Patterns, Math and Thought

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Academic Setting

Harrison Middle School

Though things are changing for the better, Harrison Middle School currently has the reputation—largely unmerited—of being a tough school. Much of that reputation is due to its location in Albuquerque's rather maligned, rural south valley. The school itself is bordered by homes with chickens and horses, a park, and the Rio Grande. The valley, despite its pastoral aspect and the many generations who have lived there peacefully, is renowned for poverty and crime. The school serves a predominately poor, Hispanic, somewhat transient population. A majority of students qualify for free and reduced lunches, and many will not go on to graduate from high school. According to recent statistics, the student population is 81.6% Hispanic, 14.2% Anglo, 2.4% Native American, 1.3% African American, 0.3% Asian and 0.1 % "other." 74.4% of the students qualify for the free or reduced lunch program. Currently, due to low test scores, the school has been targeted by the state as a "school in need of improvement," as have all the middle schools in the south valley, and is on probation.

Class for Which the Unit is Designed

This unit is designed for use in a sixth grade bilingual math class. The classes will be taught in 90-minute blocks, every other day. Typically, students entering sixth grade at Harrison vary greatly in mathematical background; while some have already mastered addition and subtraction of fractions, others have barely learned to multiply. Similarly, students' proficiency in academic language skills in both English and Spanish tends to cover the spectrum of levels. While language may not seem to affect success as much in a math class, since numbers are universal, Harrison has adopted a math program, Math In Context (MIC) that is very heavily literacy based.

Goals and Objectives

Most broadly, the goal of this unit is to foster a positive attitude toward math in general and the MIC program in particular (by supplementing the program with material more geared toward my students' needs). I want my students to realize, through learning about patterns, that math is part of daily life and, perhaps more significantly, that it is not beyond their grasp.

More pointedly, the purpose of the unit is to lay the foundation for the study of algebra. To that end, the objectives of the unit are that students will be able to:

- identify and describe various types of patterns.
- predict pattern extensions.
- generalize about patterns.
- use a variety of problem solving skills.
- explain strategies used in problem solving.

Context and Background

Rationale

"The more abstract the truth you want to teach, the more thoroughly you must seduce the senses to accept it." --Friedrich Nietzsche

According to many, including the editor of *Pattern in the Teaching and Learning of Mathematics*, math *is* essentially the study of patterns, of finding order within chaos, and, therefore, "no justification is needed for making pattern the unifying theme of a collection of specialized studies in children’s learning". For those who might be skeptical of that view, however, there are many defensible reasons for studying patterns.

Probably the least compelling, yet not insignificant, rationale for a unit on patterns is that it is one of the required sixth grade mathematics standards and benchmarks, as well as one of the sixth grade MIC units. According to the MIC schedule in use at Harrison, the unit entitled Patterns and Symbols is also the first one to be covered and, therefore, influences students’ attitudes toward the program (and math in general) for the remainder of the year. Last year, by the time we were ready to move to the second unit, my students were already complaining about the "boring" books and were begging for worksheets or something else to do. That lack of enthusiasm toward subject matter is, of course, not an entirely unusual characteristic for middle school students to display, but it seemed more pronounced and damaging than usual. Because it is imperative that the year start off as positively as possible, my aim is to present my students with a unit on patterns more tailored to their experiences and needs.

More importantly than the requisite standards, I chose this specific unit because patterns present an accessible avenue for introducing significant mathematical concepts to students at various ability levels.

In bilingual classes, as in other types of classes, students' proficiency levels in two languages tend to vary considerably. If material is language-based, as it is in the MIC program, students with lower-level language proficiencies are often lost. Patterns, however, lend themselves to a variety of presentations and activities that are easily comprehensible to students regardless of language background.

