Deductive Proof in Geometry

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Academic Setting

Cibola High School Environment

Cibola High School serves a large and diverse student population. The student body is mostly made up of children from middle class families. The student body, although mostly Anglo, includes a large Hispanic population, as well as African Americans, Native Americans, Asian Americans and a variety of newly arrived immigrants. The school has about 2,400 students and has recently created a Freshman Academy to help the new students adjust to the demands of High School and to minimize first year dropouts. The 2001-2002 school year will be the second year of the freshman academy.

The Freshman Academy consists of five core teacher teams. The core subjects that all freshmen take are English (honors or regular), health, technology (computers), biology and math (pre-algebra, algebra 1, honors algebra 1 and honors geometry). Each team of core subject teachers has the same group of students; that is, every student in a particular team has the same set of core subject teachers. This allows the teachers and students to form a closer student-teacher relationship and allows more individual interaction with students. The Cibola administration has also obtained grant money to reduce the student teacher ratio, in the Academy, to about 25 students per teacher per class. For the typical math class this would average about ten fewer students per class. Normally a teacher with five classes would be assigned more than 160 students, but in the Freshman Academy each teacher has about 100-120 students.

I have emphasized the role of the Freshman Academy because this curriculum unit will be used for my freshman Honors Geometry class. The prerequisite for Honors Geometry is a grade of B or better in Honors Algebra or a grade of 80 or better on the Honors Algebra Final.

Proposed Curriculum Unit

Ideally a math class should include information and experiences that will help the student appreciate the relationship of mathematics to their lives. If possible the development of the mathematical ideas should be presented with as much social and historical context as possible. For example, we might discuss how ancient Egyptians could have calculated an accurate value for pi. Mathematics is also an excellent vehicle for developing critical thinking skills. For most of the centuries since Euclid and Aristotle, geometrical proofs have been used to teach formal logical thinking. Indeed, I believe that the geometry class is the only place where the average high school student will be introduced to the concept of formal logical thought. Geometry, in particular, provides many opportunities to teach students how to reason mathematically. In geometry mathematical reasoning focuses on making conjectures and then either proving or disproving them. Traditionally conjectures for student proofs come from Euclid's postulates, however my Honors Geometry students will also be asked to make geometrical conjectures prior to the presentation of the formal Euclidian concept.

Teaching students to make mathematical conjectures requires a spirit of exploration and experimentation in the classroom. Students will be asked to make statements that they believe are true about a geometrical situation. Their suppositions can be made via inductive reasoning, that is, looking at special cases, observing patterns in the special cases and then generalizing from the pattern to make a conjecture. Once the conjecture is stated the students then have a problem to solve. The conjecture needs to be proved by deductive logic or disproved by counterexample. Developing the student's ability to reason correctly, experiment with new ideas, and to solve problems is our fundamental goal. Essentially we want the students to learn how to reason logically as a foundation for more advanced math classes and to improve their ability to think critically about the whole array of life experiences.

This curriculum unit will focus on the use of deductive reasoning in the form of "if-then" statements and proofs. The unit will be presented during the second week of the first semester, and the methods will be applied for the duration of the course. The lesson plans for the unit will focus on deductive reasoning. The objectives of the unit include teaching students to:

1. recognize the hypothesis and conclusion of an if-then statement,
2. use a counterexample to disprove an if-then statement,
3. use properties from algebra in proofs,
4. recognize the kinds of reasons that can be used in proofs,
5. plans proofs and write them in two-column form.

The background material for the unit will examine the nature of deductive proof in some detail as well as disproof by counter-example. We will also look at developing a sound conjecture by inductive reasoning. Finally we will provide some historical context for the development of geometry and logic and why those ancient disciplines are relevant today.

Background

Euclid's Geometry

Euclid was born around 365 BC in Alexandria, Egypt, and died around 300 BC. We know little of his life other than that he taught mathematics in Alexandria. Euclid wrote a number of treatises, but the most famous is his *Elements*. Euclid worked with other mathematicians and also built on works done in the past. Rather than representing Euclid's original work, the *Elements* are now thought to be a summary of the geometrical knowledge current during his time. However, they represent one of the earliest uses of proof in the history of mathematics. The book was a compilation of knowledge that became the center of mathematical teaching for 2000 years.

