sustaining fission reaction. We will put this discussion in a historical context beginning with the famous University of Chicago "graphite pile reactor" experiment done by Enrico Fermi in 1942. We will also discuss what the Germans were doing. It will also be a good time to discuss New Mexico's role in the emerging bomb project. We may also link the concept of an uncontrolled fission reaction (bomb) to controlled fission reaction in a nuclear power reactor. So we can put the algebraic concept of exponential growth into a more exciting concept than how fast cockroaches breed in the school cafeteria. We will present the basic equation for exponential growth and show what it means for a nuclear reaction, and show some other areas of relevance (compound interest for example).

Next we will look at exponential decay. Bombs and reactors generate radioactive materials. We have lots of both here in New Mexico and lots of radioactive waste as well. We will discuss what radioactive decay is and how it can be modeled with an exponential decay function. We will discuss the concept of half-life and what that means to decay rate and potential danger from the radioactive materials. We can discuss this in light of the fact that the United States is still cleaning up the environmental mess from the nuclear weapons program that started in New Mexico 58 years ago. The WIPP facility will be the resting-place for much of the defense-related wastes produced at Los Alamos and Sandia National Laboratories.

The unit will incorporate an historical overview, an overview of the physics of the bomb and its byproducts, and how we use exponential functions and logarithms to model the physics of the reactions. This curriculum unit will take about a week to present. Since Cibola High School operate, on a "flex" schedule the unit will start on the "flex day" when we will have a double length class and can have the time for an extended discussion of the historical and scientific background. Cibola’s flex schedule is a normal six period day for three days of the week. The two flex days will have double length class periods - the odd period classes one day and the even class periods the next.

Historical Background

During World War II and the Cold War, the United States developed a massive industrial complex to research, produce, and test nuclear weapons. This nuclear weapons complex included nuclear reactors, chemical processing buildings, metal machining plants, laboratories, and maintenance facilities that manufactured tens of thousands of nuclear warheads, and conducted more than one thousand nuclear explosion tests. Weapons production stopped in the late 1980s, initially to correct widespread environmental and safety problems, and was later ended indefinitely because of the end of the Cold War. What remains is the legacy of thousands of contaminated areas and buildings, and large volumes of waste and nuclear materials requiring treatment and disposal. Approximately one-half million cubic meters of radioactive waste must be stabilized, safeguarded, and disposed of, including enough plutonium to fabricate several thousand nuclear weapons. (DOE 1996)

The monumental effort to produce the first atomic bomb was inspired by the fear that Germany might invent it first. The Manhattan Project, more correctly the Manhattan Engineering District, was an enormous effort involving vast resources and the best scientific minds in the world. The workers on the Manhattan Project took on an amazing challenge to create a brand new industry from scratch.

From its beginning with Enrico Fermi's graphite-pile reactor under the bleachers of Stagg Field at the University of Chicago to the Trinity Site explosion of the first atomic bomb near Alamogordo, New Mexico, the Manhattan Project took a little less than three years to create a working atomic bomb. During that time, the U.S. Army Corps of Engineers managed the construction of huge industrial facilities to enrich uranium, three plutonium production reactors, and two reprocessing plants to extract plutonium. (DOE weapons)

Shortly after World War II, relations between the United States and the Soviet Union began to sour, and the 45-year long Cold War followed. Its most enduring legacy was the nuclear arms race. It began during the Manhattan Project, when the Soviet Union began developing its own atomic bomb.

In the United States, the nuclear arms race resulted in the development of a huge research, production, and testing network known as the nuclear weapons complex. Some idea of the scale of this enterprise can be understood from the cost: from the Manhattan Project to the present, the United States spent over 300 billion dollars on nuclear weapons research, production, and testing (in 1995 dollars). During half a century of operations, the complex manufactured tens of thousands of nuclear warheads and detonated more than one thousand. (NRDC).

The nuclear weapons complex ranged from tracts of isolated desert in Nevada, where weapons were tested, to its national laboratories in New Mexico and California where bombs were designed to production in Colorado, Florida, Missouri, Ohio, Tennessee, and Washington. The most notable weapons complex facilities here in New Mexico are in Los Alamos, where the first bombs were designed and built.

The Waste Isolation Pilot Plant, or WIPP, is the world's first underground repository licensed to safely and permanently dispose of transuranic radioactive waste left from the research and production of nuclear weapons. After more than 20 years of scientific study, public input, and controversy, WIPP began operations on March 26, 1999.