Similarly, patterns abound in every culture and language, as well as in nature, and are thus familiar and relevant to most students. Students will be able to build on existing foundations of knowledge. For all learners, and particularly for second language learners, the idea of "scaffolding" knowledge is essential. Because math is so often abstract, any connections students can make with prior knowledge sets them up for greater success later. For example, patterns and symbols can be used to introduce algebraic and geometric relationships early in a student's math experience so that when the student is again introduced to algebra, he or she will approach it with an open mind rather than with the fear often associated with math in the later grades. Fundamentally, the rationale for the pattern unit is that it will serve to "encourage a mind set which is curious to explore algebraic patterns and relationships" (Bourassa) by, as Nietzsche might agree, enticing the senses toward the more abstract realm of algebra.

In addition to being relevant and familiar, patterns particularly, but not limited to those of symmetry, serve many functions in nature and are widely believed to affect the perception of beauty in many cultures. "Understanding the world often comes down to discovering pattern and order" (Burger and Starbird 249) and surviving in the world often comes down to *creating* pattern. Bees and plants, for example, create the most efficient patterns for their given needs in order to survive and reproduce. Creativity itself may be due less to any sort of special talent or genius than to "cooperation between pattern recognition and selective search" (Simon 203). Humans as well use patterns in all imaginable aspects of life—from science to art and, arguably, even in the choice of a mate. Other reasons not withstanding, the pervasiveness of patterns all around us and in our thinking warrants a brief study of them.

Finally, a unit on patterns is useful because it serves to introduce math as a way of thinking, as opposed to the conception of math as limited to manipulation of numbers. In my experience, math students are often shocked when they encounter anything, such as word problems or much of the language-based explication in the MIC books, that is not number-based. They tend to think that such phenomena have nothing to do with math and are prone to asking questions in the vein of "What class is this?" and "When are we going to do *real* math?".

Students need exposure to the fact, and perhaps the beauty, that math can more effectively be thought of as a way to expand one's thinking processes. The authors of *The Heart of Mathematics* concur and have advised to "look for patterns and similarities" as one of their "lessons for life" (619). Ideally, studying patterns will empower students to comprehend that math is about thought rather than about numbers.

Background for Teachers

It is interesting to note that in the many articles and chapters of books that I have read concerning patterns, I have not encountered a single definition of pattern. Perhaps that is because they are so much a part of daily life that it is assumed no explanation is needed, or perhaps it is due to the difficult and limiting nature of defining such a word. In general, though, the type of pattern referred to in the unit is that which *Webster's New World Dictionary* refers to as "an arrangement of form; disposition of parts or elements; design." Because there is, literally, an infinite number of patterns in the world, the background information presented here is in no way intended to be comprehensive.

Patterns are present in art of every culture. It is only in modern times, in fact, that art without order has become somewhat acceptable, as disorder is often not considered aesthetically pleasing. "The beautiful designs created by different cultures mirror the uniqueness of their histories" (Natsoulas 364), and allow students and other observers to appreciate that math is a culturally rich *human* endeavor (rather than, as students are apt to think, something created by an elite group of sadists).

For instance, Islamic art and architecture—from prayer mats and mosques to axes and daggers—abound with geometric patterns and intricate designs. Such art uses mathematical concepts to convey a cultural belief. Carefully planned patterns prevail because "the sense of order is rooted in the Islamic view that everything that exists is willed by Allah and has its place in the divine scheme of things" (Brend 225). It is widely (though not universally) believed that another reason pattern plays such a great role in Islamic art is that early Islamic law prohibited the use of human or animal forms and, in turn, an alternate means of expression was sought. Also, because of the dearth of visual stimulation in the desert where Islam has its roots, the patterns grew rather complex . Like Islam, each culture has its own "mathematical variations that sustain a sense of beauty" (Ashvo-Munoz and Rewerts 27) and serve to identify and enrich it.

As part of culture, patterns are also present in languages. Were languages pattern-less, it would be impossible to learn them through any means other than memorization. Luckily, however, patterns abound even in languages, like English, that are fond of breaking their own rules. They also abound in the language of math. Adding ‘ty’ as a suffix changes the meaning of a number from one digit to two digits or, in other words, multiplies the represented quantity by ten. The suffix of ‘th’ signifies a decimal. The words ‘to’ and ‘out of’ often indicate ratios. Comprehension of patterns such as these affect one’s comprehension of math, and vice versa. Thus, "translating number words into the language of mathematics also gives students a basis for reasoning about and extending the patterns they find." (Kliman and Janssen 799). Capitalizing on the patterns in language and math can reinforce students’ skills in both those areas.

