Mathematical Problem Solving Strategies

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Academic Setting

School Setting

My current teaching assignment is a part of the Special Programs Department at Sandia High School. Sandia High School, grades 9 - 12, had an enrollment of 2019 for the 2000-2001 school year. The ethnic breakdown was 68.7% Anglo, 2.3% Black, 21.5% Hispanic, 3.0% Indian, 2.7% Asian, and 0.8% "Other." The economic breakdown for families who have students attending Sandia High School was 11% at <$10,000, 27% from $10,000-25,000, 35% from $25,000-50,000, and 27% above $50,000.

Classroom Setting

I teach in a C/D-Level self-contained classroom. There are two certified teachers in the classroom and our class numbers range from 20-30 students. I teach all subject areas (English, Math, Science, History, and Communications). I have all grades (9-12) and the students' academic levels range from 4th grade to gifted. Most of the students are in this "at-risk" program because of behavior and attendance problems. Often the students are not in class or school on a consistent basis. This, along with learning disabilities that mainly involve information processing problems, makes it difficult for the students to gain and retain information, especially in math. When they are in class and are able to work one-on-one with staff, they understand; but if they can't use it (due to not being at school), they lose it.

Curriculum Unit

The curriculum I am interested in doing involves problem solving strategies. If these students are given some strategies they can use in many different situations, then they might not have to spend so much time "re-learning" things they already have been taught. So often when working with these students in math, each problem seems completely new to them, even if they did one just like it the day before. I believe they can have more success if I approach math as "learning the tricks or strategies" instead of learning each math problem as a new and separate problem. Students' with learning disabilities have more success when they can find things in common with what they already know instead of everything being new and

scary. I also think teaching these strategies through fun math problems will peak their interest. Learning strategies through "games" and competition (Who can solve it first?) is a lot more effective than rote learning.

This unit is designed with flexibility. It can be used with students who are working on basic mathematical computations all the way up through algebra and geometry. The teacher can pick a problem to solve according to the students ability level and the type of math being learned. This unit can also cross over into a Language Arts curriculum, using the mystery solving section of the unit.

Goals

The goals of this unit are:

- to help students develop a pattern for problem solving.
- to give students confidence in their abilities to solve problems.
- to help students see a link between new math problems and math problems already solved.
- to help students breakdown math into smaller, manageable parts.

Context and Background

Rationale

The topic for this curriculum is basic problem solving strategies for students with learning disabilities. This unit is also targeted for those students with attendance problems. Students with learning difficulties have a hard time retrieving information learned from day to day. Students with attendance issues miss out on what is learned from day to day. Focusing on a few basic problem-solving strategies, which can be used in a variety of math settings, can help students re-use something they already know from one problem to the next. Showing the similarities between problems, and that each new problem has something in common with one they have already found an answer for, will give the students confidence in their math and learning skills. Students with learning disabilities have more success when they find things in common with what they already know, instead of everything being new and scary. George Polya, author of *How to Sove it*, says:

> One of the first and foremost duties of the teacher is not to give his students the impression that mathematical problems have little connection with each other, and no connection at all with anything else. We

have a natural opportunity to investigate the connections of a problem when looking back at its solution.

Another focus of this curriculum is to look at the process rather than the result. Often times we as teachers, as well as the students, are consumed with the answers and whether they are right or wrong instead of how we find the answers. Try approaching math as "thinking how to think" and "finding different ways to think" instead of just finding the answers to problems. We have to be careful not to fall into the routine of thinking that there is only one way to find an answer to a problem. In problem solving, there might be more than one way to find the solution. This can be a great opportunity for students and teachers to discover a "new way" to solve a problem. It can also be an opportunity for the student to become the teacher when explaining his or her way of finding the solution. Finding out "how" instead of "what" focuses on the thinking process. We can then show the students that math is about more than just coming up with the answers; it is about how to come up with them. We are not just teaching them to be better math students, we are teaching them to be better thinkers. Students who struggle with school can find relevance in being a better thinker, even when they can't always find relevance in being a better math student.

Background

Polya's Problem Solving Process

The first step in helping students with word problems is the introduction of Polya's problem solving process. George Polya felt that teachers should pose mathematical questions to students to encourage independent thinking. This problem solving approach can be applied to many different types of problems, and if practiced often, can help the student gain problem solving experience and confidence. This four-step process will help breakdown the math problem into something that is more manageable:

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Looking back.

Step one is important because the student cannot devise a plan if he or she does not know what they are trying to solve. The student must first breakdown what is being asked. The second step is the thinking step. The student needs to think about how to attack or solve the problem. Step three is the manipulation stage. The student carries out

the strategy chosen in step two. In reality, step three is where the most time is spent, although in actuality, the most time and effort should be spent in step two, devising the plan. And finally step four is spent checking to see if the result is the one that was desired. If the result is not what was desired, the student must re-work step two.