The Elements is divided into 13 books. Books one through six deal with plane geometry. Books one and two set out basic properties of triangles, parallel lines, parallelograms, rectangles and squares. Book three examines the properties of the circle while book four deals with problems about circles. Heath suggests that book four lays out the work of the followers of Pythagoras. Book five lays out the work of Eudoxus on proportion. Heath says, "Greek mathematics can boast no finer discovery than this theory, which put on a sound footing so much of geometry as depended on the use of proportion." (Heath 1931).

Book six looks at applications of the results of book five to plane geometry. Euclid's Elements is remarkable for the clarity with which the theorems are stated and proved. The standard of rigor set the bar for Newton and Liebnitz, for the inventors of the calculus centuries later. As Heath writes:

> Almost from the time of its writing and lasting almost to the present, the Elements has exerted a continuous and major influence on human affairs. It was the primary source of

geometric reasoning, theorems, and methods at least until the advent of non-Euclidean geometry in the 19th century. It is sometimes said that, next to *The Bible*, the *Elements* may be the most translated, published, and studied of all the books produced in the Western world.(Heath 1931).

The Elements begins with 23 definitions and five postulates. The first three postulates are postulates of construction; for example the first postulate states that it is possible to draw a straight line between any two points. These postulates assume the existence of points, lines and circles; the existence of other geometric objects are deduced from the fact that these postulates exist. There are other assumptions in the postulates that are not explicit. For example it is assumed that there is a unique line joining any two points.

The fourth and fifth postulates are of a different nature. Postulate four states that all right angles are equal. This may seem obvious, but it actually assumes that space is the same in all directions - by this we mean that a figure will be independent of the position in space in which it is placed. This idea is central to most spatial concepts in physics. When I taught physics, we spent a great deal of time examining the idea of a frame of reference, and the idea that a problem could be properly viewed in any one of many possible reference frames. The fifth, or parallel, postulate states that one and only one line can be drawn through a point parallel to a given line. It was not until the 19th century that this postulate was determined to be valid only for plane flat surfaces. For surfaces that were not flat, non-Euclidian geometry was developed in the 19^{th} century.

There are also axioms which Euclid calls "common notions." These are not specific geometrical properties but rather general assumptions that allow mathematics to proceed as a deductive science. For example: "Things which are equal to the same thing are equal to each other."

In his Elements, Euclid begins with a list of twenty-three definitions describing things like points, lines, plane surfaces, circles, obtuse and acute angles and so on. Euclid's definitions are neither true nor false: they simply act as a kind of dictionary, explaining what is meant by the various terms he will be using. He then presents a set of ten assumptions. Five of these are not specific to geometry, and he calls them common notions:

1. Things, which are equal to the same thing, are also equal to one another.
2. If equals were added to equals, the wholes are equal.
3. If equals were subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

The other five assumptions are specifically geometric, and he calls them postulates:

1. It is possible to draw a straight line from any point to any point.
2. It is possible to produce a finite straight line continuously in a straight line.
3. It is possible to describe a circle with any center and distance.
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Together, these common notions and postulates represent the axioms of Euclid's geometry. An axiom is a logical principle which is assumed to be true rather than proven, and which can be used as a premise in a deductive argument. Euclid's set of axioms, or axiomatic system, represents a collection of "first principles" from which other principles can be produced using deductive reasoning. Of course, any deductive arguments are only sound if Euclid's common notions and postulates really are true. Euclid goes on in his Elements to present various geometric propositions, and shows them to be true using deductive reasoning within his axiomatic system (Heath 1956).

Many attempts to prove the fifth postulate in this manner were made, and often a proposed proof would be accepted for a long period before being shown to be flawed. Typically, the flawed proofs contained a "circular argument:" in one way or another, they assumed that what they were trying to prove (the fifth postulate) was true in order to prove it. In fact, the fifth postulate is not derivable from the other postulates and notions, and nor is it universally true. The fifth postulate can be shown to be true in a plane (or Euclidian) geometry. However, there are many other geometries where it is not true. Surprisingly enough, this is easy to illustrate. Consider the simple case of a sphere's surface. It is impossible to draw a true straight line on a sphere without leaving the surface, So in spherical geometry the Euclidean idea of a line becomes a great circle. Thinking of the Earth, any line of longitude is a great circle - as is the equator. In fact the shortest path between any two points on a sphere is a great circle.