One of the most ubiquitous visual patterns in both art and nature is that of symmetry. As Marilyn Burns stated, "Chances are, if mathematicians hadn’t noticed all the symmetry around, someone else would have" because it is "pretty hard *not* to notice." Symmetry is defined as the "motion of an object such that the appearance of the object is unchanged." (Natsoulas 364). In other words, as "an invariant quality in the face of a transformation." (Wertenbaker 6). Symmetry is often delineated into two categories: reflectional and rotational (reflectional symmetry is also known as mirror symmetry). For there to be reflectional symmetry, an object is reflected about an axis, or line, which is referred to as the axis (or line) of symmetry. The axis, and thus the symmetry, can be oriented any possible way, although studies show that while children generally recognize "vertical" symmetry at age seven, they do not recognize "horizontal" symmetry until age eleven (Orton 152). Rather than an axis, rotational symmetry is determined by a fixed point, referred to as the rotocenter or center of symmetry. In rotational symmetry an object is rotated around the rotocenter. Degrees of (clockwise) rotation can be used to describe the rotation, or it can be described as a fraction of a complete turn of the object. As in art, examples of both reflectional and rotational symmetry abound in nature. Any child who has made a paper snowflake has seen both types of symmetry. More complex examples can be found in examinations of the structure of atoms in minerals.

Probably the name most commonly associated with symmetry is M.C. Escher. Escher, whose given name was Mauritis Cornelis, was a Dutch artist who began training in architecture but switched to the study of graphic arts. He developed an interest in the designs, especially of the Moors, he saw during travels around Europe and started his own work experimenting with planes and symmetry.(Doornek) Escher’s work often incorporates animals or human figures and lends itself to a study of symmetry (I looked at countless Escher prints during the course of a college mineralogy class).

Fibonacci is another name associated with patterns, although with numerical rather than symmetrical patterns. Fibonacci (Leonardo de Pisa) was a 13th century mathematician after whom the Fibonacci sequence of numbers was named. To add a new number to the sequence (which begins with the number one repeated twice), one must add up the previous two consecutive numbers. So, the sequence begins with the numbers: 1, 1, 2, 3, 5, 8, 13, 21…What is intriguing about the sequence is not so much the numbers or the pattern it contains, but rather *where* the sequence occurs naturally. The Fibonacci sequence can be found in the spirals of a pinecone, pineapple, or in the florets of a flower. The spirals of pinecones and pineapples, for example, are oriented in three directions. The number of spirals in each direction is usually a number from the Fibonacci sequence.

Numerical patterns, such as the Fibonacci sequence, can be used to help children learn to generalize, make predictions and select effective problem-solving techniques. Those abilities are important not only as precursors to algebra but in life beyond formal education also. With regard to numerical patterns, Hargreaves, Shorrocks-Taylor and Threlfall found that children between the ages of seven and 11 tend to rely predominately on the strategy of looking for differences between terms.

Other strategies that children employ are looking at the nature of the differences, looking at the nature of the numbers, looking for multiplication tables, and combining terms to make other terms. Hargreaves concluded that working with an assortment of types of patterns would lead children to not only rely less on the strategy of looking for differences but also to "use more than one strategy and choose information from an appropriate one to make a generalisation, and be more persistent in searching for patterns within data." (315).

Modular arithmetic is one means of investigating and creating numerical (and visual) patterns. In the 19th century, Gauss introduced modular, or clock, arithmetic as a method for examining the divisibility of numbers. Modular arithmetic requires that the answer to a given operation is always expressed as a remainder after division by the modulus (mod), which must be a positive integer. Thus, only integers ranging from 0 to mod-1 are acceptable answers. For example, in mod 5, acceptable answers to any operation range from 0 to 4 (because it is not possible to obtain 5 as a remainder when dividing by 5). In mod 5, if one wishes to express the number 13, one would write 3 (because there is a remainder of 3 after 13 is divided by 5). Tables of modular arithmetic, laid out on and transformed grids, with each acceptable number replaced by a corresponding symbol, can be used to create "math art."