3 R's of Problem Solving

Another way to breakdown the problem solving process is to have the students learn the 3 R's of problem solving: request, response, result. Request is figuring out what is being asked or what do you need to find. Response is what are you going to do or how are you going to solve the problem. And result is checking the answer and making sure it answers the request.

Basic Problem Solving Strategies

Strategy 1: Look for a Pattern

The first problem solving strategy involves looking for a pattern in the information that is given. The student uses a table to see if there is a pattern that can lead to the answer.

Problem 1: Look for a Pattern

John raises cows for sale. He sold 2 cows the first month, 5 cows the second month, 8 cows the third month, and so on. How many cows did he sell in the sixth month?

Month	1	2	3	4	5	6
Cows	2	5	8	?	?	?

The table shows that each month the number of cows sold increases by 3. With that pattern, in the sixth month, John will have sold 17 cows.

Problem 2: Look for a Pattern

What number comes next in the sequence: 1, 7, 49, 343…?

Term	Value
1	1
2	7 (1x7)
3	49 (7x7)
4	343 (49x7)
5	? (343x7)

The table shows that the pattern is multiplying each proceeding number by 7 to get the next number. So the next number in the sequence is 2401 or 343 x 7.

Strategy 2: Draw a Picture

The next problem solving strategy involves drawing a picture with the given information. Drawing a picture can help the student see the solution.

Problem 1: Draw a Picture

There are 4 stores in a row in the mall. The shoe store is east of the jewelry store. The hat

store
is
west
of
the
jewelry
store.
The
bookstore
is
between
the
jewelry
store
and
the
shoe
store.
Which
store
is
farthest
east?

West ---------- East

Fact # 1: Jewelry Shoe

Fact # 2: Hats Jewelry

Fact # 3: Jewelry Books Shoes

Solution: Hats Jewelry Books Shoes

The shoe store is farthest east in the mall. Drawing the shops out, gives the student a chance to visualize all the information.

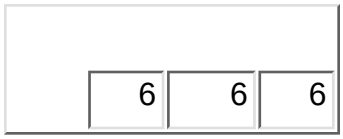
<u>Problem 2: Draw a Picture</u>

Joe
spends
$\frac{1}{4}$
of
his
money
at

the record store. Then he spends $\frac{1}{3}$ of what is left at the mall. Next he spends $\frac{1}{4}$ of what is left at the movies. He has $18 left. How much money did he start out with?

Spent $\frac{1}{4}$ at the record store

Spent $\frac{1}{3}$ of what's left at the mall



Spent $\frac{1}{4}$ of what's left at the movies

After blacking out all the money spent and figuring out what is left over, the student can see that the rectangle can be divided into 8 parts, each valuing $6. So Joe started out with $48 (6 x 8).

Strategy 3: Work Backwards

The next problem solving strategy is working backwards. The student will organize what they know and work backwards to find the answer.

Problem 1: Work Backwards

There were 4 pizzas left after the party. 12 pizzas were eaten at the party. The

birthday girl took 2 pizzas home with her. How many pizzas were there to start with?

- Pizzas eaten and taken home: 12 + 2 = 14
- Pizzas left over:14 + 4 = 18
- Total pizzas:18

Problem 2: Work Backwards

Anita found a treasure chest full of gold pieces. To get herself

and
the
gold
back
home,
she
had
to
cross
three
bridges.
Each
bridge
had
a
troll
residing
under
it
who
demanded
a
third
of
the
pieces
she
was
carrying.
When
she
finally
got
home,
she
found
she
had
40
pieces
left.
How
many
did
she

start out with?

- Bridge # 3: $\frac{2}{3}$ x = 40

  x = 60
- Bridge # 2: $\frac{2}{3}$ x = 60

  x = 90
- Bridge # 1: $\frac{2}{3}$ x = 90

  x = 135

  Anita had 135 pieces when she started.

Strategy 4: Make a Simple Case

The last problem solving strategy involves finding a simpler pattern to the problem, rather than working it all the way out.

Problem 1: Make a Simple Case

If

10
toppings
are
available
(i.e.
mushroom,
pepperoni,
sausage,
etc...)
to
top
the
basic
tomato-cheese
pizza,
how
many
different
pizzas
can
be
made?

Writing out all the different pizza combinations, which would take a lot of time and paper, can solve this problem. Or the student can make a simpler case and discover a pattern that can solve the problem.

Number of toppings	Different pizzas
1	2 (cheese or mushroom)
2	4 (cheese, mushroom, pepperoni, mush/pep)
3	8 (ch, mush, pep, sau, m/p, p/s, m/s/p)
4	16

Looking closely, a pattern starts to develop. Every time a topping is added the number of pizzas increases by multiplying 2 by itself the same number of times as the toppings.

