

Math and Reality: A Sense of Numbers

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The Academic Setting

Truman Middle School is on Albuquerque's far west side. It houses students in grades 6th, 7th and 8th, with a population of about 850 students. Truman is a very diverse middle school, with 80.4% of the students being Hispanic, but also includes Anglo, Black, Asian and Native American students, among others. Students who receive free lunch make up 76.7% of the student population, signifying a high percentage of poverty. Another telling statistic is that 19% of adults in the Truman community have no high school diploma, and another 10% have less than a 9th grade education. Every day attendance reflects the Albuquerque Public Schools' average at around 92%.

This unit is being developed for learning disabled students in middle school, particularly those enrolled in a CBI (community based instruction) program. This is classified as a D-level class, meaning the services the students receive is that of eight students to one teacher, and maybe an educational assistant. These students receive maximum services. The students are in a contained setting, with the special education teacher giving instruction for all core subject areas. The students are enrolled in the general student population for their elective classes. These students typically are academically low achieving and socially immature for their age group. The goal is that the class will bond with each other and become a unit unto itself. Collaboration with other special ed and/or general ed teachers is encouraged, in particular for development of the students' social skills. Instruction tends to be hands-on and geared toward the student being prepared for life in the real world, (i.e., a life skills curriculum). This program at Truman has traditionally operated some type of "store" during school lunches, and in so doing teaches curriculum as well as life skills. The student store sells items such as candy, popcorn, and popsicles, as well as student made foodstuffs like cookies and rice krispie treats. The teacher supports the students as they sell, but they interact with student customers and make change.

Truman students' math scores on the state-mandated Terra Nova test linger at the bottom. Eighth grade student scores in the Math Total subtests are 30 for 1998 and 31 for 1999 (the last year for which I have data). Sixth grade scores for that timeframe are 20 and 30, respectively. As shown, there are not many gains made in math scores. In comparison, Albuquerque Public Schools district scores for 8th graders were 52 in 1998 and 53 in 1999; for sixth graders those scores on the same subtests are 46 in 1998 and 47 in 1999. The gap between students at Truman and the rest of the schools in the district is wide.

Unit Goals

My unit has been developed as a semester long unit. As such I will give a lesson plan for each section of the unit. I began with number sense, then place value, money, number patterns and finally, geometry. My lesson plans will follow this same sequence.

My goals for this unit are to develop a sense of math mastery in each of my students. It seems that students who feel "smart" in math have a much greater sense of self-worth, and I would like to kindle this in all my students. I want them to see the patterns in math, and also in the world around them. I want them to be able to use math.

Rationale

I believe that my students need to strengthen their basic knowledge of math, and in so doing so, the higher skills will come. Math is so fundamental in our life, yet many students feel uncomfortable with math, and this stays with them throughout their life. Middle school is a unique time in a student's academic life. The gap between male and female students in math becomes wider during this time so that by high school many girls do not feel competent in math, yet to succeed in the adult world it is necessary not only to be competent but to excel in math. Particularly for the girls, but for all students, math skills are essential to higher paying jobs. I feel that if a student excels in math, he or she feels competent in everything else. Students are very proud of the fact that they understand and can do math.

Many times special education students are taught the same skills yearly, never quite mastering them, so in math they are never moved beyond the most rudimentary of skills. Math classes tend to be repetitive year after year, focusing on the basic computational math functions. To master these, students need memorization skills, something that my students lack due to short-term memory difficulties. Math then becomes drudgery and students begin to believe that they can not succeed in math. The groans are everywhere when math class begins. When students are younger, they do not necessarily have this same aversion to math. I believe this is a learned phenomena and that students begin to perceive themselves as "dumb." I would like to show the students techniques they can use in different math situations. I want to show them how to think about the problems, and that there are different ways of solving those same problems. I think this will carry over into other subjects, giving them a confidence that they may not have experienced before. I want them to see there is no escaping math in our world because it is all around us, and that it is not something to fear.

Context and Background

My unit is geared for a semester, but with a different population students could be moved through it more quickly. I like to work in thematic units, and for this semester, I will be using a theme of patterns. I envision this unit as something that we will be using in all subject areas. The fact that I have my students for all content areas lends itself to using themes which carry over into language arts, social studies and science, as well as math.