Mathematics itself may be said to have evolved through the act of "progressively finding patterns hidden in apparent randomness, at new levels of abstraction." (Westenbaker 6).

Background for Students

Because topics in math are often sequential—one cannot generally master algebra without knowing arithmetic—each unit has its own prerequisites. Though the pattern unit is intended to be accessible to students at various levels, students should be familiar with the English alphabet (and the Spanish if the unit is to be done bilingually). Students should have mastered basic arithmetic skills and should be familiar with various shapes.

Implementation

This unit is designed to take approximately two weeks (on a block schedule meeting every other day), though it could be adapted to be shorter or longer. As previously mentioned, patterns pervade the subject of math, and this unit is by no means intended to be comprehensive.

Because one of the goals of the unit is to inspire students to view math as a way of dealing with ideas in a creative yet very precise way, a variety of teaching and learning strategies have been incorporated. I would like to dissuade my students from owning the all too common belief that math is about manipulating numbers according to formulas someone bestows upon the unlucky recipients. For that reason, the emphasis in this unit is on inductive reasoning and problem solving. Students will work individually and cooperatively in order to predict, generalize, analyze texts and graphics, and problem-solve.

The unit begins with a general exploration of patterns. Next, students will merge arithmetic and art by making "math art," which will later be revisited. From math art, the class will explore patterns in language and then investigate word problems and puzzles by looking for patterns. Students will then be introduced informally to the concept of exponential numbers through reading and analyzing the pattern in the story "The King’s Chessboard." Finally, students will explore symmetry and create their own patterns.

The unit addresses the following standards and benchmarks from the Albuquerque Public Schools 2001 draft entitled *K-12 Mathematics*

Content and Performance Standards:

- Strand III Standard: The student demonstrates an understanding of concepts, properties, and relationships of geometry and measurement through experiences with meaningful mathematical problems that focus on identifying, describing, classifying, visualizing, comparing, estimating, and measuring various aspects of shapes and objects.
- Benchmarks: The student understands the relationships between two- and three-dimensional shapes and identifies, builds, and transforms shapes. The student uses inductive and deductive arguments to solve problems.
- Strand V Standard: The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.
- Benchmark: The student uses tables, graphs, and symbolic representations of patterns.

The New Mexico State Department of Education's standards and benchmarks addressed by each lesson are listed within the individual lesson plans (with "CS" denoting a content standard and "B" denoting the corresponding benchmark).

Lesson Plans

Lesson 1: Introduction to Patterns (1 day)

- CS 2: Students will understand and use mathematics in communication.

B: Students will interpret and explain personal mathematical thinking to make conjectures and convincing arguments.

- CS 12: Students will understand and use patterns and functions.

B: Students will describe, extend analyze, and create a wide variety of patterns.

Materials: pictures with various patterns

Procedures: Students will first do individual quick-writes defining the term "pattern" in their own words, and explaining what they believe it has to do with math. Volunteers will share their ideas. In groups, students will then examine different patterns and classify the patterns as they see fit. Groups will share and explain their classification systems. Groups will also define pattern, and together the class will establish a class definition to be posted in the room.

Assessment: Quick-writes and (informal) group presentations.

Lesson 2: Math Art (2 days)

- CS 7: Students will understand and use computation and estimation.

B: Students will solve problems through computation with whole numbers, fractions, decimals, rational and irrational numbers.

Materials: grids, rulers, colored pencils, markers, crayons, other art materials

Procedures: If available, the teacher will show the class examples of math art, in order to promote enthusiasm. The teacher will explain modular arithmetic (addition and multiplication). Students will complete a practice grid. The teacher will then model how to turn the grid into art. Students will use grids to create their own math art.

Assessment: Practice grids and completed art.