Number of toppings	Different pizzas
1	2 or 2^1
2	4 or 2^2
3	8 or 2^3

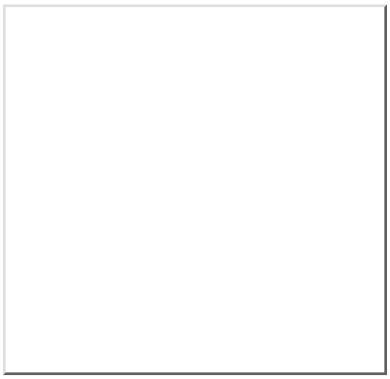
4	16 or 2^4

So how many 10 topping pizzas can be made? 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 1024. Figuring 2^{10} is simpler than writing all 1024 possible pizza combinations.

Problem 2: Make a Simple Case

How many squares of all sizes are in a checkerboard that is 8 x 8 squares?

The student can try and work out every possible square combination or they can just start with the simple combinations and see if a pattern develops.



Size Total number of squares

1 by 1 $1 = 1^2$

2 by 2 $4 = 2^2$

3 by 3 $9 = 3^2$

4 by 4 $16 = 4^2$

5 by 5 $25 = 5^2$

6 by 6 $36 = 6^2$

7 by 7 $49 = 7^2$

8 by 8 $64 = 8^2$

The simple pattern starts to develop. Each size square is itself squared. Then all the possible combinations are added together to get 204 possible squares. Seeing the simpler pattern keeps the student from having to work the problem all the way out.

Solving mystery stories and mathematical problem solving have significant connections. The questions a problem solver asks (Polya's problem solving process) and the questions a detective asks when solving a mystery are similar:

Mathematical Problem Solver	Detective
1. Understand the problem	1. Understand the case
2. Devise a plan	2. Investigate the case
3. Carry out the plan	3. Analyze the facts/data
4. Look back	4. Reexamine the facts

(Polya) (Crouse and Bassett)

Using mysteries can help students analyze and focus on details. It is also a way to help sharpen mathematical problem solving skills in a "non-math" setting. Students start to weed through the unnecessary and inconsistent information and instead stick with the facts needed to solve the mystery.

The Case of the Lookout

Dr. Haledjian was the only customer in the little drugstore when the shooting started. He had just taken his first sip of black coffee when three men dashed from the bank across the street, guns blazing. As the holdup men jumped into a waiting car, a nun and a chauffeur sought refuge in the drugstore. "You're both upset," said Haledjian. "Let me buy you a cup of coffee." They thanked him. The nun ordered black coffee, the chauffeur a glass of root beer. The three fell to talking about the flying bullets and had barely touched their drinks when sirens sounded. The robbers had been captured and were being returned to the bank for identification. Haledjian moved to a front window to watch. As he returned to the counter, the nun and chauffeur thanked him again and departed. The counterman had cleared the glass and cups. "Sorry, mister," he said to Haledjian. "I didn't know you weren't done." The counterman looked at the two coffee cups he had just removed from the counter, and passing Haledjian the

one without lipstick, said, "What do you think a chauffeur was doing around here? There isn't a limousine on the street." Haledjian thought a moment. "Good grief!" he cried. "We had the gang's lookout right here!" And he dashed out to make the capture. *What aroused Haledjian's suspicion?*

After analyzing the details of the story there is one piece of information that doesn't fit. Looking at the facts shows us that one of the cups had lipstick on it. Nuns don't wear makeup. The nun was really a lookout for the bank robbers.

When analyzing the facts, have the students look for what information is important or not important. Which pieces of information don't seem to fit together? Which data is inconsistent with other information given?

The Case of the Attempted Murder

"Jack Alden's account of the attempted strangling of Mrs. McHenry is pretty farfetched," Inspector Winters told Dr. Haledjian. "Yet he passed the lie detector test. Alden drives a delivery truck for Best Cleaners," explained the inspector. "At five minutes before noon Tuesday he drove to the McHenry House and stopped the truck in the driveway. He spent about two minutes filling out his delivery reports for the morning. Then he got out with a dress and two suits. As he closed the cab door, he noticed his front wheels were parked on the garden hose which ran from an outlet by the garage around to the back of the house. Alden claims he got back into the truck and drove forward a few feet so that his engine was in the McHenry's empty garage. Here he noticed the door between the kitchen and the garage was open. He saw Mrs. McHenry lying on the floor by the stove. He rushed to her, he says, and was trying to revive her when Mr. McHenry came through the open door of the garage. McHenry had taken the day off to water his backyard. He had been hosing down his flowerbeds and hedges for

half an hour when he noticed the truck in his garage. He walked over to investigate. We can't get McHenry," concluded the inspector, "to state definitely whether he thinks Alden was trying to choke his wife or revive her." "No wonder the lie detector test didn't trap Alden!" said Haledjian. *Why didn't he think Aldon was trying to kill the wife?*

The students must work through the unnecessary information given in the story. The open garage is mentioned many times, but it is not relevant. The piece of information that sticks out is when Mr. McHenry finally came into the house. If he had really been watering the flowers all that time, why didn't he come to investigate why the water stopped flowing? The truck was parked on top of the hose for about two minutes.