I would like to begin my unit with basic number sense. I am making the assumption that most of this beginning will be review for them. My students will be 7th and 8th graders. They should have some experience with basic number skills, such as counting in sequences, decimal/monetary system, and the four computational skills. They may not have proficiency in any of these and they may not have had much exposure to geometry.

I want to review reading and writing of numbers, which is generally a skill my students lack. I envision doing drills as relay-type games reading numbers, individually and in teams. I will give the students written three and four digit numbers and ask them to read them. I will also give them numbers written with words. Next, we will talk about sequences and the patterns of numbers. I will have the students count by twos, fives, and tens, something they have all done. We will use manipulatives to show the patterns. I would also like to have them look at some number sequences and see if they can figure out the patterns.

Discovering the pattern for counting in English is difficult. In *Using Language to Teach*, Jean Cotter compares the math skills of young children from Asian countries to those in the United States in terms of language. Counting to 100 in an Asian language

requires learning a total of 11 words, but that skill in English requires a total of 28 words. The Japanese Council of Mathematics Education states that children who are taught with a focus on counting do well in computation but have more difficulties with word problems. These children tend to develop a unitary concept of numbers which interferes with their understanding of place value concepts. To understand our base-ten number system, children must experience the pattern of trading: ten ones for one ten, ten tens for one hundred, etc... Cotter advocates the use of manipulatives, in particular an AL abacus, because children who can visualize quantities and understand patterns have developed a good number sense and will search for more patterns which leads to increasing abstract thought (Cotter 108, 114).

After counting and sequencing we will begin place value. This is also a basic skill, something that in another class would warrant only a small amount of time. Generally, students at Truman in maximum services (eight students to one teacher) have difficulty reading numbers greater than four digits. They tend to read the digits as individual numbers, not using any place value (i.e., hundred thousand, ten thousand, thousand). I intend to play games dealing with place value and to use unit cubes to demonstrate to the students the concept of trading in place value as the numbers get larger. According to the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), understanding place value is a crucial step in student comprehension of number concepts. Our numerical system is based on four basic properties that constitute place value:

1. The value of a number is determined by its placement.
2. The values of the digits increase by powers of ten from right to left.
3. The value of a digit within a given numeral is determined by multiplying the digit by the assigned place value.
4. The value of the whole number is the sum of values assigned to each digit. (Bove 542-546.)

Sandra Bove advocates the use of a vertical number line because on the traditional line numbers increase in place value from right to left, but numbers get larger on a horizontal number line from left to right. On the vertical number line, numbers increase in value as the student moves up the line. This does not conflict with the above-mentioned four properties. The vertical number line shows visually the shifting to the left as the second digit is needed. The numbers 0 through 9 are repeated, and when necessary to show more than 9, numbers must be repeated in the same pattern with the second digit shifted to the left and beginning again with 0 on the right (Bove 544).

Research studies have shown that students who have familiarity with place value are able to grasp the concept of decimals much more rapidly (Sowder 448-453.) Money is key to surviving in our world. My students will have responsibility for running a store during our school's lunch periods. They will need to make change, set prices and make the sale to student customers. In addition, field trips will be taken to stores to buy products to sell in the Truman store, or for ingredients necessary to make foodstuffs to be sold. In these situations they will need to understand money. It is necessary for them to be able to identify coins readily, as well as their monetary value. The students will need to be able to pay with coins to ride the city bus to the grocery store. Once at the grocery, they will need to purchase quantities necessary to stock the school store. Many times the store sells candy or popsicles to the students. Some items are seasonal (popsicles) with candy forming a staple throughout the school year. Other times students will purchase items to be used at school to cook items for sale in the store. An example of this would be ingredients necessary to make rice krispie bars, an especially

welcome item for our customers.

Our monetary system is based on decimals, and to this end I want the students to understand the system in addition to being able to use it. Sowder's study showed a curriculum unit that uses base-ten materials to give students familiarity first with whole numbers and then with decimals. Initially students saw decimals as two distinct numbers: those on either side of the decimal point as seen as separate, not one number. In the study, students were shown the relationships between numbers using the base-ten blocks. Even whole numbers could be represented as decimals; for example, 60 is 60 ones and 6 is 60 tenths. This might be confusing at first, but with practice students were able to make sense of the relationship between whole numbers and decimals.