Lesson 3: Patterns in Language (1 day)

- CS 2: Students will understand and use mathematics in communication.

B: Students will interpret and explain personal mathematical thinking to make conjectures and convincing arguments.

Materials: worksheets or charts (with columns for number words, numbers and translations)

Procedures: The teacher will teach students Japanese (or any other language teacher knows) number words from 1-20 and then multiples of 10, up to and including 100. The teacher will ask students if they note any patterns and, if so, what they think those patterns signify. The teacher will ask students to predict how they think some other numbers are said or written, based on the patterns they have found. The teacher will then ask students if they know of any such patterns in their own language(s). Students will then complete worksheets or charts analyzing number patterns. In this case, students will look at the patterns of the suffixes ‘ty’ and ‘th’ in English and ‘ésimo’ in Spanish, and will practice pronunciation in both languages. Students will be asked to explain, in writing, any other patterns they can think that are involved in math.

Assessment: Participation and completed worksheets.

Lesson 4: Pattern Problems (1-2days)

- CS 1: Students will understand and use mathematics in problem solving.

B: Students will differentiate among problem-solving approaches to investigate and understand mathematical content.

- CS 12: Students will understand and use patterns and functions.

B: Students will describe, extend, analyze, and create a wide variety of patterns.

Materials: copies of pattern (number or word) problems, handout with more problems

Procedures: The teacher will distribute different problems (requiring different strategies for solving) involving patterns to groups. Each group will attempt to solve the problem. For example, the teacher might give the group the first five numbers of the Fibonacci sequence and ask the group to find the following three numbers in the sequence as well as the rule for the sequence. The groups will then explain how they found, or attempted to find, their answers. The class will then, with the teacher's guidance, categorize the different strategies used and the teacher will provide examples or fill in other useful strategies. The strategies may include guessing and checking, diagramming, looking for differences, making a simpler case, making a table, etc. The teacher will post the strategies. The students will then work independently on solving several more problems, which (along with work shown) will be collected. Some examples of problems to use can be found in *Math Through Children's Literature* and "Middle Level Mathematics: Encouraging an Algebraic Mindset."

Assessment: Group presentations (informal) and collected problems (the collected problems will not be graded on whether or not the correct answer was found but on the strategies and effort that were used).

Lesson 5: The King's Chessboard (2-3 days)

- CS 3: Students will understand and use mathematics in reasoning.

B: Students will use a variety of reasoning processes to explain mathematical thinking and to solve problems.

- CS 4: Students will understand and use mathematical connections.

B: Students will use mathematical foundations as a basis for more complex mathematics.

Materials: copies of "The King's Chessboard." rice, measuring cups and spoons, calculators, chessboard (optional)

Procedures: The class will read part of the story together. (The story deals with a king who agrees to reward his wise man by doubling the amount of grains of rice each day, starting with one on the first day, which the wise man will receive after 64 days.) On a chessboard (if available) the teacher will pause to ask and record how many grains there were on the first few days. The teacher will ask the students if they see a pattern and, if so, to explain it. Students will then predict, in writing, an estimation of how many grains there will be on the 64^{th} day. Students will use a strategy to make their estimation and will need to be able to explain it. (The teacher will collect this.) Students will share their predictions and explanations. The class will then finish reading the story. The teacher will then ask the students how many people they think the final amount of rice could feed if each person ate a half of a cup of cooked rice. In groups, students will solve that problem, including figuring out what information they need in order to solve it (usually rice triples in size after it is cooked, so a single serving would be a third of a cup, and students would have to figure out how many grains of rice were in a third of a cup). The groups will compare information along the way, such as how many grains of rice are in a serving, and they will compare their final calculations. The class will discuss what factors probably account for the differences in their calculations. The teacher will collect the groups' work.

Assessment: Individual and group solutions.

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*Lesson 6: Symmetry (*2days)

- CS 4: Students will understand and use mathematical connections.

B: Students will describe how mathematics is integrated throughout the school and surrounding environment.

- CS 8: Students will have a foundation in geometric concepts.