Located near Carlsbad, New Mexico, the project facilities include disposal rooms mined 2,150 feet underground in a 2,000-foot thick salt formation that has been stable for more than 200 million years. Transuranic waste is currently stored at 23 locations nationwide. Over the next 35 years, WIPP is expected to receive about 37,000 shipments (DOE Weapons).

In 1998 the Brookings Institution published *The Atomic Audit: The Costs and Consequences of U.S. Nuclear Weapons Since 1940.* According to Brookings the cumulative cost of U.S. nuclear weapons is nearly $5.5 trillion (in constant 1996 dollars). If we include the estimated future costs for dismantling of nuclear weapons and the management and disposal of nuclear waste, the total rises to more than $5.8 trillion. Among the book's significant findings:

> Since World War II the cost of the nuclear weapons program exceeded " Federal spending on education, training, employment, and social services; agriculture; natural resources and the environment; general science and space research; community and regional development (including disaster relief); law enforcement; and energy production and regulation.
>
> "Eighty six percent of the nuclear weapons budget went for U.S. nuclear weapons expenditures went toward deploying offensive and defensive weapons and building and maintaining command, control, communications, and intelligence systems to facilitate their use. Only seven percent of the total went toward developing and manufacturing the actual nuclear explosives—more than 70,000 nuclear warheads in all.
>
> "Although the Cold War has ended, U.S. nuclear weapons expenditures remain significant. The United States currently spends $35 billion a year, or 14 percent of the defense budget, on these efforts. This amounts to more than $96 million a day. About $25 billion of the total goes toward operating and maintaining the nuclear arsenal, with the remainder used for managing and cleaning up nuclear waste, verifying arms control agreements, and conducting research into ballistic missile defenses."

Good, bad or indifferent, the real life impact of nuclear weapons on America in general and New Mexico in particular continues to be monumental.

Chronology of Key Events (DOE)

- June 3, 1920 - Ernest Rutherford postulate the existence and properties of the neutron.
- February 7 to 17, 1932 - James Chadwick demonstrates the existence of the neutron.
- September 12, 1933 - Leo Szilard conceives the idea of using a chain reaction of neutron collisions with atomic nuclei to release energy.
- December 21, 1938 - Otto Hahn submits paper to showing conclusive evidence of fission.
- October 11, 1939 - Szilard's and Alexander Sachs presents Pres. Franklin D. Roosevelt with the "Einstein Letter". The letter, signed by Einstein but drafted by Szilard in consultation with Einstein, warns the President of the possibility of nuclear weapons.
- May 19, 1942 - Oppenheimer writes Lawrence that the atomic bomb problem was solved in principle.
- July to September 1942 - Oppenheimer assembles theoretical study group in Berkeley to examine the principles of bomb design.
- September 17, 1942 - Col. Leslie Groves is assigned to command of the Manhattan Engineer District.
- December 1, 1942 - Fermi's group completes CP-1, which produces the first sustained controlled nuclear fission process.
- July 16, 1945 - At 5:29:45 a.m. the first atomic bomb, named Gadget, is detonated in the first atomic explosion in history.
- August 6, 1945 - 0850 - Flying at 31,000 ft Enola Gay drops Little Boy from 31060 feet. Little Boy explodes at an altitude of 1850 feet.
- August 9, 1945 - Fat Man explodes at 1950 feet near the perimeter of the city of Nagasaki.

Physical Principles

The Nature of the Fission Process

The nucleus of an atom can interact with a neutron that travels nearby in two basic ways. It can scatter the neutron - deflecting the neutron in a different direction while robbing it of some of its kinetic energy. Or it can capture the neutron, which in turn can affect the nucleus in several ways - absorption and fission being most important here.

