

**It's in the Cards:
Teaching Mathematical Concepts with Card Games**

Jacqueline Robison

The Academic Setting

As teachers at West Mesa High School, we have the challenge of making mathematics meaningful to all of the student population. The population at West Mesa High school is approximately 74% Hispanic, 3% each of Native American and African American, and almost 20% Anglo or Caucasian. One of the Albuquerque Public School districts goals is to reduce the dropout rate. Unfortunately, the majority of dropouts in our district are students of Hispanic and Native American background (who compromise 77% of our population). West Mesa High School has identified a sub goal to accomplish the district goal of lowering the dropout rate. This goal is to increase the success rate of the course Algebra IA. Algebra IA is a class that takes the curriculum of the regular Algebra I class and divides it into halves; the first year is IA and the second year is IB. West Mesa High School offers 60 mathematics classes; 24 of them are freshman classes varying in types from pre-Algebra to Honors Algebra I. Unfortunately, 62% of the classes offered to freshman are classes below the projected academic standards for freshman in a regular Algebra I class. The data gets more convoluted. Since March of 2000, ninth grade students are mandated to take the standardized Terra Nova test. West Mesa High School students are scoring very low overall; only 8% of the freshman in the last two years scored in the range of "proficient or better" in mathematics. With this data in hand, it is imperative that teachers not only address the issue of students passing Algebra IA, but that students take with them the content knowledge that correlates to a passing grade as well as being considered "proficient or better" in the standardized tests.

In whatever way the state or district tries to translate, identify or label the problems at West Mesa High School; the students remain clever, spirited, and able-bodied students. They have not, however, found mathematics to be fun and exciting. In general, they haven't discovered that learning mathematics can create many opportunities for them that otherwise wouldn't be available. This unit focuses on the act of playing cards as a means of learning mathematics and having fun at the same time.

This unit is designed for the freshman mathematics classroom, but not limited to the ninth grade. The objective of this unit is to encourage critical thinking, reasoning and problem solving using relevant applications that are enjoyable, and yet have students subliminally learning many processes of thought. The act of playing cards is sometimes dismissed as just a game. This unit should dispel this myth and allow the students to grasp the sometimes difficult concepts of statistics and probability.

Typically, the average class size at West Mesa High School could be considered to be very large, thirty students or more, and a hindrance to student learning. The teacher needs to use this to their advantage - larger amounts of data can be obtained at a much quicker pace. Consequently, when a class is structured in fifteen groups of two, collecting data and making comparisons with each other will be obtained quickly. Along with this the students will need clear guidelines of how to play each game and what is expected of them after every session of play. In the beginning, students should be allowed to just learn and become proficient at each game before analyzing the outcomes of their play. Next, the students will determine as a class and/or small groups the following: the least amount of plays to win, the strategies to win, and ultimately, the processes of counting cards and what are the probabilities of being able to win.

Another concern that will be addressed is the type of questions asked on the standardized tests. Often, these tests include questions that originate from different types of games questions such as, "What is the probability of getting a certain answer." Many of West Mesa High School students have either not played enough of these types of games, or worse have never even seen the game. The first aim of this unit is to introduce some of these games and to help students learn in a cooperative manner. Ultimately, this unit will expose them to understanding the difference between chance and the ability to predict what is going to happen next. The hopeful outcome of this unit is to give students the opportunity to learn how to play the game(s) with other peers and to realize that not only did they have fun, but they also had the opportunity to increase their sense of numbers. Another goal is to be able to demonstrate that with practice they can increase their own success in the games in several different ways.

This unit's purpose is to have the student create data and be able to make predictions from their data. The questions the students should be able to ask and answer are:

- What is the least amount of plays needed to win?
- How many cards are left of any particular suit at any time in the same game?
- Given a particular hand, what are the chances that the student will be the winner?

This information will not be obtained from one or two games. Each group of students will need to pool their results for an opportunity to get answers and predictions at a faster rate. Students will be able to interpret the data collected and make conclusions. The results will be shown in both the play of the game and in table or graph form.

The unit will cover several of the Albuquerque Public School District's Content and Performance Standards. Primarily, the strand that will be covered in depth will be Data Analysis, Statistics and Probability. Several of the other strands will be interwoven within the activities and the assessment process of the unit. The notation recognizing the district's Content and Standards will be in parenthesis. A list of the standards and benchmarks met will be cited in the documentation portion of the unit.

Background

The act of playing card games goes back at least 14 centuries. Many questions come to mind concerning the longevity of card games. Why do people have such a fascination with the various types of games? Why have some games lasted longer than others? Why does the typical deck consist of 52 cards? In some instances there isn't an absolute, finite answer but only speculation. For many people, the unknown is the fascination of the game. In any event, the history of cards is very interesting, and provides conversational trivia to entice students into possibly researching for themselves the history of card playing.

Humans have a fascination with the unknown, especially the future; we always want to know what the future will bring before it happens. The pastime of divination led to tarot cards. Dice is one of the oldest games of chance, and dominoes are just an extension of dice (the format of markings is the same or similar but flat in nature). With the combination of dice and tarot cards available, there was a natural progression towards blending cards with chance.

Our typical deck of 52 cards with four suits took centuries to fine-tune. It is been found that in the beginning there were four suits. What took a long time to fine-tune was the amount of cards in each suit; it had tended to vary from one to twenty. The first recorded deck of 52 cards was in France in 1516, designed by Antoine Janin. It is speculated that the amount of 52 cards coincided with the number of weeks in a year, and the four suits with the four seasons of the year.

