Problem Solving in Geometry

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Academic Setting

This curriculum unit is created for use in teaching seventh grade math students at Harrison Middle School, which is located in the south valley of Albuquerque, New Mexico. Harrison is located in a semi-rural area within walking distance of the Rio Grande River and Bosque.

Early Hispanic settlers built their farms along the Bosque and some of the students who attend Harrison are the descendants of these settlers. Other students are of more recent arrival. Many are immigrants from Mexico, a few Native Americans have moved into the area from surrounding pueblos, and others are the children of people who have come to the area from other parts of the city in pursuit of moderately inexpensive housing. The ethnic breakdown for the 1996-1997 school year was as follows: Hispanic 81.6%, Anglo 14.2%, Native American 2.4%, African American 1.3%, Asian 0.3%, "other" 0.1%. Most of the students come from households that range from the lower to middle socioeconomic levels. During the 1996-1997 school year, 74.4% of students participated in the free-lunch program.

There is a very wide range in math literacy levels among these students. Many of these students read at third or fourth grade levels (if not lower) and compute at equally low levels, while a few place in enriched math classes. The Harrison administration has decided to discontinue offering enriched math classes with the purpose of having all its students involved in the Math In Context (MIC) program. According to the publishers, *"Mathematics in Context* is a comprehensive curriculum for the middle grades," which was "designed to reflect [the NCTM standards] and to ground mathematical content in a variety of real world contexts." Furthermore, instead of relying on the teacher to explain or give definitions, the program is designed so that students investigate questions in context.

Many Harrison students are bilingual, while some are monolingual (English and non-English) and some have other special needs. A high percentage of middle school students in the South Valley eventually drop out of school, while a few complete high school (even fewer move on to college). Within the past four years, Harrison Middle school was designated as a "School in Need of Improvement."

Each of my classes will consist of approximately 25 to 30 seventh grade, regular education, students. Each group of students will be with me for 98 minutes of math on alternating days. One of my challenges, in implementing this curriculum unit, will be providing a sense of continuation despite this alternating day schedule.

Goals

The intent of the unit will be to work in conjunction with the Mathematics In Context (MIC) program being implemented at Harrison Middle School. The particular focus of this unit is to supplement the Triangles and Beyond unit, which is contained in the Mathematics in Context geometry strand, using problem solving processes and strategies. However, this unit may also be utilized as a "stand alone" unit if the MIC program is not being used. As such, it would serve primarily as a source of problems that might be solved using the problem solving process and strategies presented. This unit provides students with one problem solving model, strategies that they may use when attempting to solve problems, and additional problem solving opportunities that, specifically, focus on geometric concepts, such as spacial reasoning, etc…

It is my intent to provide my students with problems that create an awareness of how mathematics is relevant to real life situations, and to help them see that the process of solving mathematical problems can be engaging and enjoyable. Although the Math In Context program provides problems that are geared to do this, there are some areas that leave much to be desired. Often students just do not find the problems engaging or relevant. Sometimes a particular problem will be "beaten to death." From observation in the classroom, I have learned that when problem solving models are presented, students simply view these models as "just more information." I would like to teach and implement techniques for facilitating "need to know" situations. It will also incorporate mathematical literature and journal writing where appropriate. Journal writing will be used as a reflective, explanatory and assessment tool.

This unit addresses the following NCTM Curriculum and Evaluation Standards for School Mathematics:

NCTM Standard 12: Geometry (Triangles inside cover)

In grades 5-8, the mathematics curriculum should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can-

- identify, describe, compare, and classify geometric figures;

- visualize and represent geometric figures with special attention to developing spatial sense;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- develop and appreciation of geometry as a means of describing the physical world.

NCTM Standard 13: Measurement (Triangles inside cover)

In grades 5-8, the mathematics curriculum should include extensive concrete experiences using measurement sot the students can-

- estimate, make, and use measurements to describe and compare phenomena.

(NCTM Standards correlate to *Albuquerque Public Schools District Core Curriculum and Scope and Sequence* Mathematics Standard 8: A, B, C, D, E, F and Standard 9: C)

This unit also fulfills the following *Albuquerque Public Schools District Core Curriculum and Scope and Sequence* Mathematics:

NM State Content Standard 1 (1)

A. Differentiate among problem-solving approaches to investigate and understand mathematical content.

> 1. Use a variety of reasoning processes to investigate and understand the mathematical content of given problems, and explain which strategies are more efficient than others for certain types of problems.