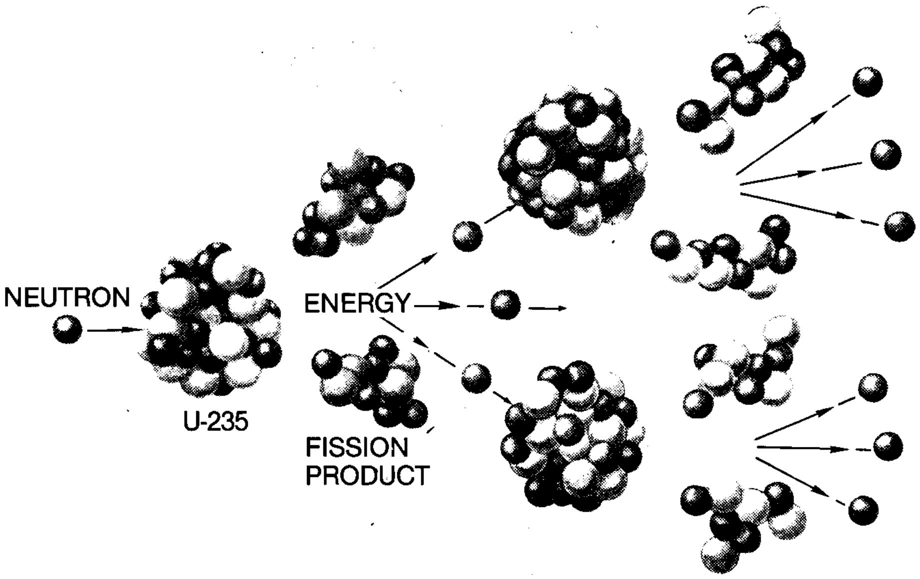
The stability of an atomic nucleus is determined by its binding energy - the amount of energy required to disrupt it. Any time a neutron or proton is captured by an atomic nucleus, the nucleus rearranges its structure. If energy is released by the rearrangement, the binding energy decreases. If energy is absorbed, the binding energy increases.

The isotopes important for the large-scale release of energy through fission are uranium-235 (U-235), plutonium-239 (Pu-239), and uranium- 233 (U-233). The binding energy of these three isotopes is so low that when a neutron is captured, the energy released by rearrangement exceeds it. The nucleus is then no longer stable and must either shed the excess energy or split into two pieces.

By contrast, when the abundant isotope uranium-238 captures a neutron and not fission after internal rearrangement. However if it captures a neutron with enough kinetic energy, then this energy plus the energy released by rearrangement can overcome the binding energy and cause fission. Since a fast neutron with a large kinetic energy is required, this is called "fast fission."

A general trend among the elements is that the ratio of neutrons to protons in an atomic nucleus increases with the element's atomic number (the number of protons the nucleus contains, which determines which element it is). Heavier elements require relatively more neutrons to stabilize the nucleus. When the nucleus of a heavy element like uranium (atomic number 92) is split, the fragments, having lower atomic numbers will tend to have excess neutrons. The excited fragments emit these neutrons very rapidly. More neutrons are produced on average than are consumed in fission.

Fission is a statistical process. The nucleus rarely splits into pieces with the same mass and atomic number. Instead both the size and atomic numbers of the fragments have probability distributions (called Gaussian distributions) around two means, one for the lighter fragment around 95 and one for the heavier around 135. Similarly, the number of neutrons produced varies from zero to six or more (Foster&Wright).



Criticality

What do we mean by criticality? When enough neutrons are available to generate a constant fission rate then we say that we have a critical mass. In a sub critical mass there are not enough neutrons being captured by the fissile material to keep a reaction going by itself. In a super-critical assembly we get enough neutrons produced to multiply the energy and the power output climbs exponentially. The first example I ever saw was a movie my junior high school science teacher showed. Imagine a large table full of mousetraps, with some sort of screen over it. Each mousetrap has two Ping-Pong balls on its arm. If we drop a Ping-Pong ball onto the table it sets off a trap and two more balls are sent off. Each of those two balls sets off a new trap and two more balls go flying off. If we think of the balls as neutrons and the traps as uranium atoms, we have a reasonable analog to the fission process in a bomb.

A neutron entering a pure chunk of one of the fissionable isotopes would have a high probability of causing fission compared with the chance of unproductive absorption. If the chunk is large and compact enough, then the rate of neutron escape from its surface will be so low that it becomes a "critical mass," a mass in which a selfsustaining chain reaction occurs. Typical figures for critical masses for bare spheres of fissionable materials are about 16 kg of U-233, 52 kg for U-235 and about 10 kg for U239. These critical masses can be reduced using various neutron reflectors (Foster&Wright).

Basic Principles of Fission Weapon Design

The principle issues that must solved to construct a fission weapon are:

Keeping the fissionable material in a sub-critical state before detonation;
Bringing the fissionable material into a supercritical mass while keeping it free of neutrons;
Introducing neutrons into the critical mass when it is at the optimum configuration (i.e. at maximum super-criticality);
Keeping the mass together until a substantial portion of the material has fissioned.