Historians who have deliberated what was put on the cards and why agree that the cards depicted customs and life of their time. A deck of cards in 1516 had the following connections: the coeurs, or hearts, are hearts; the carreaux, or arrowheads, were diamonds; the trefles were clovers or clubs; and the piques were points or lances, now known as spades. The description of these cards in terms of the types of suits has varied only slightly in the last five centuries, but the types of pictures have changed a great deal. The cards from 1516 through the late 19th century generally represented the rulers and costumes of their respective countries. There are documented types and complete sets of cards from China, Japan, India, France, Germany, Belgium, Holland, England, Italy, Spain, Switzerland, Denmark, Sweden, Russia and America.

During the last 500 years extensive changes in types of card games have occurred. In the early 1500's, the Duke and Duchess of Bavaria retired from "Le Grand Ball" and sat down at a table to play cards. In 1643 Cardinal Mazarin, who was in charge of educating the young Louis XIV, devised a series of card games to interest the young king in his studies. In 1728, a German monk recorded in his memoirs that he used card games for his pupils to surmount the difficulties of intricate study. The monk felt that the pleasure of playing cards contributed to their success in their rigorous studies.

However, all of the recorded history of cards is not positive. As early as 1397 an ordinance was issued that forbade people to play cards. It was

felt that if the commoners had time to play cards then they could get more work done. In 1633, the Puritans of the New World fined people two pounds each for playing cards because they considered it sinful. Queen Hadland of London, in 1767, had the same advice "to forego the friends whose company is Cards, Dice, and Queanes."

For centuries people have understood the pitfalls of playing cards. In our society today people play the lottery, slot machines and cards. Possibly hundreds and thousands of these people shouldn't be spending their money in this manner because they don't understand the odds. David Spanier explains very succinctly about poker: "Poker *appears* to be about gambling, but it isn't...Let the losers gamble." Spanier continues to describe a good poker player:

The winners do not really *gamble* at all in the strict sense of the word-wagering money on a sporting event of uncertain outcome-because they know with a degree of confidence that amounts virtually to certainty that they will wind up winning, in the average run, regardless of the distribution of the cards... Their skills give them the winning edge. Of course one can be lucky or unlucky with the cards, missing a vital draw, or hitting a case ace. In the long haul, however, everyone gets the same cards, good, bad, and indifferent, just as at bridge, if you aggregate all the hands,...It's how you play them that counts.

Our society is changing; many young people play only computer games and they don't understand card games. Most of us have no clue of what it takes to be successful. The question that non-card players ask repeatedly is, "If you aren't a winner, why keep playing?" David Spanier quotes Ralph Greening:

The atmosphere of excitement in gambling is visible in the trembling and sweating of players and in their restlessness, and is audible in the noise and the hushed silences. But that is only part of it. There is a rhythm of tension-discharge, which is constantly repeated. At the beginning of play it is quiet, gradually there is a crescendo of excitement until a peak is reached, and finally there is a period of quiet.

Rationale

The rationale for this unit has two points of focus. First is to learn how to find the probability of a winning hand or game. Although this unit is not going to focus on poker games, students will learn how to count a deck of cards and learn how to figure the chances of a good play or bad play. If a person feels as though they are having a lucky streak, they need to be clear about the hazards of playing without the knowledge of odds and probability. Secondly, we will focus on healthy competitiveness and camaraderie between students. These are understated in the classroom. Unfortunately, unless students play sports or compete for a role in a play, they may never develop or experience the development of these skills. Practical social skills and being able to verbalize pleasure and disappointment cannot be overstressed as a need for our students and society today. (S I:1-4, 8-10; S IV: ehs 7,8; lhs 8,9)

This unit will include three different types of card games to emphasize specific learning processes. The first type of card game will be similar to the childhood game of Memory. The primary difference will be that instead of memorizing pictures and placement on a board, students will have to match arithmetic and algebraic rules of operations. The second game that will be taught will be a type of Rummy. This will focus on learning about the types of cards in the typical 52-card deck, and on what the odds are of a certain card appearing next in a draw. The last game that will be learned is Hearts. This game builds on one's playing strategy (used in the beginning of each hand) and the follow-through (during the play of the game).

Hearts is a round game (every person for himself), and three to six players are allowed. One of the benefits of this game is that you can have larger groups. The winner of the game is the person who earns the least amount of points. In order for the students to be able to find data for comparison, the students will have to record the deal and their pass, quickly on a piece of paper. After each game the students will be able to analyze who received points, the different leads that were made during the game, and the turning point of the game (if known). The students will also analyze the distribution of cards amongst all of the players to see if they can find patterns in the play of the game. If all goes well (and there isn't any parent outcry), different types of poker could also be introduced and played throughout the school year.

In a mathematics class, teachers should not skip any content area that the district or state standards require. This particular unit is an extension of what needs to be taught, statistics and probability, along with problem solving, communication and reasoning. If a teacher has never played cards then the whole unit would be quite difficult. The memory game, however, could be used by anyone without any prior knowledge. The other two games require the teacher's background and familiarity of the games to be strong. Record keeping by the students will be a very important component within the classroom and that is where the teacher's focus should be (not on the rules of play). The student's need for competency in mathematics before teaching the games is minimal. Students, and children in general, have a knack at being able to pick up on how a game is played. Students will need to know how to count without a calculator, but even the range of numbers will not be very high. In the memory game the student will need to know the different properties of addition and multiplication, but the emphasis on this game is that the student will learn immediate recognition of different properties.

