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Mathematical Patterns: Fibonacci Numbers in Nature

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Academic Setting

Truman Middle school is located at 9400 Benavidez Road in the southwest part of Albuquerque. It is predominantly Hispanic, at 80% of the student population. Approximately 4% are Native American, 4% African American, 10.6% Anglo, 0.5% Asian and 0.4% are of "other" backgrounds. The average for all middle schools in Albuquerque is 48.4% Hispanic and 41.6% Anglo. Many of our parents are working in low paying jobs and as a result, about ¾ of our student population receives free or reduced lunches provided by APS Food Services. Eligibility for free or reduced meals is based on family size and income. For example, a family of four with income less than $21,300 qualifies a student for free meals, and income between $21,300 and $30,400 qualifies the same size family for reduced meals. The percentage of these students is considered an indicator of socio-economic status.

Test scores at Truman show that most of our students are four-five years below grade level. Even though we bring up the scores on the Terra Nova from 6th grade when they enter to 8th grade when they leave, it is not enough. We are currently on probation and will be taken over by the state at the beginning of this next school year.

I teach math and science in an 88-minute block. My students are 6th graders with a lot of enthusiasm (from most of them) for any task that I ask of them. This unit is being designed with math and science being explored together, although it can stand alone in the mathematics classroom. In my eight years with Truman, I have observed that students here have minimal exposure to educational activities at home. Whether because of single parent situations or both parents working all the time to make ends meet, they do not have the time or money, or possibly even the knowledge that educational activities exist.

Rationale

Mathematics surrounds us in our daily lives. Paying bills, calculating prices, and the like are a part of our daily lives. My students know that they will be doing this type of mathematics when they are adults, and book mathematics in the classroom. What I hope to introduce to them is something special, beautiful, and maybe a little mysterious. During the school year 2000-2001, my team took our students on a field trip to the top of Sandia Peak via the Tram. Nearly all of them said they had never been on the tram before, and for many of the students it was their first time to go to the top of the Sandias. With this in mind, this unit will concentrate on Fibonacci numbers that are found in nature.

According to the standards set forth by the National Council of Teachers of Mathematics (NCTM), all students should be able to recognize, extend and generalize patterns in the middle grades: "Exploring patterns helps students develop mathematical power and instills in them an appreciation for the beauty of mathematics." All my research has stressed that the key to mathematical growth is developing this ability to analyze patterns. "The ability to recognize patterns is the key to mathematical thinking. Patterns are basic to the understanding of all concepts in mathematics."(Burns 92).

To understand this topic it must be presented to mid-schoolers in a simple, practical, and realistic way that is relevant to their age group. If students learn to recognize patterns in mathematics, they can learn to recognize patterns in other academic areas. Hopefully, by studying mathematical patterns students can begin to learn how to recognize, analyze, and make connections and conclusions to problems in subjects other than math. Analyzing patterns is a higher level thinking skill that students should be expected to perform at this age level.

Also, while studying mathematical patterns, it will be important to integrate this concept with language arts, social studies and science. Students are consistently asked in language arts to draw conclusions, inferences and connections in stories, novels and other writings. They need to learn to draw connections between very different stories and make connections to their own lives. What were the patterns and characteristics of this character that caused them to do what they did? What were their patterns of behavior? Could they have been different and if so, what would have been the repercussions of a change of behavior? Hopefully, by examining fictional characters, students will examine the patterns of their own lives and consequences of their decisions. For social studies, students will readily see the patterns of history repeating itself over and over again. They will recognize the patterns that brought about the Revolutionary War, the Civil War and WWII specifically. They can examine the patterns of hate, prejudice and greed that led up to these wars. In science, through careful study, events in the universe occur in consistent, comprehensible patterns. "Scientists believe that through the use of the intellect and with the aid of instruments that extend the senses, people can discover patterns in all of nature."(Science and Science Education 1) Science is about seeking the truth through observation, hypotheses and experiments where it is possible to find general principles about the natural world. From the shapes of snowflakes to the formation of crystals, from plant growth to animal behavior, mathematics will play a role. It can be any type of mathematics from gathering data and graphing it to looking for patterns in a pine cone.

Background and Context

My goal is to broaden the scope of mathematics for my students; this in turn will improve their number sense. They will do this through the study of patterns and specifically, the Fibonacci sequence.

Student background required is knowledge of basic operations, addition, subtraction, multiplication and division. Also, knowledge of T-charts, coordinate graphing, decimals, ratios, rounding, and the use of a calculator will be helpful. Teacher must do some research on patterns for examples.

The first thing that needs to be done is to define the word "pattern." According to the dictionary, a pattern is:

> An ideal worthy of imitation, a model to be followed, an artistic design, traits, or characteristics.