Solving the first three issues together is greatly complicated by the presence of naturally occurring neutrons. Although cosmic rays generate neutrons at a low rate, almost all of these "background" neutrons originate from the fissionable material itself through the process of spontaneous fission. Due to the low stability of the nuclei of fissionable elements, these nuclei will occasionally split without being hit by a neutron. This means that the fissionable material itself periodically emits neutrons.

The process of assembling the supercritical mass must occur in significantly less time than the average interval between spontaneous fissions to have a reasonable chance of succeeding. This problem is difficult to accomplish due to the very rapid heat and energy generated in going from a sub-critical state to a supercritical one. The time required to go from sub-critical to super-critical is called the reactivity insertion time, or simply insertion time (Nuclear Weapons)

Radioactive Decay

After the bomb is manufactured or detonated we get radioactive waste from the industrial processes and radioactive fallout from the detonation. These radioactive by-products are quite dangerous. They emit radiation by a process called radioactive decay. Radioactive decay is the spontaneous release of energy in the form of radioactive particles or waves. This decay results in a decrease over time of the original amount of radioactive material. Half-life is used to describe the time required for 1/2 of the original amount of radioactive material to decrease. That is, the radioactive material is decreased to half its initial amount.

Most atoms are stable; a carbon-12 atom for example remains a carbon-12 atom forever, and an oxygen-16 atom remains an oxygen-16 atom forever, but certain atoms eventually disintegrate into a totally new atom. These atoms are said to be ‘unstable’ or ‘radioactive’. An unstable atom has excess internal energy, with the result that the nucleus can undergo a spontaneous change to a more stable form. This is called ‘radioactive decay’. Each kind of atom is called an isotope, and unstable ones (which are thus radioactive) are called radioisotopes. Some elements, like uranium, have no stable isotopes.

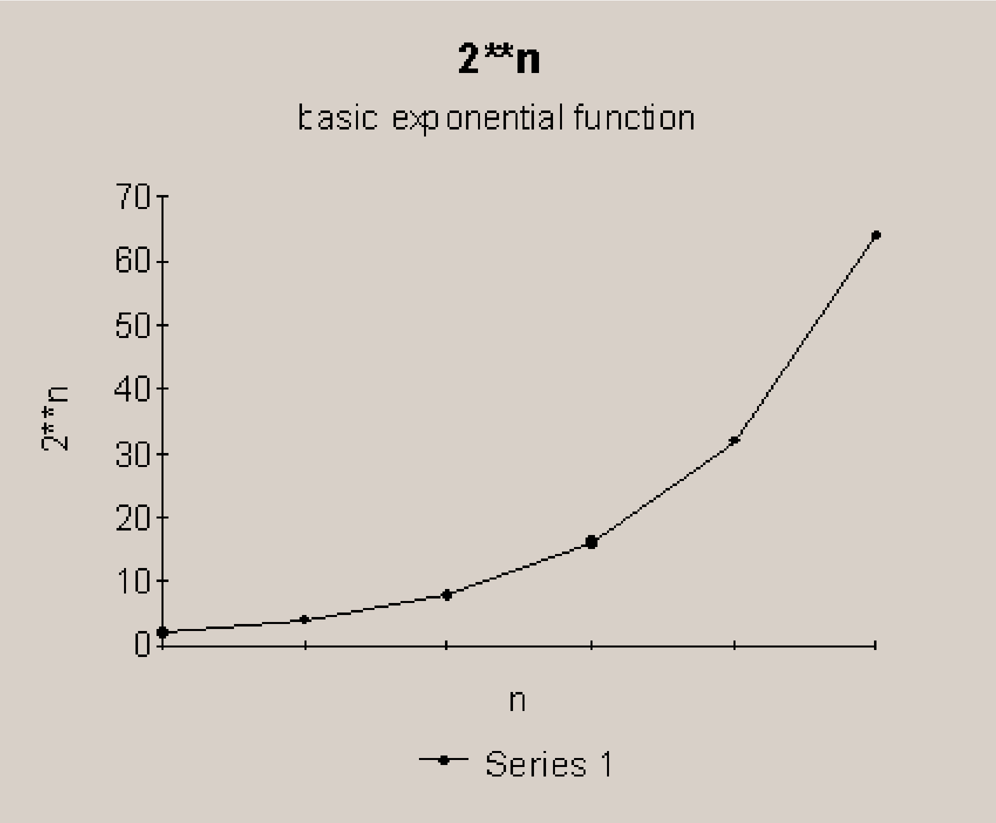
When an atom of a radioisotope decays, it gives off some of its excess energy as radiation in the form of gamma rays or fast-moving particles. If it decays with alpha or beta emission, it becomes a new element. All the time, the atom is progressing to a stable state where it is no longer radioactive. Another source of nuclear radioactivity is when one form of a radioisotope changes into another form, or isomer, releasing a gamma ray in the process. Atoms in a radioactive substance decay in a random fashion but at a characteristic rate. The length of time this takes, the number of steps required and the kinds of radiation released at each step are well known.