Context

Memory Game

In the game of Memory, the objective is to review the different rules of operations. Needless to say, in the course of Algebra I we really want the student to understand the application of rules, not just be able to regurgitate a formula. In this situation, it would be a great accomplishment if the students could remember the rules (instead of forgetting that they even have these rules). The textbook West Mesa choose for Algebra I has presented all of the basic axioms by Chapter 2.5; this is very early in comparison to some books. This causes some of the students feel a little overwhelmed with the content of the curriculum very early in the year. By having a memory game, they are asked to be competitive with each other and to sharpen their minds for the process of prediction. Student will be watching other students make their move and recollecting which rules of operations are left to match. Question to ask in the beginning will be, "If there are 20 pairs, then what is the least amount of moves that someone could make and possibly win? Are there any playing strategies that the students could share with each other?" This memory game could be over or underplayed; it could be reinforced with a matching quiz after a day of play. The student who gets the most pairs wins or the pair of students in the classroom who wins in the least amount of turns are the class winners. (See Fig. 1 for list of properties.)

Memory Games Components

$A + B = B + A$	Commutative Property of Addition
$3 + (-1) = (-1) + 3$	Commutative Property of Addition
$(A + B) + C = A + (B + C)$	Associative Property of Addition
$(-2 + 4) + 5 = -2 + (4 + 5)$	Associative Property of Addition
$A + 0 = A$	Identity Property of Addition
$6 + 0 = 6$	Identity Property of Addition
$A + (-A) = 0$	Inverse Property of Addition
$7 + (-7) = 0$	Inverse Property of Addition
$A \cdot B = B \cdot A$	Commutative Property of Multiplication
$2 \cdot 3 = 3 \cdot 2$	Commutative Property of Multiplication

$(A \bullet B) \bullet C = A \bullet (B \bullet C)$	Associative Property of Multiplication
$(3 \bullet 4) \bullet 5 = 3 \bullet (4 \bullet 5)$	Associative Property of Multiplication
$1 \bullet A = A$	Identity Property of Multiplication
$1 \bullet 6 = 6$	Identity Property of Multiplication
$A \bullet 0 = 0$	Zero Property of Multiplication
$3 \bullet 0 = 0$	Zero Property of Multiplication
$(-1) \bullet A = -A$	Multiplication Property of Opposites
$(-1) \bullet 4 = -4$	Multiplication Property of Opposites
$A(B + C) = AB + AC$	Distributive Property
$3(B + 5) = 3B + 15$	Distributive Property

Figure 1 (S I:4,5 8-10,16; S V: ehs 9,10 lhs 9)

Gin Rummy

The next game played will be Gin Rummy. The game uses all 52 cards from a typical deck of American cards. In the beginning, wild cards or jokers should not be allowed. The students need to understand that there are four suits, each having 13 cards; the ace is considered to be a low or high card. At the teachers discretion the choice between Gin Rummy and Rummy could be made. The major difference between the two games is that Gin only allows two players, so if the teacher does not want a lot of small groups the better choice would be rummy (up to 4 players are allowed). The advantage of Gin Rummy is that the game includes two options for closing the hand. The disadvantage is not being able to watch the play at different tables. The game is more complicated, and a class may get off task without the teacher being aware. The basic goal is to get all of the cards in one's hand, ten cards in either sequences or series, or a combination of the two. A series is a group of three or four of a kind. A sequence is when three or four cards have two characteristics: they must all be the same suit and the cards must be all in a row (for example: four, five and six of hearts). Until the cardholder has all of their cards spoken for in any of these ways, players continue to draw and discard from the unturned deck. When a player has all of their cards spoken for they lay their hand down in a "gin rummy" and points are awarded for being able to lay down all of the cards at one time (and by the other persons hand). The other way to end the play of the game is to go down early, also known as "knock." This means that instead of having at least one of the groups be a set of four, the player would have three sets of three and go down for a certain amount of points. If a player is successful at this they win, but they do not earn as many points for the win.

During the play of the game each player has to do several things to have a better chance of winning. First of all, the player has to decide which types of sequences or series they should go after in his or her own hand. Second, the player has to watch the discard pile and remember what cards have been played; the rule is once the card is covered by another card then it cannot be played again (it is not an option to be able to pick up the card later on). Next, and probably the most important, the player has to judge from the first two steps what the chances are of getting the card(s) needed. This is the act of counting cards. What will be most difficult for this type of unit will be the recording of data. It will be imperative that students record their opening hand at the beginning, to keep the cards in order in the discard pile, and then to record the points earned in each hand. If the students don't keep good records, the class does not have to be given the opportunity to play. At this point the

students will see the difference between having good luck and making good choices. (S I: 1-19; S II: ehs 8-11 lhs 2-4,6; S IV: ehs 3-8 lhs 1-3,5,6,8-10)

Hearts

Once the students are adept at counting cards, writing down the data, and most importantly, analyzing and predicting the data, it will be time to show them how to play Hearts. This game will be best if played with groups of four. As the explanation of the play of the game is given, this fact will be pointed out. Unlike Gin Rummy the object of the game is to not get any points. Again all 52 cards are used, ace being high in this game. You pass out all of the cards; 52 is divisible by four evenly, so everyone would receive thirteen cards. Each player should sort cards by suits and assess their hand. The queen of spades is worth 13 points and each heart is worth one point. The rest of the cards in the deck do not have any point value. The first play is to decide which three cards each player doesn't want to keep. You can set up one's hand to avoid certain problems during a game. The pitfall is that you will also receive three cards from someone else. The pass is done in rotation to the right, left across (if there are exactly four people) and hold. The hold round can be the most difficult to play as you have no choice but to play the cards as dealt. Next is the choice of plays. Since you don't want any points, you want to make sure that you don't have to pick up the cards that have point value. Each player must follow suit; if they don't have any of the suit that was lead by the first player, they may pass off a point card or any card that they don't want to continue to hold. The person who played the highest card of the lead suit takes the round, or "trick" as it is called. Every play can make a difference in who will get the most points. A good and/or practiced player learns when and how to play each card in their hand.