B. Formulate problems from community mathematical situations.

> 1. Find examples of numerical and geometric concepts to interpret the environment and culture of their community or state.
>
> 2. Describe each example in a variety of ways (e.g., orally, in writing, in pictures, in graphs and tables, with concrete materials, and/or algebraic notation).

C. Develop and apply strategies to solve a wide variety of problems with an emphasis on multi-step and non-routine problems.

D. Verify and interpret results with respect to the original problem situation.

> 1. Check to see that the solution of a problem is reasonable and demonstrate why it is reasonable through models and/or manipulatives.

F. Generalize solutions and strategies to new problem situations.

> 1. Adapt previously use solutions and strategies to new problem situation by using pictures and physical models.
>
> 2. Adapt previously used solutions and strategies to new problem situations by making conjectures, gathering evidence, and building an argument to support mathematical concepts.

NM State Content Standard 2 (3)

A. Interpret and explain personal mathematical thinking to make conjectures and convincing arguments.

1. Use language to communicate mathematical ideas.

2. Reach agreement about work meanings and recognize the importance of commonly shared definitions.

3. Write about mathematical ideas in journals.

Context and Background

Rationale

Students need to learn geometric concepts and how to solve problems effectively. These needs are not mutually exclusive. Rather, investigation of each may facilitate the acquisition of the other. According to *Principles and Standards for School Mathematics* (NCTM 41), geometric concepts "are useful in representing and solving problems in other areas of mathematics and in real-world situations." Some simple questions that geometry can answer are questions such as, How much carpet is needed to carpet my room? How much paint do I need to paint a given area? What is the distance I will have to run in a race around the aforementioned triangular park? How far do I live from the mall? How will my house look from this perspective? However, geometry is more than definitions or formulas for arriving at numbers, "it is about describing relationships and reasoning...it has long been regarded as the place in the school mathematics curriculum where students learn to reason." These reasoning skills should also culminate with the capability of working " in proof in the secondary grades." (NCTM 41).

The logic inherent in geometry can be used to arrive at solutions to many types of problems. For example, when investigating geometry, one often uses the "if - then" logic; *if* the measure of each side of an equilateral triangle is the same, *then* side AB is the measures the same as side BC, and so on. This reasoning can often be extrapolated to other situations; if the sides of a triangular park are equal in measure, then the corners must be equidistant from each other. Therefore, the amount of concrete needed to pave each side should be equal, and so on.

Strategies used in solving geometry problems may be used in other situations as well. For example, many geometry problems may be solved by looking for patterns. Looking for patterns can also be used in identifying scientific concepts or social trends, economic trends, etc. Problem solving is defined, as "a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understanding." (52).

Middle school students should be able to relate to the need for problem solving. The average middle school student is at a developmental stage in his or her life in which they face many problems. Some are as trivial as, "which dress code violations can I get away with?" Others are of a more serious nature. While understanding of mathematical concepts and problem solving skills may not solve all of our students problems, practice at problem solving and the familiarity of geometric math concepts can foster confidence that may help students in many areas of their lives. What better place to begin teaching the art of problem solving than in a geometry setting. Geometry has its foundations in the tangible. Geometry surrounds us, we cannot escape from it. Geometry defines where we exist in space.

Our job as math teachers must be to provide problem solving techniques, processes and strategies that may eventually be used in other areas of our students' lives. Although there is nothing we can do about some things (such as students home lives, lack of value placed upon education and raging hormones), we can help our students to get some training and, possibly, develop some confidence in problem solving. In helping our students to cope with math problems, we can help save them from becoming victims of circumstance. Students will acquire "ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom," by engaging in problem solving situations in the classroom. Not only this, but they will be better prepared for "life and in the workplace" where "being a good problem solver can lead to great advantages." (52) This life preparation will occur while they learn geometry content through problem solving. What a deal!

Often, I have had students exhibit what has been called "learned helplessness." Learned helplessness manifests in a student deciding not to invest effort into finding a solution for a given situation. This is usually accompanied by the idea that someone or something will come to the "rescue" by providing an answer or solution for them. Perhaps this can be attributed to student beliefs that they are incapable of finding solutions for themselves. However learned helplessness works, it is both frustrating for me and, in the long run, disastrous for the student. Often, students have learned throughout the course of their education that they "can't" solve problems and, even if they can, that it is easier to wait for someone to provide solutions and answers for them. Furthermore, students often consider any teacher who has expectations of them as hard, mean, or as "picking" on them. This is due to conditioning that effective problem posing and opportunity in the classroom may serve to eliminate. Effective teaching "conveys a belief that each student can and is expected to understand mathematics and that each will be supported in his or her efforts to accomplish this goal ...Teachers' actions are what encourage students to think, question, solve problems and discuss their ideas, strategies, and solutions." We are challenged to create an environment where "serious mathematical thinking is the norm." We can do this by utilizing well-chosen tasks which "can pique students curiosity and draw them into mathematics." These tasks may be "connected to the real-world experiences of students, or they may arise in contexts that are purely mathematical." (18,19).