The half-life is the time taken for half of the atoms of a radioactive substance to decay. Half-lives can range from less than a millionth of a second to millions of years depending on the element concerned. After one half-life the level of radioactivity of a substance is halved, after two half-lives it is reduced to one quarter, after three half-lives to one-eighth and so on.

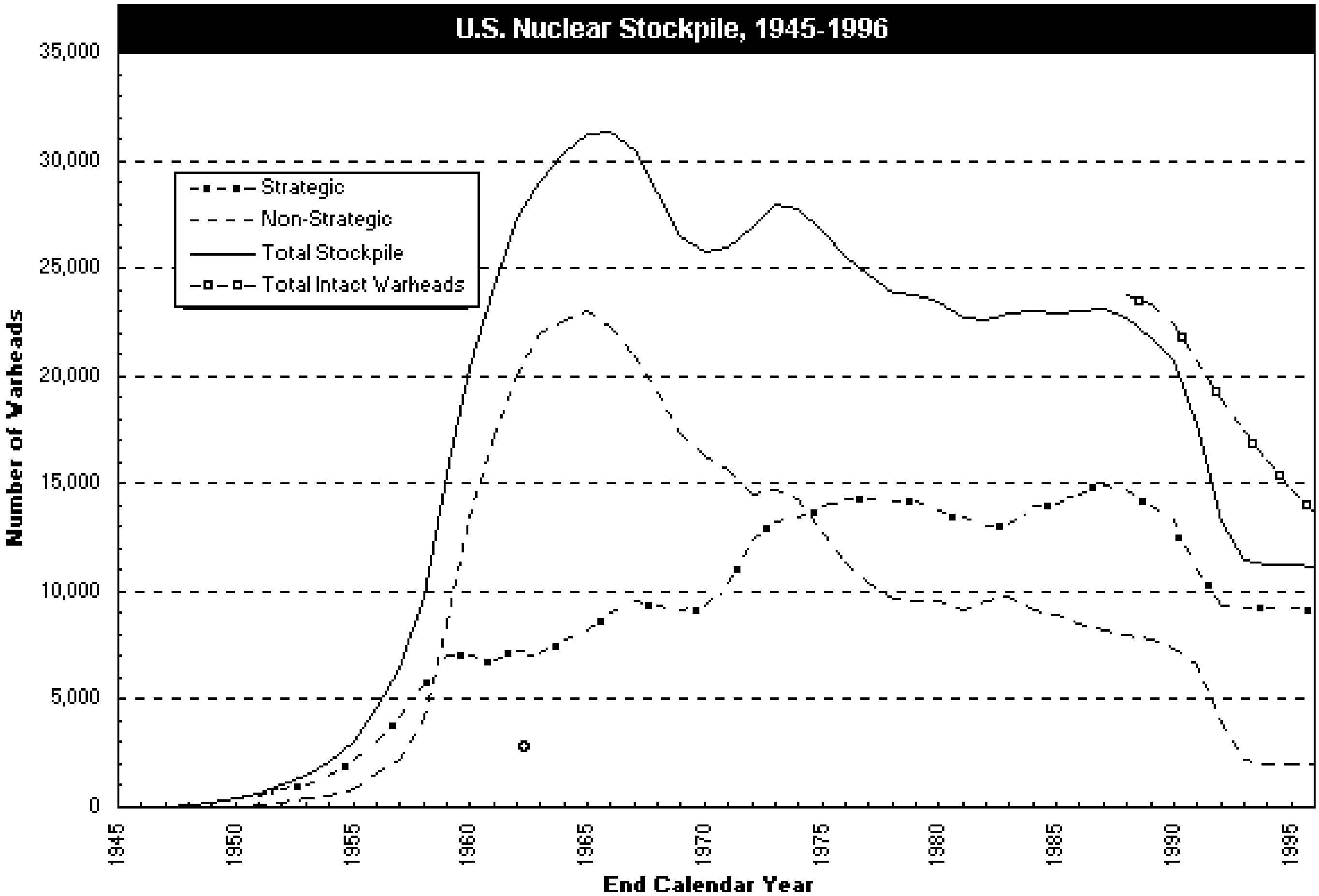
All uranium atoms are mildly radioactive. The following table for uranium-238 shows the various changes, the type of radiation given off at each step and the ‘half-life’ of each step that U-238 goes through in its change into stable, non-radioactive lead-206. The shorter-lived each kind of radioisotope, the more radiation it emits per unit mass (H.Cember).