One more strategy that makes the game exciting, and a little twisted, is if one of the players can get all of the points, then that person actually gets zero points and everyone else gets 26 points. This leads to actually having to work as a team to stop the person from "running" the game. Usually the person running doesn't just announce this plan, because he or she may have cards that are holes in their plan. If they can finesse them, they might succeed. The person who wins is the person who is able to count cards the best and predict what cards will entice people to play certain cards. It is imperative, for strategic reasons, that a person counts how many of each suit has been played. In short, whether or not a person has the lead or needs to finesse a card that they don't want to hold, a player always needs to know how many of each type of suit are left during each play of the game. The player who will win most often not only counts how many of each suit have been played, but what card of each suit has been played. A beginning player has a difficult time deciding what to do in the first few hands. Since the object of the game is to not get points, understanding how the queen of spades plays out is a major objective in the game. A couple of questions that students will start to ask are: "If you don't have the queen in your own hand, who does? If you have the ace and/or king of spades in your hand, is this a benefit or a hazard?" The students will struggle for a while learning all of the little subtleties. The questions and problems that will arise will vary with each student.

Hearts is a game of chance to a lesser degree than Gin Rummy. When a student plays the game they will have to ask themselves one question all of the time: "If I lay this card down, what will happen?" Recording one's hand in the very beginning, and writing down which cards were passed and received are going to be a very important components of learning the strategies of play. Several votaries who are mathematicians have proven that you don't learn the trade of playing cards well by playing hundreds of hands. In 2000, 37 year old Chris Ferguson won 1.5 million dollars at the Las Vegas Nevada Poker Championship. "The use of mathematics, probability and online poker resources can greatly accelerate the learning curve. So while there's no substitute for experience, you can certainly come up to speed faster," said Zimmers, a graduate student and former software developer. You still have to understand how to read your opponent, but using the facts of the cards can increase one's chances of winning. (S I:1-19; S II ehs 8-11 lhs 2-6; S IV ehs 8-11 lhs 9-11; S V; ehs 5,9,10 lhs 7,9)

The ability to predict what others might do based on one's own hand and what has been played before is the key to success in Hearts. A person will calculate and commit to a plan, and that becomes their own strategy. This is a goal and a lesson, too in math. As teachers, we want students to be able to solve a problem, but more importantly we want students to develop a plan on how to solve the problem. If a student can develop the processes of reasoning and make the connection during the game, then the unit is a success. Not everyone will win, but if the student can say after the game: "if I hadn't led that low diamond I wouldn't have lost so badly," the student has:

- a. communicated their understanding of the game and

- b. they have applied reason to their mistakes.

It is a goal that every math teacher hopes for that a student can find their own errors; if they can, they won't make them as often or, better, ever again. The final outcome of this unit is to be able to put together data on the distribution of cards when first dealt and on how the play unfolds. Are there correlations and predictions that can be made from the original hands?

Implementation

In the following lessons it may look like there are only three days worth of work, but as with any hands on manipulative, it may take more than a day to master the objectives. In order to be proficient in both the data collection and the play of the games several days could be spent playing. A section on probability and odds is at the end of chapter two, which coincides with the properties of addition and multiplication. This unit could be taught as a two week unit, or broken up and attached to the relevant chapters in the textbook being used. Also, these games could be used as a reward for the classroom. Students like to feel as though they are the masters of their own destiny.

Memory game: A game using skill and chance to review mathematical concepts.

Materials:

- Enough sets of card stock so that the class can be divided up into pairs. Use the list of properties to make a blackline master. Suggested material to make cards would be 24 lb. Paper, and then laminate. When cutting out the cards the dimensions are lengthwise is $1 \frac{3}{8}$ " each, width is $3 \frac{7}{8}$ " , with a total of two columns.
- Instruction sheet and tally sheet (piece of paper) for each student.
- Pencil (student should have).

Introduction:

Focus: Recognition of the addition and multiplication properties that have been discussed.

Objective: The student will be able to remember what each one of the properties looks like and be able to match this to the name of the property.

Purpose: To allow students to go through the thought process of remembering the last play(s). As in the children's game of Memory, the student will discover that they can win if they remember where the other cards that have been turned over are.

Guided Practice: Have students set up game and play. Make sure students keep track of how many attempts each one makes before the game is over.

Closure: In a large group, discuss the activity. For example:

- Can the students tell the difference between chance and skill?
- What is the least amount of moves possible? If someone were to do this, would this be chance or skill?
- Find out and share the different strategies that students may have used to win.
- Record the amount of turns each pair of students took to win. Make comparisons.
- Can the students find any patterns in the data that they took?
- Is there any connection between who goes first and who wins the game?