Mysteries can be a fun way to get the students thinking in a problem solving way. The kind of thinking that it takes to solve mysteries is the same kind of thinking that it takes to solve a mathematical problem. Using mysteries is a fun way to help develop these problem-solving skills.

Implementation

This unit is designed with flexibility of pace and ability level. The problem solving strategies are to be used by any level math student. The teacher will need to modify the types of math problems used. It is recommended that the teacher stay with each strategy until the students can use it on problems with little help. After there has been some demonstration of mastery, the teacher can move on to other strategies. It is also recommended that the problem solving strategies and use of mysteries be revisited on a regular basis throughout the rest of the school year.

Content Performance Standards Met

- 1-A: Students will analyze problem-solving approaches to investigate and understand mathematical content.
- 1-B: Students will formulate problems from global mathematical situations.
- 1-C: Students will select the best strategies to solve a wide variety of problems in diverse contexts.
- 1-F: Students will analyze solutions and strategies for use in mathematical modeling.

- 3-B: Students will evaluate reasoning strategies to select the most appropriate reasoning method to solve a given problem.
- 5-B: Students will apply number-sense skills within the real number system.
- 6-A: Students will use order relations within the real number system.
- 10-A: Students will construct and draw inferences from charts, tables, and graphs that summarize data from inside and outside the school environment.
- 11-B: Students will use experimental or theoretical probability to represent and solve problems involving uncertainty.
- 12-B: Students will represent and analyze relationships using tables, graphs, rules, and equations.

Lesson 1 - Polya's Problem Solving Process

Purpose: To teach the students a four-step guide for solving mathematical problems.

Materials: Poster board, word problems

Procedure:

1. Present a mathematical problem
2. Have the students try and solve in small groups without teacher help
3. Record all group answers
4. Present Polya's 4-step process
5. Have the same small groups solve a similar problem using the process
6. Create a classroom poster of the process for display

Lesson 2 - 3R's

Purpose: To teach students another problem-solving strategy.

Materials: Poster board, word problems

Procedure:

1. Introduce 3R's of problem-solving
2. Work problems together on the board
3. Give students simple word problems having them illustrate where they used the 3R's
4. Create a classroom poster of the 3R's for display

Lesson 3 - Look for a Pattern

Purpose: To show students how problems can be solved by recognizing a pattern in the information that is already known. Also to show students how to use tables to reveal that given information.

Materials: Word Problems

Procedure:

1. Present a problem to be solved
2. Demonstrate how to use a table to organize the information that is known
3. Reveal the pattern and solve for the unknown
4. Have the students work on similar problems

Lesson 4 - Draw a Picture

Purpose: To teach the students that problems can be solved by drawing a picture with the information that is known.

Materials: Word Problems

Procedure:

1. Present a problem to solve
2. Demonstrate how to draw a picture with the information that is known
3. Have the students work on similar problems - have them include their drawings
4. Present 2 problems and have the students decide which can better be solved finding a pattern and which can better be solved by drawing a picture.

Lesson 5 - Work Backwards

Purpose: To teach the students how to work backwards using the information they know to solve a problem.

Materials: Word problems

Procedure:

1. Present a word problem to solve
2. Demonstrate working backwards to solve

3. Have students work in pairs to solve similar problems

Lesson 6 - Make a Simple Case

Purpose: To teach the students how to solve a problem by finding a simpler pattern rather than working out the problem the long way.

Materials: Word problems

Procedure:

1. Present a word problem to solve
2. Solve it the long way
3. Solve it the simple way finding a pattern
4. Discuss the differences and similarities
5. Have the students solve similar problems both ways

Lesson 7 - Multiple Strategies

Purpose: To present students with problems that can be solved with previously learned strategies to see if they can see which strategy works best with which problem.

Materials: A variety of word problems

Procedure:

1. Review problem solving strategies
2. Have students solve a variety of word problems using each of the strategies
3. Have each student demonstrate one of the problems they solved explaining the strategy used, why they used it and how they found the solution

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Lesson 8 - Mysteries

Purpose: To show the connections between mathematical problem solving and the solving of mystery stories.

Materials: Short mystery stories

Procedure:

1. Read a mystery and have the students try to solve it
2. Present a math word problem and have the students solve it
3. In small groups, have the students list the similar strategies used in solving both

4. Present the 4-step Polya process and the 4-step Crouse/Bassett process and examine the similarities
5. Have the students create their own mystery to be solved by a classmate

Bibliography

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