The Math

Exponential functions and logarithms are mathematical functions that can be used to model the physics of nuclear fusion and decay. The growth of the nuclear stockpile and the growth of spending on the nuclear weapons program can also be modeled by exponential functions. What is an exponential function any way? It is a way of multiplying a number by itself as many times as we want. We can even multiply a number by itself a fractional number of times. Let's take a number, oh, two would be wonderful. Let's multiply two times two five times; we would write that as 2*2*2*2*2 or 2^5 and call it two to the fifth power or more simply two to the fifth. In general we would take some number and call it *a*; a^n is a multiplied by itself *n* number of times. Whenever n is greater than one we have an exponential growth function. Exponential growth functions can also be used to model money earned from interest or the growth of undesirable pests such as cockroaches in the school cafeteria.



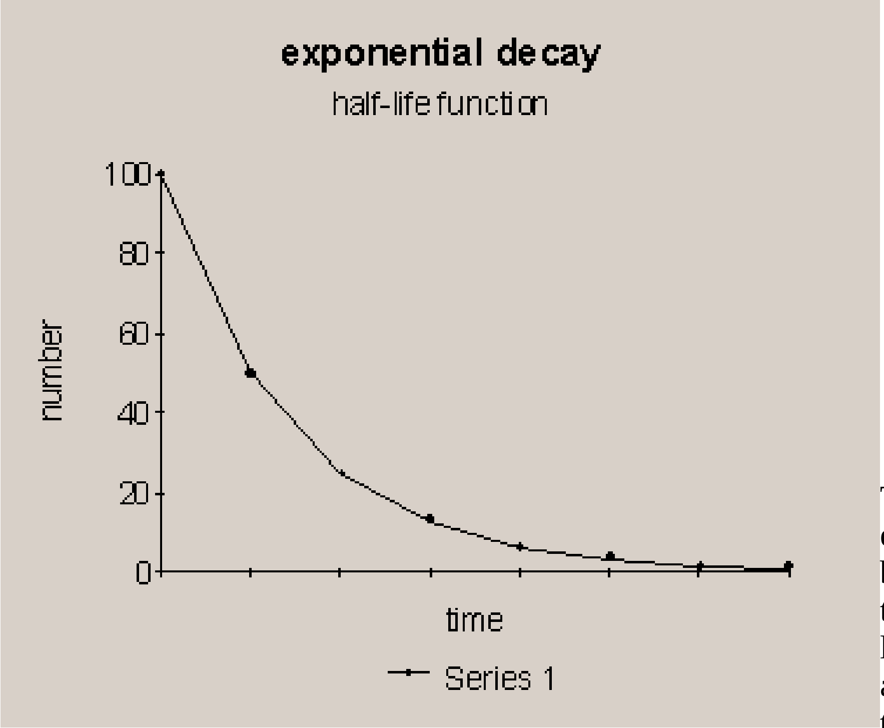
The graph below shows the growth of the U.S. nuclear stockpile since 1945. The growth of the nuclear stockpile from 1945 to 1960 looks very much like an exponential function. In Lesson Plan X we will use this as an example of how to calculate a rate of growth.



Lets look at exponential decay now. Let's say I start on day 1 with a million atoms of radioactive iodine (I131). I131 has a half-life of about eight days. After eight days I will have 500,000 atoms of iodine; after 16 days I will have 250,000 atoms of iodine; after 24 days I will have 124,000 atoms and so on. This is an exponentially decreasing function.

The initial radioactive nuclide in any decay mode is called the parent. The decay product nuclei are called the daughter. If several generations of daughters are radioactive we call it a radioactive decay chain. Think of a system containing many nuclei of the same species at some initial time. The nuclei decay at the decay rate, which we will label R (often called the decay constant). The basic experimental fact of radioactive decay is that the probability for any one nucleus to decay within a small time interval is independent of any external influence including the decay of another nucleus. All nuclei of a given nuclide have identical decay probability. If there are some number "*N*" undecayed nuclei at time *t* then the number decaying in the following a time interval Δt is ΔN. $R\Delta t$ is the probability that a nucleus will decay in Δt $NR\Delta t$ is the probability that any nucleus will decay in Δt. The basic exponential decay function for *N* radioactive nuclei is:

$$N=N_0e^{-Rt}$$



The basic equations of exponential functions are listed below; we will explain all of these rules in the context of the lesson plans. I presume the average high school math teacher is comfortable with the basic equations. This section will simply address specific applications.