Assessment: When you devise hands-on manipulatives for the classroom, assessment is not always clear-cut. During the activity, you can always judge if the students are staying on task. If you feel that you can't do the activity without written assessment, have the students take a matching quiz before and after playing Memory. Then, if you're really brave, have a fill-in-the-blank exam! Remember, another important feature to this game is that the students start to memorize by being able to recall a match by the card position.

Directions for the game of Memory:

1. Place all cards (20 pairs), randomly, upside down on a flat surface.
2. Flip a coin to see who goes first; heads wins.
3. Students will take turns picking up two cards.
4. If the student gets a pair turning over two cards, then the student gets another turn.
5. Record how many turns each person takes.
6. The game is over when there aren't any more pairs left.
7. Count and record how many pairs each person won; winner is the student who took the most pairs.
8. Classroom winners are the pair of students who won the game in the least amount of turns.

Gin Rummy: Sequences and series - Learning to count cards

Materials:

- Decks of American playing cards for groups of two.
- Instructions and tally sheets for all students (fig. 2); make sure that there are plenty.
- Pencil
- Teachers may want the book, *According to Hoyle*

Introduction:

Focus: Learning the types of cards that are in a typical deck of cards and what is a sequence and series in the game.

Objective: Students will be able to plan and manipulate their hand in the most efficient manner possible to get the necessary matching cards. The winner is whoever gets 100 points first. This may take one hand or ten.

Purpose: This game will allow students to learn to watch the opponent's moves so that they can understand the advantages of a gin or a knock, and to plan their game accordingly. It is an excellent way for the student to understand the advantage of counting all of the cards played by both.

Guided Practice: Have students read instructions to the game. Make sure that everyone has a good understanding of the point system, three or four of a kind, three or four in a sequence, and what happens to the unmatched cards. Observe the play of the game and interaction to check for understanding and different strategies used. Check that the students are recording all of the hands on the tally sheets.

Closure: In a large group, discuss the activity. For example:

- Is it possible to not have to draw any cards at all?
- If the above happens, what is the probability of this happening?
- Is it possible to go through the whole deck of unturned cards without anyone winning?
- Is it important to watch the other player's discards?
- Is it important to count how many times the other player picks up the card showing?
- Does it make any difference as to which person the winner will be if the players already has a group(s) in their original hand.
- Should you collect series over sequences or vice versa?
- When and what are the advantages of 'knocking' instead of Gin?
- Should you discard high cards early?

- What is the average amount of turns taken per game for the class?
- Is there any relationship between going first and winning?
- If the ace was not both a low or high card would the game be slower or faster? Why?
- Did each player choose the right sequence or series to go after? If not, did they recognize which one they should have chosen?
- Was the "wrong choice" a from lack of information, or taking the riskier choice?
- What is the average amount of points that each person is getting per hand?
- Record the data, decide on the type of analysis the class wants to get, arrange the data, generate results, and display the information found. Decide if it's relevant.

Assessment: In the playing of Gin Rummy, the teacher should collect the data and give a grade for good data collection. The measure of "good" would be if they answered the questions from above. During the first day of play, asking the students to have the experience to answer all of the above would be too much. The teacher could start with a few questions and, with time, could expect the student to write a short synopsis regarding the games that they played for the day. At the end of a couple of days of playing Gin Rummy, the class should start to have a sense of cards. They should be able to tell their partner how many are left of the cards they are looking for. This is also something that could be recorded at the end of every game before counting how many cards are left in the draw pile.

This game should be practiced until the students can answer most of the questions in the closure section. Tables or graphs would be a good way to display the results of the student's predictions.

The following rubric should be used as a guideline for both the teachers and the students to understand that there will be a grade given to each session of play. The rubric is generic in nature; modifications should be made with each type of game and data collected.

Teacher name: Ms. Robison				
Student Name: _____				
CATEGORY	4	3	2	1
Works with Others	Almost always listens to, shares with, and supports the efforts of others. Tries to keep people working well together.	Usually listens to, shares, with, and supports the efforts of others. Does not cause "waves" in the group.	Often listens to, shares with, and supports the efforts of others, but sometimes is not a good team member.	Rarely listens to, shares with, and supports the efforts of others. Often is not a good team player.
Monitors Group Effectiveness	Routinely monitors the effectiveness of the group, and makes suggestions to make it more effective.	Routinely monitors the effectiveness of the group and works to make the group more effective.	Occasionally monitors the effectiveness of the group and works to make the group more effective.	Rarely monitors the effectiveness of the group and does not work to make it more effective.