One of the consequences of Euclid's first four postulates is that if two different lines cross, they meet at a single point. This presents a small problem on the sphere, since distinct great circles always cross at two opposite points. Two lines of longitude always cross at both the North and the South Pole. According to Euclid's definition number 23, "Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction." Given these definitions, it is easy to see that Euclid's first four postulates still make good sense. The fifth postulate, however, fails because it is impossible to draw two different lines that do not meet. In spherical geometry there are *no* parallel lines.

Euclid's Definitions:

1. A point is that which has no part.
2. A line is breadthless length.
3. The extremities of a line are points.
4. A straight line is a line which lies evenly with the points on itself.
5. A surface is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A plane surface is a surface that lies evenly with the straight lines on itself.
8. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line:
9. And when the lines containing the angle are straight, the angle is called rectilinear.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.
11. An obtuse angle is an angle greater than a right angle.
12. An acute angle is an angle less than a right angle.
13. A boundary is that which is an extremity of anything.
14. A figure is that which is contained by any boundary or boundaries.
15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;
16. And the point is called the center of the circle.
17. A diameter of the circle is any straight line drawn through the center and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.

18. A semicircle is the figure contained by the diameter and the circumference cut off by it; and the center of the semicircle is the same as that of the circle.
19. Rectilinear figures are those which are contained by straight lines, trilateral figures being those contained by three, quadrilateral those contained by four, and multilateral those contained by more than four straight lines.
20. Of trilateral figures, an equilateral triangle is that which has its three sides equal, an isosceles triangle that which has two of its sides alone equal, and a scalene triangle that which has its three sides unequal.
21. Further, of trilateral figures, a right-angled triangle is that which has a right angle, an obtuse-angled triangle that which has an obtuse angle, and an acute-angled triangle that which has its three angles acute.
22. Of quadrilateral figures, a square is that which is both equilateral and right-angled; an oblong that which is right-angled but not equilateral; a rhombus that which is equilateral but not right-angled; and a rhomboid that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these are called trapezoids.
23. Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction. (Heath 1956)

Deductive Reasoning

If we take a set of facts that are known or assumed to be true, deductive reasoning is a powerful way of extending that set of facts. In deductive reasoning, we say (argue) that if certain premises are known or assumed, a conclusion necessarily follows from these. For example, if we are given the following premises: A) All men are mortal, B) and Socrates is a man then C) the conclusion Socrates is mortal follows from deductive reasoning. In this case, the deductive step is based on the logical principle that "if A implies B, and A is true, then B is true." Medieval philosophers called this principal "modus ponens" (Macaphee). Of course, deductive reasoning is not infallible: the premises may not be true, or the line of reasoning itself may be wrong (This is an example of an "if-then" statement). They are also referred to as conditional statements or simply conditionals. For example lets see if we can prove that 1 = 2:

Let $a = b$
Then
$a^2 = ab$
$a^2 + a^2 = a^2 + ab$
$2a^2 = a^2 + ab$
$2a^2 – 2ab = a^2 + ab - 2ab$
$2a^2 – 2ab = a^2 – ab$
$2(a^2 – ab) = 1(a^2 – ab)$
divide both sides by $(a^2 – ab)$ and
2 = 1 QED

I have used this example in my algebra classes and ask the students to find the error (division by zero in the last step). Now, if a conclusion doesn't follow from its premises, the argument is said to be invalid and no reliable judgement can be made about whether the conclusion is true, regardless of whether or not the premises are true. Using the example above, a conclusion that "Socrates was bald" would be invalid because the premises of the argument do not support the conclusion.

If the argument is valid but the premises are not true, then again the conclusion may or may not be true, but the argument can't help us decide this.

Finally, if the argument is valid and the premises are true, then the argument is described as sound, and we deem the conclusion to be true. From a pragmatic point of view, we can be said to have proved something if we can find a sound argument for it.

Table 1 summarizes these different kinds of deductive arguments, and an example of each is shown below.

	C doesn't follow	C follows
P untrue	Invalid	Valid, unsound
P true	Invalid	Valid, sound

Table 1

Examples of different kinds of deductive arguments.

Invalid, false premises:

P: Fish are mammals.
P: Fish are warm-blooded.
C: Mammals are warm-blooded.

Valid, unsound premises:

P: Mammals are cold-blooded.
P: Humans are mammals.
C: Humans are cold-blooded.

Invalid, true premises:

P: Fish are cold-blooded.
P: Humans are not fish.
C: Humans are warm-blooded.