Lesson Plan

Day 1

On the Cibola schedule, day 1 will be a flex day or double length period. We will begin with a discussion of the nuclear weapons program and the cold war. We will give a brief description of nuclear fusion and radioactive decay. We will then show a number of example problems related to nuclear issues.

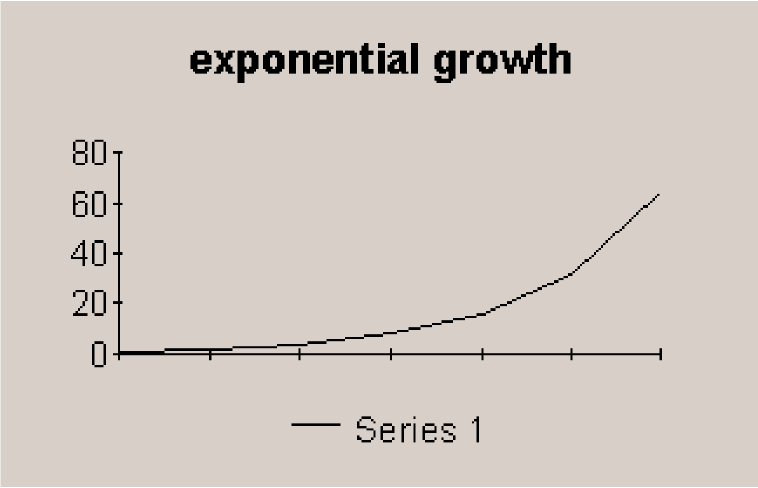
Objective: Students will understand the application of exponential relationships in real world situations.

The lesson will begin with a 30 minute discussion of the reasons for developing the atomic bomb and New Mexico's unique role in that project. We will discuss the work at LANL, SNL and WIPP. We will also discuss the process of nuclear fission using the mousetrap example and illustrations presented earlier in this paper. The teacher will invite the students to voice their thoughts on nuclear issues, or the Cold War, or the impacts on New Mexico.

We will show the basic exponential growth and decay equations. We will first define what an exponent is. It is the representation of a number in the form:

X^{y}

The integer *X* is called the base and *Y* is called the exponent. Exponents are used as a short way to represent a number. The exponent is the number of times the base is multiplied by itself. Let's graph a simple exponential growth function, $y=2^{x}$:



Here are some examples:

27 can be represented as 3^3 , 8 can be represented by 2^3 ; this is same as 3 multiplied by itself 3 times or 2 multiplied by itself 3 times.

Advantages of Exponents

Easier representation of large numbers; large numbers when represented in the form of exponents look simpler for and easier for reading, writing and calculation purposes

Multiplication is made simpler

Example:

$$64 * 32 = 2^6 * 2^5 = 2^{11}$$

Division is made simpler.

Example:

$$64 / 32 = 2^6 / 2^5 = 2^1 = 2$$

$$27 = 3^3 = 3 \times 3 \times 3$$

32 can be represented as 2^5 ; this can also be represented as 2 multiplied by itself 5 times.

$$32 = 2^5 = 2 * 2 * 2 * 2 * 2$$

1,000,000 can be represented as 10^6. That is a short way to represent 10 multiplied by itself 6 times.

$$1{,}000{,}000 = 10^6 = 10*10*10*10*10*10$$

Independent Practice: Assign about 20 problems illustrating these concepts.

Day 2 & 3

We will review the properties of exponents and work several examples of each property and combinations of properties.

Property 1: Reciprocal Property

A number in exponential notation can also be expressed in terms of its reciprocal. The reciprocal of a number raised to the exponent a is that number raised to the exponent -a. Therefore,

$X^a = 1 / X^{-a}$

and

$X^{-a} = 1 / X^a$

Examples:

$2^{-3} = 1 / 2^3 = 1 / 8$

$5^2 = 1 / 5^{-2} = 1 / (1 / 25) = 25$

Property 2: Multiplication Property

When two exponential numbers having same base are multipled, their exponents are added. If *k* and *t* are numbers, then:

$X^k * X^t = X^{k+t}$

Example:

$243 * 81 = 3^5 * 3^4 = 3^{5+4} = 3^9 = 19{,}683$

It is important to note that the two exponential numbers to be multiplied should have the same base.