Quality of Work	Provides work of the highest quality. Keeps clear and concise records of data.	Provides high quality work. Has collected all of the data; not as easy to read.	Provides work that sometimes needs to be checked by other group members to ensure quality. Some parts of the data is missing.	Provides work that usually needs to be checked/redone by others to ensure quality. Needs clarification of data and must rewrite data information.
Attitude	Never is publicly critical of the project or the work of others.	Rarely is publicly critical of the project or the work of others.	Occasionally is publicly critical of the project or the work of other members of the group.	Often is publicly critical of the project or the work of other members of the group.
Time-management	Routinely uses time well throughout the project to ensure things get done on time. Group does not have to adjust deadlines or work responsibilities because of this person's procrastination.	Usually uses time well throughout the project, but may have procrastinated on one thing. Group does not have to adjust deadlines or work responsibilities because of this person's procrastination.	Tends to procrastinate, but always gets things done by the deadlines. Group does not have to adjust deadlines or work responsibilities because of this person's procrastination.	Rarely gets things done by the deadlines AND group has to adjust deadlines or work responsibilities because of this person's inadequate time management.
Contributions	Routinely provides useful ideas. A definite leader! Creates and asks good questions that the group or class may want to find answers to.	Usually provides useful ideas. A strong group member. A strong member to find answers to related problems.	Sometimes provides useful ideas. A satisfactory group member. Limited help to find answers to questions.	Rarely provides useful ideas. Does not work as a team player to calculate data for answers.
Problem-solving	Actively looks for and suggests solutions to problems.	Refines solutions suggested by others.	Does not suggest or refine solutions, but is willing to try out solutions suggested by others.	Does not try to solve problems or help others solve problems. Lets others do the work.
Focus on the Task	Almost always stays focused on the task and what needs to be done. Very self-directed.	Focuses on the task and what needs to be done most of the time. Other group members can count on this person.	Focuses on the task and what needs to be done some of the time. Other group members must sometimes nag, prod, and remind to keep this person on-task.	Rarely focuses on the task and what needs to be done. Lets others do the work.

Data collection example:

The student has two sets of three of a kind, and is trying to fill a sequence. There are 15 cards left in the draw pile and the student has been very careful to watch and count what has been discarded. The student is holding the cards 7, 8, and 9 of diamonds and the 7 of clubs. The student needs to draw a 6 or 10 of diamonds, or one more card to fill the two series to win. The student draws a 7 of spades. The student knows that the 6 or 10 of diamonds has not been played, but doesn't know if the other player is holding either card. What should the student do:

- Go for the series instead of the sequence?
- Throw away the 7 of spades and continue to try for the sequence.

The odds are easy to calculate in favor of going for the sequence:

- a. player has seen 27 cards, 25 cards unknown.
- b. you know you have four possible winners, 4:25 or 16% chance of drawing on the next turn.

The odds for going for any of the series: three possible winners, or 3:25 or 12% chance of drawing another match to either of the series on the next turn. Note that these odds are for the possibility of winning on the next turn (unlike the odds that are given in horse racing where the odds are shown in terms of the chance that the horse won't win).

Probability event will occur (how many cards will be a winner.)

1 – (probability event will occur)

What the student needs to realize is that, even though changing to the series appears to be the winner, statistically you do not look at this information by itself. The student will also need to watch and consider what the other player has been saving. If the other player appears to have the 6 or 10 of diamonds, then the change to collect to the series might be the better choice. The student will have to weigh the possibilities that the other player may be very close to a Gin. By choosing to collect the series the player is choosing the safer bet. These different possibilities may need explanation, and the teacher should stop the play periodically and explain as there are many variables to consider.

Directions for Gin Rummy:

1. Choose who will go first: each person selects a card from the deck; the student with the high card can choose if they want to deal first or not.
2. Use a regular deck of cards, all 52, and pass out ten cards to each person, face down.
3. Turn up the next card and put all the other cards (31) face down in a pile next to the card with its face up.
4. Each pair of students should take the time to sort their own hand.
5. The person who did not deal has first choice of the turned up card; they may either take it to help their hand or let the dealer have the same choice.
6. If either person takes it they must discard a different card down, face up.
7. It is now the other person's turn.
8. That person has the choice of the card that is face up or the pile that has all cards face down (only the last card that is showing in the face up pile is available to be picked up).
9. Again, that person must discard one card to the face up pile.
10. This goes on until a person knocks or Gins.
11. The amount of points that a person receives depends on the type of closing of the hand:

Gin: The person receives 20 points for a correct Gin, and all of the points from the other person's unmatched cards. This could vary from two to 100.

Points are: 2-10 are counted as face value; J, Q & K = 10 pts; the ace is 15 pts.

Knock: The person who knocks is not getting the three complete matching sets; they have three sets of three and one leftover card. This card must be between 2-10. Whatever card the person knocks with will be the point value to determine the winner. If the other person has a point value of more than the person who knocked, then the person who knocked wins. The points they earn is the amount of point difference between the person who knocked and the other person. If the person has fewer points from unmatched cards than the person who knocked, they win. They receive the

difference in points and 25 points as a bonus for under cutting the other person.

12. The play continues until a player reaches 100. Alternate the deal.

Hearts: Strategic planning and finesse! (Calamity Jane version)

Materials:

- Decks of American playing cards, enough for groups of four
- Instructions and tally sheets for all students (fig. 3), make sure there are plenty.
- Pencil
- Teachers may want the book, *According to Hoyle*

Introduction:

Focus: Learning the strategies of setting up one's own hand to be either on the offensive or defensive during the play of the game.

Objective: The student will need to plan their hand for the necessary strategy of not getting any points, or to get all of the points. The winner is determined by who has the lowest amount of points when one of the players reaches 100 points.

Purpose: This game will let students choose which cards to play. The deal of the hand is luck, but how the student plays the hand is not. The students will be able to count cards and be able to predict what other students will have in their own hands as well as what cards will be played during the play of the game.

Guided Practice: Have students read the instructions to the game. Make sure that everyone has a good understanding of the point system and that getting points is bad. The queen of spades is worth thirteen points, and each heart is worth a point. Observe the play of the game, and students' interactions with each other, to check for understanding. This game is a little more difficult: if you have a group of four or two groups of four that have previously played the game, let them play a couple of rounds with the other students watching from behind. This will allow for the shy or uneasy student to get a better understanding of the game and provide for a positive comfort level amongst the students.