From learning about the base-ten system, and decimals and money, my students will move to looking at patterns in the world around them. Often the beauty we see in nature is the hidden order of nature. Young children search for patterns as they learn about their world. Children make great advances in their learning because of the fact that patterns are predictable. Many of the patterns found in nature are repeated, such as interlocking spirals -- two sets of spirals, one clockwise and the other counterclockwise. Examples are the yellow center of a daisy or sunflower. The observation of these spirals can be written in mathematical terms; by counting the number of spirals in each set a pattern is generated. There is a definite pattern to this sequence of numbers found in nature (Burger and Starbird 47).

A strong understanding of geometry is a necessary component of a child's mathematical foundation. Geometry is a basis to represent and describe our world. Spatial structuring is constructing in the mind an organization of parts and how they are related. A strong spatial sense allows students to formulate image-based solutions to math problems. In this information age, the ability to conceptualize and create takes on great importance, thus increasing the importance of spatial sense.

A child's introduction to geometry begins in infancy with mobiles, blocks, puzzles, and sorting toys. Most of this is without direct instruction, only observation and play. Most early childhood programs include the study of shapes. Spatial structuring plays a crucial role in geometric reasoning. To cultivate the development of geometric strategies, students should be encouraged to focus on spatial structuring. In all problems, students should first make predictions then check these predictions by using manipulatives or concrete materials (i.e., boxes). For young children, geometry is a skill of the hands, eyes and mind, and therefore instruction should focus on the manipulation of familiar objects. Unit blocks are a mainstay of early primary grades and playing nurtures children's geometric thinking. Building with blocks allows the development of ideas of size and shape, and the measurement of height, length, volume and angles. Students learn to generalize about balance, gravity and space. Geometry is the branch of mathematics that is most visible in our world. It has application in our everyday life, from figuring out patterns and quantities for a tile floor to industrial work situations. I would like to have my students use pattern blocks to explore geometric shapes. Initially we would create large triangles using a set number of shapes: triangles, rhombuses and trapezoids. Many different triangles can be made using the pattern blocks of those shapes. Next the students could be shown how to make other triangles using fractions, i.e., "create a triangle that is $\frac{4}{9}$ green, $\frac{2}{9}$ blue and $\frac{1}{3}$ red.

Mathematics Standards and Benchmarks

In developing my curriculum unit, I have relied on the Albuquerque Public School standards and benchmarks which were adopted in 2001.

Strand II deals with Number Sense and Operation. The content standard states that this will be demonstrated through problems focusing on number meanings and relationships, and place value concepts. For the middle school student (grades six - eight), the benchmarks include understanding fractions, decimals and percents, a use of variety of methods to solve problems.

Strand III is Geometry, Spatial Sense and Measurement. The content standard says the student understands concepts, properties and relationships of geometry, focusing on identifying, describing, comparing and measuring various shapes and sizes. Benchmarks for the middle school include understanding the relationship between two and three dimensional shapes.

My goals for this math unit are twofold, involving both math and social skills. Students will develop number sense, learning to read and write numbers. We will use counting and the vertical number line in developing sequencing and identifying patterns. The students will write four, five and six digit numbers and also write them in words.

When moving to money, the students will identify coins and be able to count change. Place value will help them as they work with decimals. Again we will look at identifying patterns in decimals and money. In looking at patterns, students will learn about symmetry and use this knowledge to identify shapes and see geometry in the world around them. I feel geometry is something that may not have been a large part of their math experience, and therefore I want to include it in this unit. But I also realize they need some more basic math skills before we jump into geometry. By beginning with number sense and making them comfortable in their own knowledge I hope they will be able to make the leap into more complex ideas.

I believe through the use of manipulatives and crafts my students will remember concepts important in life. Students will develop social skills through game playing and art projects. Many of these activities will be in groups. I intend to let the students work on concepts and the thinking that is involved in coming to conclusions. I hope to give them a variety of strategies and ways of looking at math so they are not without tools to solve problems, not only in math but in their lives. I want my students to see math in the world around them understand and that math is not only computation. I want them to learn to think for themselves, and I will use math to accomplish this life skill goal.