History of Geometry

The word geometry comes from the Greek words *geo* "earth" and *metrein* "to measure." (Greenberg 6). Although the word used to describe this discipline originated with the Greeks, geometry has been around since before the ancient Greek civilization. Much of geometry has been around for thousands of years. In the 5th century, the Greek historian Herodotus wrote that ancient Egyptians had surveyors who invented geometry (Greenberg 6). One can easily deduce the fact that geometric knowledge was necessary in the construction of the great pyramids of Egypt. In fact, surveyors in many civilizations other than those of the Egyptians and Greeks used geometry to map out land for agriculture and architecture. The Babylonians, Hindus and Chinese possessed much geometric knowledge. Babylonian and Chinese geometric knowledge included the Pythagorean Theorem (they did not call it by that name, however). The use of geometry was even sanctioned by the ancient Jews for use in the construction of the tabernacle of Moses. In particular, an approximation of π is given in I Kings 7:23.

The earliest named contributor to geometry was Thales, known as "the father of deductive reasoning," who lived from about 625-547 B.C. Thales was a direct descendent of the original Greek settlers. After accumulating enough wealth he devoted the rest of his life to the pursuit of academics. He founded the Ionian School. Thales discovered that "the base angles of an isosceles triangle are equal," that "two triangles are congruent if two angles and the included side are equal," and that "the sum of the three angles of a triangle equals two right angles" among other things. Thales discovered some elementary trigonometric relationships and "was able to determine the height of the pyramids by comparing shadows and employing the ideas of similar triangles. Today, Thales is remembered for employing his method of deduction in mathematics. (Lightner 15, 16).

In ancient Greece geometric knowledge was considered a religious tenet. Pythagoras (of Pythagorean theorem fame, 585-501 B.C.), was regarded by his contemporaries and disciples as a prophet. He taught that union with God could be achieved via mathematics (Greenberg 7, 8). He also believed that all numbers were rational. When irrational numbers were discovered, the Pythagoreans were reputed to resort to violence in order to suppress this new heretical teaching. A Greek historian by the name of Proclus wrote that a shipwreck was staged in order that the first man to publish the theory of irrational numbers might be silenced by death (8). When the truth could not be suppressed, the Pythagoreans insisted that the square root of two had to be equal to some fraction (Burger and Starbird 114). Pythagoras' reputation, however, is redeemed by his discovery that "the square of the length of a hypotenuse is equal to the sum of the squares of the lengths of the other two sides" in a right triangle ($a^2 + b^2 = c^2$)." (Burger and Starbird 210). So influential was Pythagoras that Plato posted the following notice over the door to his academy: "Let no one ignorant of geometry enter this door (Greenberg 1)."

Plato, known as the "public relations man for mathematics," also made significant contributions to geometric knowledge in the areas of polygon construction and polyhedra (platonic solids). His greatest contribution, however, was in his method of analysis, for he "insisted on accurate definitions, clearly stated assumptions and logical deductive proof...he also insisted that geometrical proofs be demonstrated, or constructed, with no aids other than a compass and straightedge." (Lightner 17).

The Greek philosopher-mathematician Euclid (c 365-275 B. C.), who was probably trained by one of Plato’s pupils, systematized geometry in his *Thirteen Books of the Elements* in about 300 B.C. The *Thirteen Books of the Elements* contain definitions, propositions, and postulates in which geometry is rigorously addressed. In fact, Euclid became the torch bearer of geometric knowledge. The academies of the Greek philosophers began to post, "Let no one come to our school, who has not first learned the elements of Euclid." (Heath 4). Euclid did not discover the math content in his books, but he logically arranged and unified ideas. Euclidean geometry still continues to have profound influence on geometry today.

Euclid’s geometry was based upon five fundamental principles called postulates or axioms. Postulates are basic principles or assumptions that are deemed to be self-evident. Euclid’s five postulates are as follows:

Postulate I. For every point P and for every point Q not equal to P there exists a unique line *l* that passes through P and Q.