Valid, sound:

P: Humans are warm-blooded.
P: Fishermen are human.
C: Fishermen are warm-blooded.

As the two invalid arguments illustrate, the conclusion of an invalid argument doesn't necessarily have to be false - it's just unproven by that particular argument. (http://pass.maths.org.uk/issue7/features/proof1/index.html)

The first lesson of the unit will examine if-then statements. The Second lesson will review properties from algebra. We want to emphasize that geometry involves real numbers and we want to keep the students algebra skills sharp with the use of problems that require the use of algebraic techniques. Properties from algebra are:

Addition Property	If a = b and c = d, then a + b = c + d.
Subtraction Property	If a = b and c = d, then a – b = c – d.
Multiplication Property	If a = b, then ca = cb.
Division property	If a = b and b is not zero, then a/c = b/c.
Substitution Property	If a = b then either a or b may be substituted for the other in any equation.
Reflexive Property	a = a.
Symmetric Property	If a = b, then b = a.
Transitive Property	If a = b and b = c, then a = c.
Distributive Property	a(b + c) = ab + ac.

Lets look at solving a simple algebra problem using a two column proof approach:

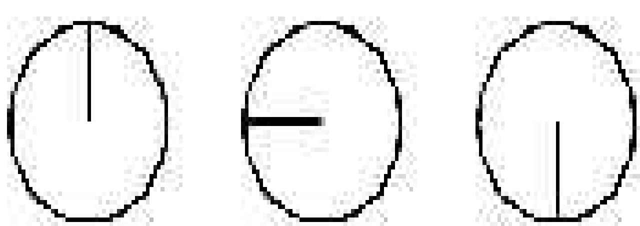
Solve 4x = 8 – 2x

Steps	Reason
4x = 8 – 2x	Given
2x = 4 - x	Division property of equality
3x = 4	Addition Property of equality
x = 4/3	Division property of equality

Geometric proof is deductive reasoning at work. Throughout a deductive proof, the statements that are made are specific examples of more general situations, as is explained in the "reasons" column. For example, you might state in a proof that two angles add to 90 degrees, and in the next line, state that they are complementary. In the reasons, column, you would write, "Two angles whose sum is 90 degrees are complementary." Think of this reason as a conditional statement: "if p, then q, " where p is "angles sum to 90 degrees," and q is "they are complementary." The first statement, that the sum of the angles is 90 degrees, is the hypothesis of the conditional statement. If the conditional statement is true, which we know, then q, the next statement in the proof, must also be true. In this example, the conditional statement is a general statement about angles, and using it, it is deduced that a specific pair of angles is complementary. In this way, a proof makes use of deductive reasoning.

Writing a proof consists of a few different steps.

1. Draw the figure that illustrates what is to be proved. The figure may already be drawn for you, or you may have to draw it yourself.
2. List the given statements, and then list the conclusion to be proved. Now you have a beginning and an end to the proof.
3. Mark the figure according to what you can deduce about it from the information given. This is the step of the proof in which you actually find out how the proof is to be made, and whether or not you are able to prove what is asked. Congruent sides, angles, etc. should all be marked so that you can see for yourself what must be written in the proof to convince the reader that you are right in your conclusion.
4. Write the steps down carefully, without skipping even the simplest one. Some of the first steps are often the given statements (but not always), and the last step is the conclusion that you set out to prove. A sample proof looks like this:



Given: **BX** is the bisector of ABC.

Prove: m ABX = ½ m ABC; m XBC = ½m ABC

Steps	Reason
BX is the bisector of ∠ABC	Given
m∠ABX + m∠ XBC = m∠ABC	Angle addition postulate
m∠ABX = m∠XBC	Definition of bisector of an angle
m∠ABX + m∠ABX = 2 mvABC; m∠ XBC = m∠XBC =2 m∠ABC	Addition property
m∠ABX = ½ m∠ABC; m∠XBC = ½ m∠ABC	Division property

Deductive Reasoning Unit Lesson Plans

This unit will focus on the use of deductive reasoning in the form of "if-then" statements and proofs. The objectives of the unit include teaching students to:

1. recognize the hypothesis and conclusion of an if-then statement,
2. use a counterexample to disprove an if-then statement,
3. use properties from algebra in proofs,
4. recognize the kinds of reasons that can be used in proofs,
5. plan proofs and write them in two-column form.