Implementation

Following are my lesson plans for this unit. I have done one lesson for each different section of my unit. In my classroom when doing math, I tend to follow Marilyn Burns' philosophy of providing students time to work with problems, searching for solutions on their own or in groups, and learning to evaluate their results. As the teacher, I try to emphasize the importance of working on the problems, not only on getting an answer. I begin the lesson explaining the problems we are working on, making sure that each student understands the problem. Then I allow the students, working in groups, ample time to explore solutions. During this time, I observe the groups, helping where needed, keeping all groups on task. Finally, we discuss each solution, summarizing the groups' work and how they reached their conclusions. Many times these explorations will last several days.

Performance standards addressed will be listed with each lesson plan, with the full APS standards written in full following this section. Assessment is generally teacher observation with group and/or individual projects.

1. The unit will begin with number sense. It will probably last for about two weeks and should be a review for most of my students. We will start by counting: 1, 2, 3, 4,

counting by ones. From here we will proceed to other counting sequences: counting by twos, 2, 4, 6, 8, ...; counting by fives, 5, 10, 15, 20, ...; counting by tens, 10, 20, 30, 40, Students will need paper and pencil. They will work in groups of two, each counting individually and writing down the sequences.

Next, the students will each make their own number line. I will utilize the vertical number line advocated by Sandra Bove, and each student will create a number line from 0 to 100. In this particular number line, students will see that after using the number nine, the next number increases by one in the tens column and the numbers in the ones column begin again with zero. We will make the number line from 0 to 100. We will use strips of white paper glued onto construction paper. In this way, the students can either tape the number line to their desk, or keep it in a folder as a tool to utilize throughout the year in math class.

Assessment for this unit will consist of teacher observation of group work, as well as completed assignments of counting sequences and the vertical number line. The purpose is for students to review basic math skills of counting, leading into our discussion of place value.

APS standards: Strand I, Strand II.

2. In discussing place value, we will begin by reading numbers. This section will run for about one month. Before we go on in this math unit, I want to be sure that they understand place value. It is an underlying tenet of our math system, and in my experience one that students do not always understand. The goals for this section are to develop an understanding of our number system by using grouping and place-value concepts and to construct number meanings through real world experiences.

I will give the students lists of numbers and we will read them as a class. Then we will break into groups of twos and each student will read the numbers to their partners. The purpose of this is to give the students experience in reading two, three and four digit numbers. With their partners they will discuss the value of each digit in the number, identifying ones, tens, hundreds, thousands. Students will make a chart, putting the digits under the correct categories:

	Thousands	Hundreds	Tens	Ones
1,390	1	3	9	0
5,028	5	0	2	8
675	0	6	7	5

They will write the numbers, separating by commas where appropriate. Students will also write the numbers using words: fifty, five hundred thousand.

In another example, we will use a chart showing place value for ones, tens, and hundreds. We will utilize unit cubes and manipulatives in this game.

Hundreds	Tens	Ones
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I am not sure where this game came from, but I first heard of it in a math class at the University of New Mexico. It was contained in a course reader compiled by my professor, Diane Torres-Velasquez. It may have come from the Cuisenaire Company. In

any event students use a "place value mat," containing the heading above. Additional materials are two dice, base ten blocks, and recording sheets. The object of the game is to reach 100 first. The first player rolls the dice and takes the amount of blocks indicated by adding together the number rolled on the dice. The players take turns. When it is the first person's turn, the dice are rolled again. The number rolled that time is added to the number rolled the time before. Again, the player takes the amount of blocks rolled. The player must now make some decisions. Blocks must be placed in the appropriate columns on the place mat. "Trades" must be made, turning in ten ones cubes for one tens block, etc... Play continues until the first person reaches 100.

The purpose of this lesson is to teach place value. Students need to know that numbers are not written in a string arbitrarily and that the place a number has in a larger number is as important as the number. This would be the perfect time to introduce some history of number systems to the students. In different ancient civilizations place value did not always have the importance that it has in our numerical system. This could also be the time to discuss different ways of writing numbers. The Roman numbers in particular are ones that even today are used. Whole sections of math classes are devoted to teaching these numbers. There are many extensions one could use in class with this particular section.

Students seem to enjoy playing the place value game, and in doing so it gives them the repetition necessary to recall place value. Materials needed for this unit on place value are paper, pencil, manipulatives, place value mat, unit and base ten cubes, and recording sheet. Assessment will be teacher observation and completion of group and individual projects.