Postulate II. For every segment AB and for every segment CD there exists a unique point E such that A is between A and E and segment CD is congruent to segment BE.

Postulate III. For every point O and every point A not equal to O there exists a circle with center O and radius OA.

Postulate IV. All right angles are congruent to each other.

Postulate V. For every line *l* and for every point P that does not lie on *l* there exists a unique line *m* through P that is parallel to *l*.

Geometry uses systematic reasoning or logic. That is, the way in which we think about geometric concepts must follow certain rules of logic. Some of the rules that we must follow deal with not using unstated assumptions to prove things, rather, we may use hypotheses, postulates, theorems, definitions, previous steps or logic rules to justify our conclusions. Other logic rules require some detailed attention to and study of symbols. Using logic, we can arrive at many discoveries based upon some basic assumptions.

The Greek Eratosthenes used geometry in about 250 B.C., to estimate, quite accurately, the distance around the earth. He did this my measuring the angles of shadows in different locales. He measured the shadows cast by poles of the same length in different geographic locations then employed geometric concepts and logic. (Try doing this with one pole in Vancouver and another in Tijuana!) Eratosthenes was also able to figure out the distances of the sun and the moon from Earth. (Concise 279).

From the beginning rigorous mathematicians saw a problem with Euclidean geometry in that the first four postulates could be readily verified, whereas, the fifth postulate could not. This was because it is humanly possible to draw line segments, whereas, it is not possible to draw actual lines. True lines would extend forever. Many mathematicians, therefore, invested much effort into proving or disproving postulate V. These efforts, eventually, culminated in the discovery of Non-Euclidean geometry (Greenberg 20, 148-162).

Non-Euclidean geometry was first discovered in the nineteenth century. Pioneers of Non-Euclidean geometry were Janos Bolyai, Carl Friedrich Gauss and Nikolai Ivanovich Lobachevsky. Non-Euclidean geometry makes use of one of two postulates that are not those of Euclid in place of Postulate V. The first postulate, discovered by Lobachevsky and Bolyai almost simultaneously says that two parallel lines may exist through any external point of a line. This leads to what is called hyperbolic geometry. The second postulate, discovered by G.F.B Riemann, allows no parallel lines through any external point. This leads to elliptic geometry. These postulates can be used to define all sorts of shapes and other useful information. The results, however, are usually identical to those attained through Euclidean geometry, except "for propositions involving parallel lines" (Concise 627).

Today, geometry continues to grow. It can, however, be defined as the study of physical forms, shapes and spacial relationships. "Geometric relationships give us a basic sense of order, coherence and beauty... geometry captures the structure and nuances of physical reality." (Burger and Starbird 209). Exploration of geometry can help us to think in new ways "that can be applied in many areas. Some fascinating concepts that can be explored in geometry include infinity, topology, Mobius bands, Klein bottles, fractals and the fourth dimension. These concepts have their computational/algebraic and geometric aspects.

In ancient Greece it was recognized that geometric knowledge was not simply something that could be "poured into a students head." When Euclid, like many math teachers of today, was asked by one of his students, Ptolemy, for a shortcut to learning geometry, he replied by saying "that there was no royal road to geometry." (Heath 1). On another occasion Euclid was asked by a student, "But what shall I get by learning these things?" (How often is this query uttered in modern classrooms!) Euclid's reply was to call his slave and command him "Give him threepence, since he must make gain out of what he learns!" (Heath 3). In the future when my students ask "why do we need to learn this?" I shall reply by loudly calling for my slave (or aid) and say, "Give him/her threepence..."

If there is no "royal road" to geometry, does this mean that path is less than pleasant? And why should my students be willing to be led down a road in which they encounter many problems? Problems, problems, PROBLEMS! "The very word 'problems suggest unpleasantness and anxiety." (Burger and Starbird ix) Students equate math with problems. That is, they equate math with unpleasantness! The truth of the matter is that math should not be viewed as just a series of problems, but as a tool in finding solutions to problems.. Conversely, as is one of the main goals of this unit, problems can be used to teach math content. Geometry can be used to solve problems and problems can be used to learn geometry.. The challenge will be to get students involved in the cyclic nature of this unit. So where do problems come from?