Lesson plans are based on the presentations from the text Geometry (Jurgensen, 2000).

Lesson 1: If-then Statements; Converses

Objectives: Recognize the hypothesis and conclusion of an if-then statement; state the converse of an if-then statement; use a counterexample to disprove an if-then statement.

Standards: New Mexico Content Standard 3: Unifying Concepts and Processes-Students will understand and use mathematics in reasoning and develop and test conjectures and mathematical arguments, construct and evaluate logical arguments, formulate counter examples to understand mathematical reasoning.

The statements, "if it is sunny, then I will jog," or, "if the point B is between the points A and C on the line **AC**, then **AB** + **BC** = **AC,**" are examples of **if-then statements**, also known as conditional statements. An if-then statement can be written as "if p, then q," where p is the **hypothesis** and q is the **conclusion**. The **converse** of a conditional statement is formed by interchanging the hypothesis and conclusion. For example, "if I jog, then it will be sunny."

An if-then statement is false if any example can be found where the hypothesis is true but the conclusion is false. So if we can find any **counterexample** the if-then statement is false. In math, finding only one counterexample of an if-then statement is sufficient to disprove it. Suppose I argue, "if my dogs are gray, then all dogs are gray." Then when I take my gray dogs for a jog and I see a yellow dog, I will have found a counterexample to my argument.

Reinforcement: Geometry, p 35 #1-35, odd.
Assessment: Written test at the end of the unit.

Lesson 2: Properties from Algebra

Objective: Use properties from algebra and properties of congruence in proofs.

Standards: Content Standard 12: Functions and Algebra Concepts-Students will understand and use patterns and functions.

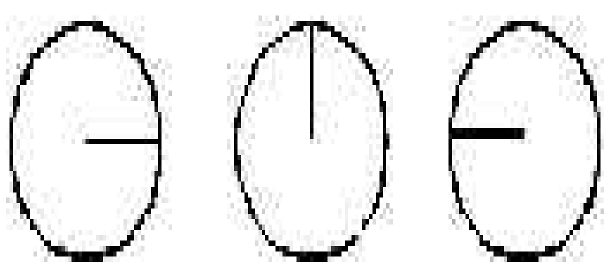
We will use algebraic concepts and facts throughout the study of geometry. We will present the following properties for the class:

Properties of Equality	
Addition Property	If a = b and c = d, then a + b = c + d.
Subtraction Property	If a = b and c = d, then a – b = c – d.
Multiplication Property	If a = b, then ca = cb.
Division property	If a = b and b not zero then a/c = b/c.
Substitution Property	If a = b, then either a or b may be substituted for the other in any equation.
Reflexive Property	a = a.
Symmetric Property	If a = b, then b = a.
Transitive Property	If a = b and b = c, then a = c.
Distributive Property	a(b + c) = ab + ac.
Properties of Congruence	
Reflexive Property	**DE** @ **DE**
Symmetric Property	if **DE** @ **FG**, then **FG** @ **DE**
Transitive Property	if **DE** @ **FG** and **FG** @ **JK**, then **DE** @ **JK**.

Example 1: Solve 4x = 8 – 2x

Steps	Reason
4x = 8 – 2x	Given
2x = 4 – x	Division property of equality
3x = 4	Addition Property of equality
x = 4/3	Division property of equality

Example 2: given that **RT** and **PQ** intersect at S such that RS = PS and ST = SQ, prove that PT = PQ.



Steps	Reason

RS = PS; ST = SQ	Given
RS + ST = PS + PQ	Addition Property of equality
RS + ST = RT; PS + SQ = PQ	Segment addition postulate
RT = PQ	Substitution proposition

Reinforcement: p41-#s 1,4,7,8,11,14.
Assessment: Written test at the end of the unit.

Lesson 3-Proving Theorems

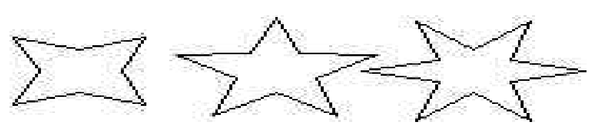
Objective: Recognize the kinds of reasons that can be used in proofs.

Standards: New Mexico Content Standard 3: Unifying Concepts and Processes-Students will understand and use mathematics in reasoning and develop and test conjectures and mathematical arguments, construct and evaluate logical arguments, formulate counter examples to understand mathematical reasoning.