APS Standards: Strand I, Strand II.

3. This section is on money and decimals. After working thoroughly on place value, I feel that my students will be better able to move into a successful unit on decimals and money. We will work on this unit for about one month. Incorporated into this unit will be our working at the school store during lunch periods and various field trips to the grocery, all the while working in the real world with money.

Again, we will begin with what should be review. We will write in words and numbers different combinations of money: \$.75 or 75¢ or seventy-five cents. A fun way of working on these skills is to write the amounts on small cards, much like a deck of cards. Working in pairs, students can draw from the pile of cards, and quiz each other on how to say or write whatever numbers are written on the cards. They can keep a tally and use it as a competition in who got the most correct.

We can use the place value game of racing to 100 in talking about money. Since the students should have a good grasp of how to play the game, they should be readily able to turn it into a money game. Instead of unit and base ten cubes, I will give them play money, coins and bills. This will be a race for \$1.00 (or you can extend it by making it \$10.00 or \$100.00). Students in a group will be given a bag with 60 pennies, 40 dimes and four pretend dollar bills (money may change if the amount is changed to a higher dollar). Students roll the dice and add the numbers. They take that amount of pennies. Once the students have enough to change the pennies to dimes they make the exchange, but only when it is their turn. The first person to have a \$1.00 is the winner (Burns 175, 177).

I will give the students many opportunities to make change in the classroom since this is a skill they need to work in the store. We will use play coins and bills and role play real situations they may have encountered in the store. We will also role play before we

embark on field trips to the grocery, giving each student the opportunity to be both the customer and the cashier. In this way, they should be confident to go to the grocery and buy whatever supplies we need.

Another game is "How Much Are You Worth?" Each student writes their name on a post it note. Each letter of the alphabet is assigned a number value: a=\$.01, b=\$.02, c=\$.03, etc. This is a good game to get a discussion going about money. I ask the students to calculate the worth of their name. We then build a bar graph using these amounts. Then we discuss what the students see. Whose name is worth the most? the least? Would their last name be worth more than their first? Who has the same value as your name? How much more or less than \$1.00 is your name? (Torres-Velasquez). The questions are endless. This can be the best part of math class: having the students actually talk about what they have learned. They can relate things to their world outside of class; they can begin to be excited about what they are learning.

Materials necessary for this section are paper, pencil, phony money. Assessment will be teacher observation.

APS Standards: Strand II.

4. In working with patterns and symmetry, the purpose is for the students to recognize, describe and create a variety of patterns; and also to be able to represent and describe mathematical relationships. In introducing patterns, I would first show the students that there are patterns in everything. I would do rhythmic clapping with them, something they had done in elementary school. Together we would identify the pattern of the claps. I would show them patterns in nature: in sunflowers, daisies or pineapples. Working in groups, they would identify and draw any patterns they find.

Because mathematics is somewhat intimidating to some students, I would begin with these type of exercises and then move the students into seeing patterns in numbers. One way to do this is by using *The Eyes Pattern*. Begin by having one student stand. Ask the class how many eyes he/she has. Record the data to a T-chart. Have a second student stand and ask the same question. Record the data. Continue in this manner asking the student and recording the data. Then discuss with the class all the patterns you can observe. The t-chart should be similar to this:

Students	Eyes
1	2
2	4
3	6
4	8
5	10

The students will record individually or in groups the data as we continue our question and answer. They should be able to recognize a pattern; and if not, we will continue with imaginary students. The discussion that follows will involve all the patterns we have observed, both in nature (flowers, fruits) and in mathematics. T-charts can be used to measure other patterns: students and noses or thumbs; cars and wheels; amount of money to dimes, etc. Again, the discussion that follows is the teaching moment, able to

excite them to seek out more (Burns 115).

The materials necessary are paper, pencil, and anything in nature to show patterns (sunflower, daisy, pineapple). I would finish this unit within a week. Assessment would be teacher observation and group work.

APS Standards: Strand V.