The nature of life involves problems. A lack of problems is a sure sign that death has occurred. Biological existence is a study in how environmental challenges have been or must be overcome. At each organismic level a host of problems or challenges has been or must be addressed, - and not only addressed, but there must be solutions reached if survival is to occur. For example, food sources have to be attained, consumed and processed in order for our physical bodies to continue in existence. Our bodies are faced with the challenge of breaking down this food into forms that may be utilized by each individual cell. Each cell must then combine oxygen and glucose correctly in order for overall body metabolism to remain in operation, etc... The organism that is more successful at solving these biological problems has a better chance of survival. Biology is not, however, the only area in which the problematic nature of life may be seen. Our social and psychological lives also involve problem solving. In Michael Crighton's novel, *Disclosure*, one of the characters makes the statement, "Everyone has a problem that they are trying to solve. We are all trying to solve some problem in life."

Polya's Problem Solving Process

There are any number of problem solving guidelines or processes. In fact, every math text seems to incorporate some version of a problem solving process in its first chapter. To the average student this is just more book knowledge. Some of these guidelines are so verbose that one can be intimidated just looking at the page. To date, the simplest and most logical procedure I have come across is George Polya's Problem Solving Process. It is as follows:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

In a nut shell, it is a simplification of the scientific method used in middle school science classes, which usually involve the following steps: 1) Identify the question or problem, 2) research the subject, 3) devise an hypothesis, 4) test your hypothesis (or do an experiment), 5) record results, 6) analyze results, 7) draw a conclusion.

Problem Solving Strategies

In an article entitled, *Problem Posing in Geometry*, Author Larry Hoehn says, "All that one needs to be a problem poser is first to be a problem solver." This tells me two things. First, that I need not always have all of the answers, because I too am a problem solver (student). Furthermore, I need not pose as an all knowing and invincible being for my students. Secondly, I can successfully pose problems to my students by following some simple guidelines. Some helpful guidelines offered in the article that may be used for posing and solving problems are as follows:

1) <u>Utilize different levels of difficulty</u>. Some problems may simply be inappropriate for certain students. Others may be adjusted to fit any age or level group. When adding to the list of problems in our teaching "repertoire," we should explore difficulty levels and identify what content may be learned in each situation (Hoehn 10).

We can also use the technique of "<u>solving a simpler problem</u>." That is, before having students tackle a certain problem, we can either provide them with, or work at arriving at the solution to a simplification of the problem. The student can then apply the same technique(s) to a more complex problem.

2) <u>Generalize a previous result.</u> For example, if the solution to a problem was "hexagon," we might choose to change the wording of the answer to "n-gon." This is called inductive reasoning. In inductive reasoning we look at many specific cases and deduce a generalization.

3) <u>Tailor a generalization to a specific problem</u>. We can also perform investigations as to whether any generalization is applicable to a given situation. This is called deductive reasoning. In deductive reasoning

we apply a generalization to a specific case. Likewise, we may have students apply and tailored solutions from previous problems to current ones.

4) Consider the converse of a problem. Before we have students tackle a certain problem we may present the problem in reverse form. For example, if we want students to demonstrate that the sum of the angles in a triangle is equal to 180 degrees by tearing of the corners of a paper triangle and placing them so that they form a 180 degree angle, we can have them work the converse of the problem first. That is, we might have them cut a semi-circle into three wedges, then have them use the corners created by doing this as the vertices of a new triangle (12).

5) Look for patterns. The teacher should help students in becoming proficient at identifying possible patterns. Once students become accustomed to looking for patterns some solutions will "leap out" at them.

6) In demonstrating situations for students or when students are attempting to solve problems, use or have students use the tangible whenever possible. Drawings, manipulatives, or objects, etc., should be used for good effect.

In an article entitled, "Realistic Problem Formulation and Problem Solving," which appeared in *The Mathematics Teacher*, authors Anthony Ralston and Stephen S. Willoughby state that "we seldom introduce students to problems that lend themselves to long-term, in-depth analysis at different levels of intellectual accomplishment. They advocate the use of games that can be used from second grade on up to college undergraduates. The game they highlight in the article is called "Roll-a- 15" (Ralston 430). While I was not completely sold on that particular game, I do appreciate the point Ralston and Willoughby are trying to make. If play is thought to enhance survival in the animal kingdom because it sharpens acuity and skill, then, perhaps it is equally beneficial for humans to play problem solving games!

Advice for Other Aspiring Math Teachers

Finally, as a "math teacher novice," it is my humble opinion that math teachers can assist students in becoming problem solvers by being problem solvers themselves. That is, that we should help students become motivated problem solvers by serving as problem solving role models.