Content Standard 8: Geometry and Measurement Concepts-Students will have a foundation in geometric concepts. Classify figures in terms of congruence and similarity and apply these relationships. Deduce properties of and relationships between figures from given assumptions.

Suppose that M is the midpoint of the line **AB**, and that **AB** = 12, then it appears obvious that **AM** = 6. We could generalize and make a conjecture that the line segment on either side of the mid point is one half of the total length of the line. In fact, this is the midpoint theorem and is stated mathematically as:
"If M is the midpoint of **AB**, then **AM** = ½ **AB,** and **MB** = ½ **AB**."

OK, lets prove it.

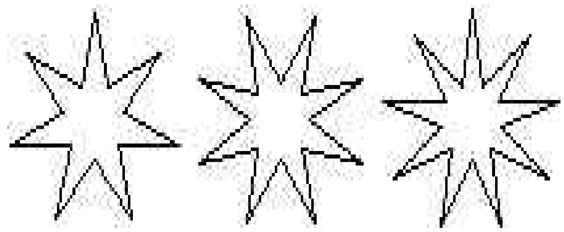


Given: M is the midpoint of **AB.**

Prove: AM = ½ AB ; MB = ½ AB

Steps	Reason
M is the midpoint of **AB**	Given
AM @ **MB** or AM = MB	Definition of a midpoint
AM + MB = AB	Segment addition postulate
AM + AM =AB, or 2AM = AB	Substitution proposition
AM = ½ AB	Division property
MB = ½ AB	Division property

Now let’s try to prove the Angle Bisector Theorem. If **BX** is the bisector of angle ABC, then the measure of angle ABX is one half the measure of angle ABC and the measure of angle XBC one half of the angle ABC.



Given: **BX** is the bisector of ∠ABC.

Prove: m∠ ABX = ½ m∠ ABC; m∠ XBC = ½m ∠ABC

Steps	Reason
BX is the bisector of ∠ ABC	Given
m∠ ABX + m∠ XBC = m∠ ABC	Angle addition postulate
m∠ ABX = m∠ XBC	Definition of bisector of an angle
m∠ ABX + m∠ ABX = 2 m∠ ABC; m ∠ XBC = m∠ XBC =2 m∠ ABC	Addition property
m∠ ABX = ½ m∠ ABC; m∠ XBC = ½ m∠ ABC	Division property

To summarize, the reasons used in proofs include given information, definitions, postulates and theorems that have already been proved.

Reinforcement: pp46-47, #s 1-8, 13, 16,18, 21,22.
Assessment: Written test at the end of the unit.

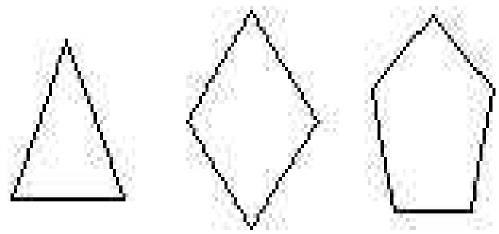
Lesson 4-Theorems About Angles and Perpendicular Lines

Objectives: Apply the definitions of complementary and supplementary angles,
State and use theorems about vertical angles.
State and apply theorems about supplementary and congruent angles,
Plan proofs and write them in two column form.

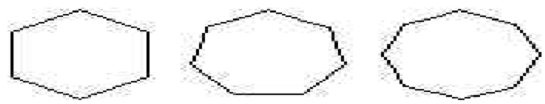
Standards: New Mexico Content Standard 3: Unifying Concepts and Processes-Students will understand and use mathematics in reasoning and develop and test conjectures and mathematical arguments, construct and evaluate logical arguments, formulate counter examples to understand mathematical reasoning.

Content Standard 8: Geometry and Measurement Concepts-Students will have a foundation in geometric concepts. Classify figures in terms of congruence and similarity and apply these relationships. Deduce properties of and relationships between figures from given assumptions.

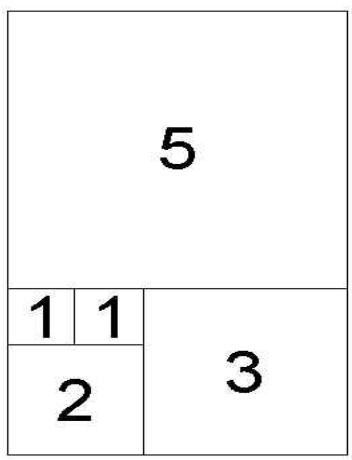
Complementary Angles are two angles whose measures have the sum of 90. Each angle is the complement of the other.