5. The last section of my unit is geometry. Following patterns and symmetry, I will begin to introduce my students to different shapes. We will discuss shapes in nature, in the classroom, in their home, and all around us. We will attempt to classify and identify any shapes they may see. In this unit I will use pattern blocks. From this beginning of geometry, there are many ways to go. We can begin to explore fractions, area, or perimeter. For my unit we will concentrate on shapes and exploring relationships between them, which may involve a discussion of fractions. This unit will last three to four weeks. Students may not have much previous experience; this may entail more detailed explanations and more teaching moments.

In the beginning, I will allow the students to experiment with the blocks - building shapes, making patterns, and exploring color combinations. After they have built a shape, I will ask them to write in their journal about what they have created. I will give them criteria to work with such as shapes and colors. We can then discuss patterns and/or geometric shapes.

Next, we will begin to study geometric shapes through studying the beadwork of native peoples. Since we are in the southwest and have many examples from the Native American pueblos, I will ask the students to choose and examine a beadwork design and write what they can observe. As a class we can then observe all the designs and discuss what we see. I will try to direct the discussion to using mathematical terms, seeing the designs in terms of geometric shapes. We will identify all shapes used. Students will then be directed to design their own beadwork using shapes we identified and other shapes from the pattern blocks (Barkley and Cruz 362). My students in 7th grade study the culture of New Mexico and this activity will allow integration between social science and math.

The next part of this unit is using geometry to show fractions. We will use Laura Cameron's pattern blocks to create shapes and show portions of the whole. The students have experimented with building shapes. We will now use that knowledge to show them that they can create the same shapes (i.e., a triangle) but using different parts (triangle, rhombus and trapezoids) in different quantities and configurations. We will begin by instructing the students to make a triangle using pattern blocks of the following configuration: three triangles, two rhombuses and three trapezoids. Then we will examine the figures and discover how to relate the figures to each other, there are 16 triangles in the large constructed triangles. Two triangles cover the same space as a rhombus and three triangles cover the same space as a trapezoid. Therefore, the small triangles are $\frac{3}{16}$ of the whole, the rhombuses are $\frac{1}{4}$ and the trapezoids are $\frac{9}{16}$. From this knowledge, the students can construct more triangles using particular colors and shapes. For example, form a triangle that is $\frac{1}{2}$ blue and $\frac{1}{2}$ green. This is moving the students into fractions using geometric shapes and properties. Once the students understand how the shapes relate to each other in terms of building shapes of particular sizes, they can create many more triangles using their own design. From this point, it would be relatively easy to move the students into a study of fractions.

My semester unit would be finished at this point, but I like to have a culminating activity. I was able to find an art project making string puppets utilizing geometric

shapes. Students will cut a big square for the body, two short rectangles for the arms and two long rectangles for the legs. Cut two small circles for the hands and two small trapezoids for the feet. Make a large circle for the head. Use pieces of string or yarn to connect the parts of the body together after punching holes in each piece. Each shape must be at least one inch wide in order to more easily connect together. Finish the puppet by drawing a face and be creative by adding details such as hair or clothes. String or yarn can connect the head and arms to a tube so that the figure can be jiggled and then watch it dance (*The Math & Literature Connection, Level C. 12*).

Documentation

Albuquerque Public Schools has drafted *Mathematics Standards, Benchmarks & Performance Standards 2001*. Following are the standards that I have addressed in the previous curriculum unit.

Strand I: Global Mathematical Process.

Content Standard: The student understands and uses mathematical processes.

Benchmarks: K-12. The student uses problem solving, reasoning and proof, communication, connections, and representations as appropriate in all mathematical experiences.

Strand II: Number Sense and Operations.

Content Standard: The student demonstrates number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

Benchmarks: 6-8. The student understands problems involving fractions, decimals, and percents and develops, analyzes, and explains a variety of algorithms and methods to solve problems.

Strand III: Geometry, Spatial Sense, and Measurement.

Content Standard: The student demonstrates an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems that focus on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and sizes.

Benchmarks: 6-8. The student understands the relationships between two- and three-dimensional shapes and identifies, builds, and transforms shapes. The student uses inductive and deductive arguments to solve problems. The student uses metric and U.S. measurement systems and selects the appropriate measurement unit for a given situation.

Strand V: Patterns, Functions, and Algebraic Concepts.

Content Standard: The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

Benchmarks: 6-8. The student uses tables, graphs, and symbolic representations of patterns. The student understands and uses variables and linear equations in algebraic problem solving.

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