At some time during my public school education, I too began to considered myself a "math victim." Perhaps it was because I received a swat on the knuckles for every multiplication fact I missed that estranged me from the subject. Later, the thought of having to solve a math problem would strike terror into my heart. If I failed at coming up with the right answer, I just knew something unpleasant would happen to me. Once I became comfortable with algorithmic problems, word or story problems became my phobia. I too have said, "I hate math," and "I'm not good at math." The types of problems that have, traditionally, been given to students of mathematics were usually "naked" numbers that were either taken out of context or had no relation to the tangible. In my school days, math homework consisted of many number or computational problems with, occasionally, one or two word problems, that students generally skipped and the teacher glossed over). Thus, I do not recall understanding as a student, what relevance math had to the "real world." It was not until well into my adult life that I became enamored of problem and puzzle solving. Why? I'm not sure. Perhaps, it was the fact that I was no longer in an academic setting in which grades depended upon using math, so my stress level was significantly reduced. Getting involved in puzzles that pique my interest, however, has turned me on to math. At last, math has become fun. At some point I also came to the realization that since I was required to teach math, I had better "make peace with my adversary." I realized that if I am to teach students to love math, I myself must learn to appreciate the role that math plays in the world. In a letter to teachers published by The National Middle School Association, we are told, "You are not only an important contributor to the learning enterprise, you are a major lesson!" By extension I am a geometry problem solving lesson. Therefore, my advice to the person who wishes to teach math? Learn to love it. This is not an easy confession to make since I have been "chief of sinners" in this area.

One way in which the teacher might gain an appreciation of problem solving is to explore problem solving situations that are interesting to the teacher, personally. Some of these might include problems that can be used in the classroom, but this need not be the case. If fact, if I am to love the subject that I teach, I must practice using problem solving skills and mathematical knowledge for "personal edification" as well as for professional use. For example, the aspiring math teacher might use the daily cryptogram in the *Albuquerque Tribune* or read music (value of whole, half, quarter, eighth notes, etc.) to gain an appreciation of math in real-world context. The nature buff might gain an appreciation by looking for how Fibonacci patterns exist in nature. Text books written for teacher professional development, such as *The Heart of Mathematics: An Invitation to Effective Thinking*, are written specifically for the purpose of turning teachers on to math and problem solving. Or as they put it, "to give ... motivation to make mathematical thought a pleasurable part of their lives." The publishers also "hope this textbook helps teachers bring mathematics to life for students." (Burger and Starbird 620).

Implementation

Essential Question: How does my knowledge of the problem solving process and strategies help me become a problem solver?

While participating in problem situations, students will be introduced to Polya's Problem Solving Process and various problem solving strategies. They will also have hands-on, minds-on practice with strategic thinking. Students will be asked to reflect upon problems, the process and strategies through writing.

This unit will take approximately four weeks to complete. One week will be spent in having students appreciate that the world may be described using geometry, recognizing that triangles play an important role in the world and becoming familiar with the relationships between the side lengths of triangles. During the second week students will continue triangle side length relationships and explore angle relationships. In the third week, students will identify how the side lengths of triangles and opposite angles are related and begin to investigate the properties of parallel lines and parallelograms. Finally, the fourth week will be devoted to investigating translations, rotations, reflections and polygons.

Objectives

- Students will make a triangular model using mini marshmallows and straws.
- Students will locate examples of triangular objects in magazines and newspapers. They will cut and paste making collages.
- Students will use Student Activity Sheet 1 (MIC) to identify spatial relationships of triangles in the frame of a bridge.
- Students will construct a model of the bridge frame using toothpicks and clay.
- Students will make drawings that show how they expect rotating triangles to look from different perspectives.
- Given an outline of stacked triangles, students will investigate how many different ways the triangles may be stacked (permutations possible).
- Students will restore triangles in which one or more angles have been torn off.

- Given certain lengths of spaghetti, students will be required to investigate which lengths are appropriate for constructing triangles.
- Students will be required to make models, using spaghetti, of acute, isosceles and equilateral triangles.
- Using a similar triangle, students will be required to calculate distance of a point.
- Students will be required to use circles to create triangles.
- Students will use paper triangles and semi-circles to construct the concept that the sum of the angles of a triangle equal 180 degrees.
- Students will be asked to determine the measure of angles after being given the measure of one angle.
- Given any triangle, students will be asked to identify the longest and shortest sides and to investigate their relationships to the largest and smallest angles of the triangles.
- Students will be asked to identify properties of parallel lines in pictures and manipulatives. They will be asked to find methods of constructing parallel lines.
- Students will use stamps and stencils to investigate the three transformations (reflections, rotations and translations).
- Students will identify with letters of the alphabet can be transformed and which have a line of symmetry.
- Students will be asked to divide polygons into similar triangles and to use rotating triangles to create polygons.
- Students will use polygons to create designs and pictures and to locate instances in the real-world where these may be used.
- Students will be required to use triangles in the construction of rhombi.