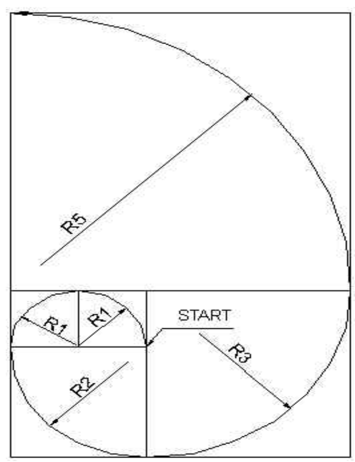


Supplementary angles are two angles whose measures have the sum of 180.



Vertical angles are two angles such that the sides of one angle are opposite rays to the sides of the other angle. When two lines intersect they form two pairs of vertical angles.





Theorem: Vertical angles are congruent.

Given: ∠1 and 2 are vertical angles.

Prove: ∠ 1 = (approx.) ∠ 2

Proof:

Statement	Reason
m∠ 1 + m∠ 3 = 180; m∠ 2 + m∠ 3 = 180	Angle addition postulate
m∠1 + m∠ 3 = m∠ 2 + m∠3	Substitution Property

m∠3 = m∠ 3	Reflexive property
m∠1 = m∠2 or ∠1 = (approx) ∠2	Subtraction property

Reinforcement: pp 52-54, # 1-8, 17,18, 22,23,28-31.

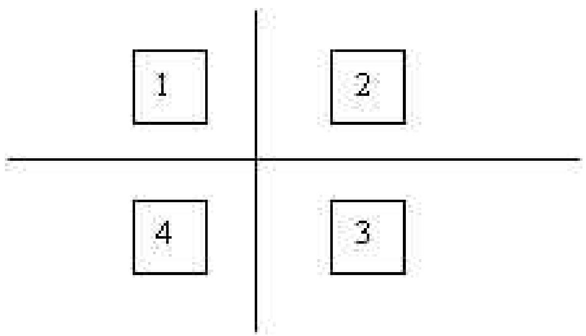
Assessment: Written test at the end of the unit

Lesson 5-Perpendicular Lines

Objective: Apply the definition and theorems about perpendicular lines,

Plan proofs and write them in two column form.

Perpendicular lines are two lines that intersect to form right angles. Since lines that form one right angle always form four right angles, by definition, you can conclude that two lines are perpendicular if any one of the angles is a right angle.



Theorem: If two lines are perpendicular, then they form congruent adjacent angles.

Given: l ⊥ n

Prove: ∠1, ∠2, ∠3, and ∠4 are congruent

Statement	Reason
l ⊥ n	Given
∠1, ∠2, ∠3, and ∠4 are 90º angles	Definition of ^ lines
∠1, ∠2, ∠3, and ∠4 are congruent	Definition of congruent angles

Theorem: If two lines form congruent adjacent angles, then they are perpendicular.

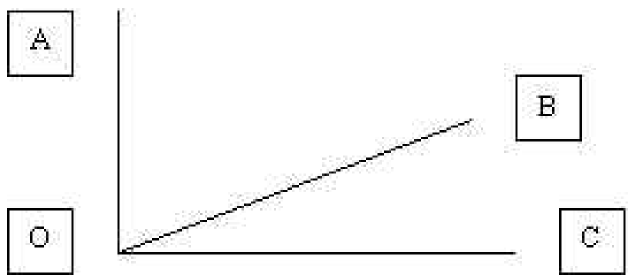
Given: ∠1 = (approx.) ∠2

Prove: l n

Statement	Reason

∠1 = (approx.) ∠2, or m∠1 = m∠2	Given
m∠1 + m∠2 = 180	Angle addition postulate
m∠2 + m∠2 = 180, or 2m∠2 = 180	Substitution postulate
m∠2 = 90	Division property
l ⊥ n	Definition of ⊥ lines

Theorem: If the exterior sides of two adjacent acute angles are perpendicular, then the angles are complementary.