Closure: In a large group, ask questions about the activity. For example:

- What kind of hand is considered to be a good offensive hand? What are the strategies for a take-all hand?
- What kind of hand is considered to be a good defensive hand? What are the strategies for a take-none hand?
- Is it important to count the cards as they are being played?
- What kind of cards should a person pass?
- During the hand which is a hold, why is this hand more difficult?
- Which type of hand does each student like to have?
- When can a person predict if a player is trying to get all of the points for a 'run'?
- How does getting a void in a suit help a hand?
- How does each player choose which card to lead?
- When is it important to take the lead? When is it important to not get the lead?
- How many times, on average, will a lead result in everyone being able to follow suit? (How many rounds of each suit are clear of any finessed cards?)
- Is having a low hand a guarantee of not taking any points?
- How many rounds does it take for someone to reach 100 points first?

- Does the person who takes the most tricks take the most points?
- What is the distribution of cards within a suit among the four players? How does this affect one's own hand? How does this affect the chances to win?
- What particular leads are best? Can a lead indicate the type of hand a person has?

Assessment: In the playing of Hearts, the teacher should collect the data and give credit for keeping good records of the game(s). A measurement of "good" would be if they recorded their own beginning hand accurately, they passed correctly, and they were able to answer questions from the closure section. The teacher should set goals for the students during each day that the game is played. Each day assign some of the questions; do not expect the students to be able to answer all of the questions daily. From the data that is recorded, take a day to analyze it. Have the students make predictions from the data and then play again to see if their analysis holds true. Can the students control the random variables with good plays of each hand. This game and unit should only continue as long as the students are recording, predicting and communicating there results both verbally and on paper.

Data collection example:

Compare how many hearts a person had before the first lead to how many points earned after the end of each hand. The students may want to compare the actual amount of hearts held, so it would be a count of one to one; or they may want to give a point value to each heart (i.e. 2=2, 3=3...J=11, Q=12, K=13 and A=14). The class may also want to compare how many spades each person had in the beginning and who took the queen.

Another type of data collection could be to determine if is there is a correlation between taking a lot of tricks and having the least or most amount of points.

Directions for Hearts:

1. Break up into groups of four. (If there is a group with only three, pull the 2 of clubs out of the deck, for five players pull out the 2 of clubs and 2 of diamonds).
2. Choose a dealer by having each person take one card; low card deals.
3. The entire deck of cards is dealt.
4. Each player looks at their hand, sorts by suit and decides which three cards they would like to pass with (the pass is to the right on the first hand, then left, across and every fourth hand it is a hold round, wherein each person keeps the cards as dealt. If there are three or five players then the pass is right, left and hold only).
5. The play is clockwise, so the person left of the dealer leads first after everyone has passed the three cards.
6. Whatever suit is lead, everyone must follow. The highest card of the same suit played will take the trick(no trump, but each time everyone plays it is called a trick).
7. If a person does not have any cards of that suit, then that person can play any card from their hand.
8. The play continues until everyone is out of cards; this should happen at the same time for everyone.
9. To count the score; the queen is worth 13 points, and every heart is worth one point.
10. Keep a tally: whoever reaches 100 points first loses, and the winner is whoever has the least amount of points.

Documentation

Albuquerque Public Schools K-12 Content and Performance Standards for Mathematics

Strand I: Global Mathematical Processes

Content Standard: The student understands and uses mathematical processes.

K-12 Benchmark: The student uses problem solving, reasoning and proof, communication, connections, and representations as appropriate in all mathematical operations.

Problem Solving and Reasoning

1. Develops resourcefulness and perseverance in problem solving in mathematics and other disciplines.
2. Recognizes when to use previously learned strategies to solve new problems.
3. Develops and uses strategies.
4. Monitors, discusses, and reflects on the process of mathematical problem solving.

Reasoning and proof

5. Makes and investigates mathematical conjectures and uses them successfully in developing and evaluating mathematical arguments and proof.
6. Uses the concept of counterexample to test the legitimacy of an argument.
7. Develops a logical sequence of arguments leading to a valid conclusion or solution to a problem.

Communication

8. Works in teams to share ideas, to develop and coordinate group approaches to problems, and to communicate findings.
9. Communicates mathematical thinking coherently and clearly to others.
10. Analyzes and evaluates mathematical thinking and strategies of others.

Connections

11. Relates applications to mathematical language in various modalities.
12. Identifies and connects functions with real-world applications.
13. Identifies how seemingly different mathematical situations may be essentially the same.
14. Investigates and explains the mathematics required for various careers.
15. Recognizes and applies mathematics in contexts outside the mathematics course.

Representations

16. Develops a repertoire of mathematical representations.
17. Selects, applies, and translates among mathematical representations to solve problems.
18. Uses representations to model and interpret physical, social, and mathematical phenomena.
19. Uses manipulatives, calculators, computers, and other tools as appropriate in order to strengthen mathematical thinking, understanding, and power to build upon foundational concepts.

Strand II: Number Sense and Operations

Content Standard: The student demonstrates number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.

9-12 Benchmark: The student understands rational, real, and complex numbers and uses a variety of means, including technology, as appropriate, to solve problems in these number systems.

Early High School:

Computation and Estimation

8. Applies ratios, proportions, and percents in more complex mathematical situations.
9. Judges the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities.
10. Uses estimation as a first step in calculations.
11. Evaluates relationships between key components in a problem situation.