(Standards denoted in parentheses previously listed under goals in the Academic Setting.)

Materials Required

- Three straws and three mini-marshmallows per student
- Enough old magazines and newspapers for each student to cut several triangles from
- Class set of scissors
- Class set of glue or paste
- One sheet of construction paper for each student
- About 20 toothpicks per student or pair of students

- Clay or gumdrops, about ten per student
- pre-made triangles for stacking permutations
- Spaghetti, about 12 per student
- String and/or compasses, class set
- Drawing paper, several sheet per student
- Copies of the alphabet, large letter size helpful
- Pictures of parallel lines from real-world, as examples
- Pre-made triangles or copies of various triangles (for side length and angle relation activity)
- Pre-made squares and rectangles, at least one per student
- Stencils and stamps, at least one of each per student

Vocabulary

Angle, compass, equilateral triangle, isosceles triangle, line of reflection, line of symmetry, parallel, parallelogram, reflection, rhombus, rotation, scalene triangle, translation, vertex.

Weekly Lesson Plans

When this unit is being used in conjunction with *Mathematics In Context Triangles and Beyond*, the students may be assigned appropriate questions from each section and Student Activity Sheets may be used. The majority of the following problems are taken from *Triangles and Beyond*. If MIC is not being used the references to MIC questions, etc…, may be ignored. Polya’s Problem Solving Process and the problem strategies should be posted or otherwise made accessible to students. Students need to be coached in selecting appropriate strategies for each problem. (Possible strategy "catch-phrases" for posting have been underlined in the "Problem Solving Strategies" section.)

Students will use Polya’s Problem Solving Process for each problem. Some of the problems may be assigned as homework or homework may be assigned from MIC. Students will write down each step in their math notebooks as they proceed. (My students will be required to maintain a notebook/journal.) Problem 1 is given as an example.

Week 1 - Introduction to Geometry and Triangles

Begin by having students identify triangles in the classroom and world at large. Ask students to describe and define triangles in their own terms and to compare their definitions with those in texts, etc... Students will make a collage of triangles using triangles from magazines and newspapers. They will be given the following problems to solve and questions from MIC will be assigned.

Problem 1

Understanding the Problem - Can you make a triangle from three straws measuring 12, 4 and 6 centimeters in length? (Triangles 3)
I have to make a triangle that has one side 12 cm's long, the 2nd side should be 4 cm's long and the 3rd should be 6 cm's long.

Devising a Plan - Students should be told to write the steps they intend to use in solving the problem. Example:

1. Use a ruler to measure and mark 1 straw at 12 cm's
2. Cut the straw. . .
7. Try to arrange the straws so that. . .

Carrying out the Plan - Have students follow their plan and record outcome.

I followed every step and found out that I could not make a triangle without bending the longest straw . . .

Looking Back - Have students evaluate whether a satisfying answer was arrived at or whether a new plan needs to be created. Students should be guided in evaluating with steps may need to be changed, etc. I was not happy with the result. I think I should measure each straw again . . .

Problem 2

Understanding the Problem - Given Student Activity Sheet 1 (Attachment 1), identify three triangles that are part of the bridges frame. Now, using toothpicks and clay (or gumdrops), make a model of the part of the frame given. (Triangles 8)

Problem 3

Understanding the Problem - Given a triangle, make five drawings showing how a triangle changes as it is rotated around one of the sides. (Triangles 10)

Problem 4

Understanding the Problem - Given an outline of three stacked triangles and three corresponding pre-made triangles, in how many different ways can they be arranged? (Triangles 14)

Problem 5

Understanding the Problem - Restore given triangles in which one or more angles have been torn off. (Triangles 16)

Problem 6

Understanding the Problem - Given spaghetti, break it into various sizes. What can you tell about which lengths are appropriate for making triangles?

Problem 7

Understanding the Problem - Using spaghetti, Construct models of an isosceles, an equilateral and a scalene triangle. (Triangles 20,24)

Week 2 - Angle/side Relationships and the Sum of the Angles of a Triangle

Students will be introduced to the concepts of equilateral, isosceles and scalene triangles. Problems 8 and 9 will be used to show that the sum of the angles of a triangle are equal to 180 degrees. Students will answer questions in MIC workbook.