Given: **OA** ⊥ **OC**

Prove: ∠AOB and ∠BOC are complementary

Statement	Reason
OA ^ OC	Given
m∠AOC = 90	Definition of ⊥ lines
m∠OAC + m∠BOC = m∠AOC	Angle addition postulate
m∠OAC + m∠BOC = 90	Substitution proposition
∠AOB and ∠BOC are complementary	Definition of complementary angles

Reinforcement: p58-60, # 2, 5-8, 11-12, 16-21, 24-28.

Assessment: Written test at the end of the unit.

Lesson 6: Planning a Proof

Objective: Recognize the kinds of reasons that can be used in proofs,

plan proofs and write them in two-column form.

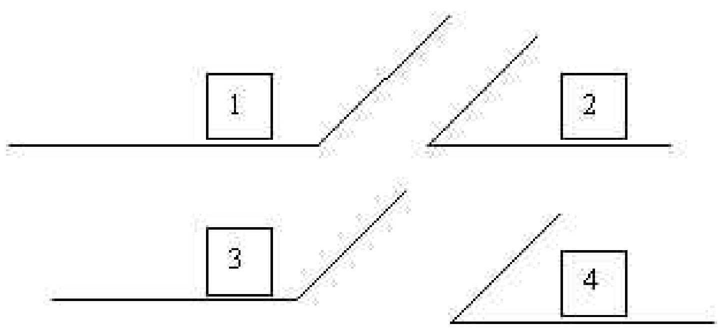
Standards: New Mexico Content Standard 3: Unifying Concepts and Processes-Students will understand and use mathematics in reasoning and develop and test conjectures and mathematical arguments, construct and evaluate logical arguments, formulate counter examples to understand mathematical reasoning.

Content Standard 8: Geometry and Measurement Concepts-Students will have a foundation in geometric concepts. Classify figures in terms of congruence and similarity and apply these relationships. Deduce properties of and relationships between figures from given assumptions.

Normally a proof of a theorem consists of five parts:

1. Statement of the theorem,
2. A diagram of the given information,
3. A list, in terms of the figure, of what is given,
4. A list, in terms of the figure, of what is to be shown,
5. A series of statements and reasons that lead from the given information to the statement is to be proved.

Theorem: If two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.



Given: ∠ 1 and ∠ 2 are supplementary;

∠ 3 and ∠4 are supplementary;

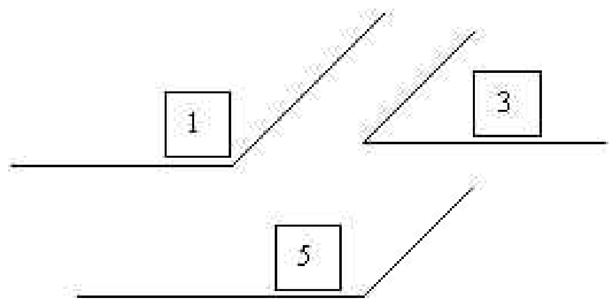
∠ 2 = (approx.) ∠ 4 .

Prove: ∠ 1 = (approx.) ∠ 3

Statement	Reason

∠ 1 and ∠ 2 are supplementary ∠ 3 and ∠ 4 are supplementary	Given
m∠ 1 + m∠ 2 = 180 m∠ 3 + m∠ 4 = 180	Definition of supplementary angle
m∠ 1 + m∠ 2 = m∠ 3 + m∠ 4	Substitution property
∠2 = (approx.) ∠ 4	Given
m∠ 1 = m∠ 3 , or ∠ 1 = (approx) ∠ 3	Subtraction property

Theorem: If two angles are complements of congruent angles (or the same angle), then the two angles are congruent.



Given: ∠ 1 and ∠ 5 are supplementary

∠ 3 and ∠ 5 are supplementary

Prove: ∠ 1 = approx. ∠ 3

Statement	Reason
∠ 1 and ∠5 are supplementary ∠ 3 and ∠5 are supplementary	Given

$m\angle 1 + m\angle 5 = 180$ $m\angle 3 + m\angle 5 = 180$	Definition of supplementary angles
$m\angle 1 + m\angle 5 = m\angle 3 + m\angle 5$	Substitution property
$m\angle 5 = m\angle 5$	Reflexive property
$m\angle 1 = m\angle 3$ or $\angle 1$ = approx.$\angle 3$	Subtraction property

Reinforcement: pp 63-65, #1-15, 17, 18, 21, 23.

Assessment: Written test at the end of the unit.

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