Late High School:

Number Systems

2. Uses models, such as number lines and Venn diagrams, to show similarities and differences among real/imaginary numbers.

Number Theory

3. Develops fluency in operations with real numbers and matrices using mental computations or paper-and-pencil calculations(for simple cases) and technology(for more complicated cases).
4. Compare and contrasts the properties of real numbers and number systems.

Computation and Estimations

5. Develops a fundamental understanding of sequences and series.

Strand IV: Data Analysis, Statistics, and Probability

Content Standard: The student identifies patterns and special features of data and events of chance through experiences with meaningful mathematical problems that focus on comparing, predicting, representing data, and making decisions to communicate mathematical understanding.

9-12 Benchmark: The student explains the characteristics of a well-designed study, describes the differences between studies, and uses a variety of probability situations and computes and interprets simple probability cases.

Early High School:

Statistics

3. Identifies and uses statistical terms.
4. Defines sampling and recognizes its role in statistical claims.
5. Constructs and draws inferences from charts, tables, and graphs.
6. Writes an equation of best fit using a set of data points and makes predictions based on the graph.

Probability

7. Uses simulations to create and interpret discrete probability distribution
8. Computes the probability of a compound event.

Late High School:

Statistics

1. Explains the differences among various kinds of studies and which types of inferences can legitimately be drawn from each.
2. Recognizes the characteristics of well-designed studies including the role of randomization in surveys and experiments.
3. Analyzes the effects of data transformation on measures of central tendency and variability.

5. Uses simulations to explore the variability of sample statistics from a know population and to construct sampling distributions.
2. Evaluates published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions.
3. Explains how basic statistical techniques are used to monitor processes in the workplace.

Probability

4. Uses the fundamental counting principle, linear and circular combinations and permutations.
5. Uses experimental and theoretical data to represent uncertainty and to solve the probability of an event occurring.
6. Applies the concepts of conditional probability and independent events.

Stand V: Patterns, Functions, and Algebraic Concepts

Content Standard: The student demonstrates an understanding of algebraic skills and concepts through experiences with meaningful mathematical problems that focus on discovering, describing, modeling, and generalizing patterns and functions, representing and analyzing relationships, and finding and supporting solutions.

9-12 Benchmark: The student represents patterns with relations and functions, investigates rates of change, and symbolically manipulates numbers.

Early High School: Algebraic Operations

3. Uses order of operations and the commutative, associative, and distributive axioms to simplify expressions.
4. Recognizes and uses the identity and inverse properties.

Late High School: Algebraic Operations

1. Validates the reasonableness of conclusions.

Bibliography

Blackwell, David and M. A. Girshick. *Theory of Games and Statistical Decisions*. New York: Dover Publications, Inc. 1954.

A textbook written for the express use of data analysis in games.

Buckhiester, Philip G. "Probability, Problem Formulation, and Two Player Games." *Mathematics Teacher* 1994 Vol. 87: 154-159.

How to teach students to find data and learn probability from games.

Colby, Anne. "The New Deal Era." *Albuquerque Journal* 10 June 2001, Sunday edition, sec. E:1, E5

How technology has helped expedite the learning curve for predicting the odds in winning at cards.

Frey, Richard L. *According To Hoyle*. New York: A Fawcett Columbine Book, 1996.

A rule book to more than 200 games, includes expert advice on how to win.

Hadland, Francis. EBSCO host via World Wide Web; <http://search.epnet.com/> (London; needs a password to get in or via Telnet);

A short notice written in approximately 1700's to Londoners.

Hargrave, Catherine Perry. *A History of Playing Cards and a Bibliography of Cards and Gaming*. New York: Dover Publications, Inc.

1966.

A thorough compilation of the various types of cards throughout history.

Jacoby, Oswald and Albert Morehead. *The Fireside Book of Cards*. New York: Simon and Schuster, 1957.

A historical guide to the different types of games and how they are played.

Kolpas, Sidney J. "David Copperfield's Orient Express Card Trick." *Mathematics Teacher*. 1992 Vol 85: 568-570.

A description of how to show and prove a card trick.

Morehead, Albert H., Richard L. Frey and Geoffrey Mott-Smith. *The New Complete Hoyle*. Garden City: Garden City Books, 1964.

A older version of a compilation of rules on how to play different card games.

Schoonmaker, Alan N. *The Psychology of Poker*. Las Vegas: Creel Printing Co., 2000.

A look at the different types of poker players and what their strategies are.

Skolnick, Jerome H. *House of Cards The Legalization and Control of Casino Gaming*. Boston: Little, Brown and Company, 1978.

An overview of gambling and the effects on society.

Spanier, David. *Total Poker*. New York: Simon and Schuster, 1977.

A thorough compilation on the how to play poker, including how and when to bluff.

Wayland, Virginia. *Apache Playing Cards*

A historical look at Apache playing cards.

Gin Rummy Tally sheet: Names:

Original Hand Points Earned:

Spades		
Hearts		
Diamonds		
Clubs		
End of game- Sequences/Series:		
Cards left in face down pile:		

Specific cards needed:		
------------------------	--	--

Figure 2

Hearts Tally Sheet Players:

Cards dealt: Pass:

Hearts				
Spades				
Diamonds				
Clubs				
Cards passed				
Cards received:				
Which # trick, first point card: Queen of Spades:				
How many tricks taken:				
How many points earned:				
Overall card distribution: (Share with each other)				
Hearts				
Spades				
Clubs				
Diamonds				

Figure 3