Problem 8

Understanding the Problem - Draw and cut out and triangle using drawing paper. Now tear off the three corners. Can you place all three corners so that together they make a straight line? No corners should overlap. (Triangles 38, 40)

Problem 9

Understanding the Problem - Using a piece of scrap or drawing paper that has one straight side, tear or cut it so that you have a semi-circle. Now cut this semi-circle into three slices. Using these slices as the angles of a triangle construct a triangle by drawing possible sides. (Triangles 42, 44)

Problem 10

Understanding the Problem - Give any triangle and one or two angle measures can you figure out the remaining angle measures?

Problem 11

Understanding the Problem - Given a set of triangles, identify the longest side and the biggest angle. Do you see any correlation. Do the same to the shortest side and the smallest angle. Can you come up with any case in which your correlation is not true?

Week 3 - Side Length and Angle Relationships Continued

Using a student volunteer and a length of string demonstrate how a circle can be delineated if the one person is the center point. This will help prepare students for problem 12. Student hint: The endpoints of the side of the triangle given can be used as the center points of two circle. What happens when the circles intersect? Students will answer the appropriate questions in the MIC workbook.

Problem 12

Understanding the Problem - Using a piece of string and the fact that one side of a triangle measures 5 inches, construct an equilateral triangle.

Problem 13

Understanding the Problem - Use a compass to create an isosceles triangle if one side measures 6 inches.

Problem 14

Understanding the Problem - Give students an equilateral triangle. Tell them that it is similar to the big triangle you will partially outline for them. Mark two points (vertices) in the classroom (or outdoors) and have students determine where the third point is. Follow this procedure with an isosceles triangle and, finally, using scalene proportions.

Week 4 - Parallel Lines, Parallelograms, Transformations and Polygons

Begin by having students identify parallel lines in pictures and in room. Have students work on problem 15. Have students use sets of parallelograms to construct parallelograms. Students will answer MIC workbook questions. Students will then proceed to problems 16 and 17 and answer appropriate questions in workbooks.

Problem 15

Understanding the Problem - Figure out a "sure-fire" method for constructing parallel lines.

Problem 16

Understanding the Problem - Cut a given polygon into triangles that all have the same side and angle measurements.

Problem 17

Understanding the Problem - Make a rhombus using two triangles that have the same side and angle measurements. (Triangles 116)

Assesment

At the end of each week, students will be allowed to choose their best problem as an assessment of concept aquisition and problem solving skills. Notebook/Journals will also be used to assess progress. At the end of the unit a formal assessment, taken from *Triangles and Beyond* (112,114), will be administered

Additional Geometry Reasoning Problems

In an article that appeared in the *Mathematics Teacher*, Author Millard E. Showalter says:

> A good mathematical problem situation is one that not only invites finding its solution, but also allows for extension beyond its immediate confines. The teacher needs to change the emphasis of problem solving from "Here’s a problem, solve it’ to ‘here’s a situation, let’s explore it!" (Showalter 5)

I have included the next three problems primarily "just for the fun" of proplem solving. They are not devoid, however, of geometry content. For as Palmer states, "it is importand to get students inside a subject, it is equally important to get the subject inside the students." (7). These problems can sharpen geometry reasoning skills and may be used at teacher discretion.

Problem A

Take an 8 ½-inch-by-11-inch sheet of typing paper and fold it eight times. The paper must be folded in half each time and rotated 90 degrees between folds. Under normal circumstances this will prove to be an impossible task. Therefore, in this case no solution will be forthcoming. However, the "why" can then be analyzed by generating a pattern between the number of folds and the number of thicknesses. (Showalter 5) For example:

#folds	#thicknesses
1	$2 = 2^1$
2	$4 = 2^2$
3	$8 = 2^3$ ...
8	$256 = 2^8$

Problem B

Suppose we start with a large piece of paper 1/500 of an inch thick. We then cut the paper in half and stack one side on top of the other. If we repeat the procedure fifty times, how high will the stack be? (Showalter 6)

1) Have students write down an estimate.
2) Record some of these estimates on the chalkboard.
3) Cooperative learning/technology can be used at this point to help students .
4) A solution that works is $(2^{50})(1/500)(1/12)(1/5280)$=35,540 miles!

Problem C

The PTA is thinking of holding a Fun Fair with lots of different activities. One activity would be to guess the number of nickels needed to stack from floor to ceiling in our classroom. We need to help them out by giving them the most accurate answer that we can. Don't forget to share with the PTA exactly how you figured out the answer so they will be confident in giving the prize to the person who has the closest guess during the fair. (Task 4)

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