B: Students will explore transformations of geometric figures.

Materials: visual examples of symmetry, symmetry worksheets

Procedure: Part I. The teacher will introduce symmetry by giving examples and "non-examples" of images that have reflectional symmetry. For example, the teacher will say (and draw), "the capital letter H but the little letter h is not an example of the concept I'm thinking about," and will continue giving examples (and listing them) until the students have figured out that the teacher is thinking about symmetry, it is not necessary that students use that exact term. If the students did not come up with it, the teacher will introduce the term.) The teacher will explain that symmetry can be reflectional or rotational, and will ask students for examples. The teacher will explain and give examples of axes of symmetry and rotocenters. Students will then complete a worksheet identifying different types of symmetry, axes and rotocenters. Part II. Next, the class will go for a nature walk. Along the way, students will look for and sketch examples of at least five symmetrical objects (including both kinds of symmetry), and will label the characteristics of the objects. Part III. The teacher will tell the class that their assignment is to persuade (on paper) an audience of skeptics, with at least one well thought out reason, why it might be important to study symmetry (in other words, what could symmetry possibly have to do with the real world, other than that it exists? Arguments such as "it's cool" or "it's easy" will not be accepted). To give the students a potential line of reasoning, the teacher will draw or show different shapes (such as circles, squares, pentagons, hexagons, etc..) to the class. The teacher will ask the class to think about why bees build their hives using hexagons rather than the other shapes. The teacher will give other suggestions as necessary. Arguments may be humorous but ultimately they must also be convincing and show some understanding of symmetry. Volunteers will share their arguments.

Assessment: Worksheets, labeled sketches, persuasive writing.

Culminating Activity/Assessment: Essay—What I Know about Patterns (1/2 day or homework)

Students will write a somewhat open-ended essay describing what they know about patterns. The essays need to include, in students' own words, a definition of the term pattern, some examples of patterns, definitions and examples of rotational and reflectional symmetry, and a description of at least one problem-solving technique used during unit. Also in their essays, students may describe activities they liked and did not like in the pattern unit.

Bibliography

Ashvo-Munoz, Alira and Ardis M. Rewerts. "Off-beat Rhythms: Patterns in Kuba's

Textiles." *Journal of Popular Culture* 32. 1998: 27-39.

This article specifically addresses three types of patterns used in the raffia cloth of the Kongo people, and briefly speaks in general to the significance of the use of pattern in textiles.

Bourassa, Ed. "Middle Level Mathematics: Encouraging an Algebraic Mindset." *University of Regina.* 1997. Math Central. 12 Jun 2001. http://mathcentral.uregina.ca/RR/database/RR.09.97/bourassa1.html

Website includes a short background on problem-solving in mathematics and some examples of problems to use, along with students' work.

Burns, Marilyn. *The I Hate Mathematics! Book.* Boston: Little, Brown and Company, 1975.

This is a children's book with information, activities and puzzles geared toward children who are not necessarily fond of math. To understand it, a child would need to be a decent reader.

Braddon, Kathryn, Nancy Hall, and Dale Taylor. *Math Through Children's Literature: Making the NCTM Standards Come Alive.* Englewood, CO: Teacher Ideas Press, 1993.

Organized according to the standards of the National Council of Teachers of Mathematics, books and activities pertaining to standards 6-13 are listed for grades K-3 and grades 4-6. An overview of standards 1-5 is included, as are some activity sheets.

Brend, Barbara. *Islamic Art.* Cambridge, MA: Harvard University Press, 1991.

Chapters in this somewhat dry, informational book are arranged by locale and empire. The introduction and conclusion include general information about Islam and Islamic art. Plenty of photographs of the art that is described.

Bresser, Rusty. *Math and Literature (Grades 4-6).* White Plains, NY: Math Solutions Publications, 1995.

Each chapter includes a summary of a children's book related to math, followed by a narrative of how the story was used with a class and examples of students' work. Good ideas for teaching, though the scenarios are a bit idealized.