Example:

$16 * 1024 = 2^4 * 4^5 = 2^4 * (2^2)^5 = 2^4 * 2^{10} = 2^{14} = 16{,}384$

Property 3: Division Property

When another exponential number with the same base divides an exponential number, their exponents are subtracted. If *k* and *t* are numbers, then

$X^k / X^t = X^{k-t}$

Example:

$243 / 81 = 3^5 / 3^4 = 3^{5-4} = 3^1 = 3$

It is important to note that the two exponential numbers to be divided should have the same base.

Example:

$64 / 16 = 2^6 / 4^2 = 2^6 / (2^2)^2 = 2^6 / 2^4 = 2^{6-4} = 2^2 = 4$

Property 4: Distributive Property over multiplication

When two bases with the same exponent are multiplied, their product is equal to the product of the bases with that exponent.

$X^a Y^a = (XY)^a$

Property 5: Distributive Property over division

When two bases with the same exponent are divided, their quotient is equal to the quotient of the bases with that exponent.

$$X^a/Y^a = (X/Y)^a$$

Independent Practice- assign about 60 problems illustrating the above principles.

Day 4 & 5

We will review the properties of logarithms and work several examples of each property and combinations of properties.

Properties of Logarithms

Property 1

If x and y are positive real numbers, b is positive and not equal to 1, then

$$\log_b xy = \log_b x + \log_b y$$

Examples:

- $\log_4 35 = \log_4(7*5) = \log_4 7 + \log_4 5$
- $\log_3 21 = \log_3(7*3) = \log_3 7 + \log_3 3$
- $\ln 18 = \ln(2*3*3) = \ln 2 + \ln 3 + \ln 3$

Property 2

If x and y are positive real numbers, then

$$\log_b x/y = \log_b x - \log_b y$$

Examples:

- $\log_{10} 10/2 = \log_{10} 10 - \log_{10} 2$
- $\log_e 50/5 = \log_e 50 - \log_e 5$
- $\log_{10} 100/3 = \log_{10} 100 - \log_{10} 3$
- $\log_{100} 10/2 = \log_{100} 10 - \log_{10} 2$

Property 3

If x and y are positive real numbers, then

$$\log_b x^p = p \log_b x$$

Examples:

- $\log 5^3 = 3\log 5$
- $\log_2 16^5 = 5\log_2 16$

Property 4

If x and y are positive real numbers, then

$$\log_b 1 = 0$$

Property 5

If x and y are positive real numbers, then

$$\log_b b = 1$$

Property 6

If x and y are positive real numbers, then

$$\log_b b^n = n$$

Independent Practice- assign about 60 problems from the text using all of the properties of logarithms.

Day 6

We will conclude the section with several problems related to the "atomic" issues. First we will look at exponential growth using the example of stockpile growth. Look at the stockpile growth illustration earlier in the paper. It shows that the stockpile of U.S. nuclear weapons grew from 5 nukes in 1945 to about 25,000 in 1960. We will start with the basic equation for growth:

$$N=N_0e^{Rt}$$

Where $N=25000$; $N_0=5$; $t= 15$ years.

Find the growth factor R:

Lets use logarithms to help solve this problem.

$\mathrm{Ln}(25{,}000)=\ln(5e^{15R})$
$\ln (25{,}000)=(15R)\ln(5)$
$15R=(\ln 25{,}000)/(\ln 5)$
$R= 0.29$

We will also do a simple exponential decay problem. I have been injected with 1,000,000 atoms of radioactive iodine. The iodine has a half-life of about 7 days. If none of the iodine leaves my body, how long will it take for the 1,000,000 atoms to decay to 100 atoms?

First let's figure out the decay constant. If the half life is 7 days then

$N/N_0=1/2$.
Since $N= N_0\, e^{rt}$ and the half life is 7 days then
$1/2 = 7r$, therefore
$r=1/14$

Next we plug into our basic decay equation

$100=1{,}000{,}000\, e^{-1/14t}$
$\ln 100 = t/14t \ln 1{,}000{,}000$
$t/14 = \ln 1{,}000{,}000/\ln 100$
and finally $t= 42$ days

Remember, when we do radioactive decay problems we must remember to use the base *e*. In this case we use base *e* because experiments show that this is the correct number to use.

Suggested Reading

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