Burger, Edward and Michael Starbird. *The Heart of Mathematics: An invitation to effective thinking.* Emeryville, CA: Key College Publishing, 2000.

A very readable text written for college students but suitable for anyone wanting to brush up on math, know how it applies to the "real world," or solve some puzzles. It is written in sustained narrative and does not include the sort of exercise sets or pages of problems typically found in math texts.

Doornek, Richard. "M.C. Escher: Beyond the Craft." *IprojectONLINE*. School Arts Magazine. 12 June 2001.

http://www.iproject.com/escher/teaching/beyondcraft.html

Web page for teachers with a brief background and biography about Escher, as well as a few ideas for activities with elementary and secondary students.

Garland, Trudi Hammel. *Fascinating Fibonaccis: Mystery and Magic in Numbers.* Dale Seymour Publications, 1987.

A simply written book, suitable for upper secondary grades and beyond, explaining what the Fibonacci numbers are as well as how and where they appear in all facets of life. Clear illustrations on almost every page.

Gorini, Catherine A. "Using Clock Arithmetic to Send Secret Messages." *The Mathematics Teacher 89 (1996): 100-104.*

Article about clock arithmetic and how it is used, as well as activities students, who are familiar with exponents and have calculators, can do

Gosler, Cathy. "Math Art." Handout for university students. University of New Mexico.

This handout designed for students in a teacher preparation program summarizes modular arithmetic and provides an art activity to do with elementary or middle school students.

Hargreaves, Melanie, Diane Shorrocks-Taylor, and John Threlfall. "Children's Strategies with Number Patterns." *Educational Studies* 24. 1998: 315-332.

Article with detailed evidence of "the importance of pattern and generalization in mathematics," with classroom implications in the conclusion.

Kliman, Marlene and Susan Janssen. "Translating Number Words into the Language of Mathematics." *Mathematics Teaching in the Middle School* 1 (1996): 798-800.

Interesting article about connections between language and math. There are ideas for classroom activities for younger grades.

Macauly, Sara Grove. "Two Views of Islam." *Arts & Activities* 128 2001: 30-34.

Article for teachers with a brief introduction to Islamic art followed by art activities suitable for the upper middle grades.

Natsoulas, Anthula. "Group Symmetries Connect Art and History with Mathematics." *Mathematics Teacher* 93 2000: 364-371.

Illustrated article with information about symmetry in general as well as the symmetry of the square and equilateral triangle. Some quickly outlined ideas for exploring symmetry in the classroom.

Nietzsche, Friedrich. "Aphorisms." *Essays in Philosophy*. Houston Peterson. New York: Pocket Books, 1959. 216-229.

Exactly as the title states, this chapter is a list of 185 of Nietzsche's aphorisms.

Orton, Anthony, ed. *Pattern in the Teaching and Learning of Mathematics.* London: Cassell, 1999.

Written for teachers of various levels, each chapter addresses a different aspect of patterns in mathematics education. Some chapters have much more dry information than practical implications. The final chapter suggests activities for classroom use.

Patterns and Nature. WQED Pittsburgh. Videocassette. Texas Instruments, 1998.

One in a series of films made for television about math and its applications in the world. Suitable for students in the upper secondary grades or beyond.

Simon, Herbert A. "Creativity in the Arts and the Sciences." *Kenyon Review* 23. 2001: 203-220.

Provocative article about what creativity is and how all people have the capacity to be creative. The author concludes that pattern recognition is one component of creativity.

Tsuruda, Gary. *Putting It Together.* Portsmouth, NH: Heinemann, 1994.

Written by a math teacher for teachers, this book documents a "paradigm shift" in teaching from a teacher-centered approach to a more student-centered, problem-solving based approach. Some innovative ideas and compelling evidence for incorporating literacy into the math curriculum.

Webster's New World Dictionary. Second College Edition. New York: Prentice Hall Press, 1986.

Wertenbaker, Christian. "Nature's Patterns." *Parabola* 24. 1999: 6-14.

Intriguing philosophical article about order and chaos in nature, and how symmetries and broken symmetries define our existence.