From this definition we can begin to explore the many possibilities of patterns in these students' daily lives. For example, coming and going to school, daily rituals, sleeping patterns, when and how they do their homework, family patterns at home, and patterns with peers and teachers to name a few.

Next we would begin to explore mathematical patterns specifically. There are two types of patterns that we see introduced in middle school mathematics: repeating patterns and growing patterns. Repeating patterns are most likely to be the easiest for students to see. There are many resources available for teachers using patterns, but problems can be made out of anything. Shapes, numbers, colors, rhythms, etc…; you name it, you can find a pattern problem. Generally, the first three to six characters of the pattern are shown and students are asked to recognize and then extend the pattern, usually for at least three more characters. An easy way for teachers to get started is with rhythmic patterns.

Example 1: Rhythmic patterns

Clap, Clap, Stomp, Stomp, Clap, Clap…

Pattern continues with …stomp, stomp, clap, clap.

Clap, snap, snap, clap…

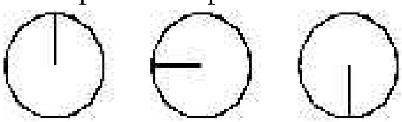
Pattern continues with…snap, snap, clap.

Example 2: Alphabet patterns

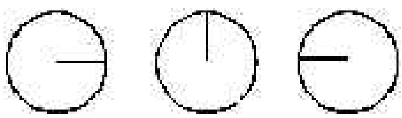
ABABAB…

The next three letters would be ABA…

Example 3: Shapes

 ...

Pattern continues with…

 ...

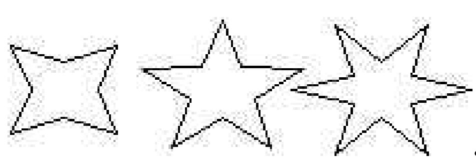
Growing patterns are usually numerical, although you can find examples in pictures, symbols etc... These patterns start with something concrete and can be extended and generalized as an introduction to algebra. There are many books on the topic of how to go from a concrete picture pattern to algebra. See teacher resources at the end of this paper.

Example 1: 0,4,8,12…

Pattern continues with…16, 20,24…
This pattern grows by adding 4 to each previous number to get the next number in the pattern.

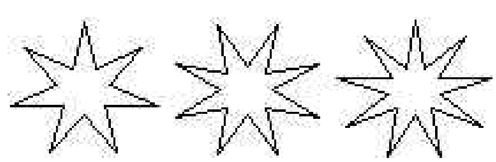
Example 2: 1,2,4,8…
Pattern continues with… 16, 32, 64…
This pattern grows by multiplying each previous number by 2 to get the next number in the pattern.

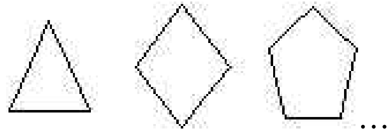
Example 3: Pattern is increasing the number of points.

 **….**

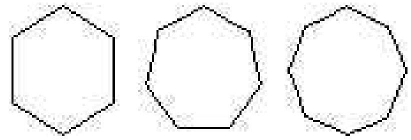
Pattern continues with…

Example 4: Pattern is increasing the number of sides.





Pattern continues with …



History

This next example is called the Fibonacci Sequence or Fibonacci Numbers. It is these numbers that will be explored further.

Example: **1, 1, 2, 3, 5, 8…**

1+1 1+2 2+3

3+5

You will notice that each successive number is found by adding the two previous numbers. The first 13 numbers of the Fibonacci sequence are as follows: **0,1,1,2,3,5,8,13,21,34,55,89,144…**

The mathematician known as Fibonacci, and for whom the pattern sequence is named was the son of Guilielmo Bonacci. Fibonacci (pronounced "Fee-buh-NOTCH-ee") is short for filius Bonacci, which means son of Bonacci. His real name was Leonardo and he was born around the year 1175 AD. He was called Leonardo de Pisa as he was from the town of Pisa, Italy (of Leaning Tower fame). He also used the names, Leonardo Pisano, Leonardo Bigollo (bigollo meaning traveller), Bonaccii, and Bonacij. He traveled with his father and grew up in North

Africa where he was educated under the Moors. His father worked in a warehouse, where candles were exported to France, as a kind of customs official maintained by Pisan merchants in Bugia, now called Bejaia. Bejaia is a Mediterranean port in northeastern Algeria. He later traveled extensively around the Mediterranean coast, and it was while on these travels he recognized the advantages of the mathematics being used in the countries he visited. He was introduced to the "Hindu-Arabic" digits, the decimal point, and a symbol for zero that we use today. He found these far superior to the Roman numerals that were prevalent in Europe at the time. He returned to Italy and published, in 1202, a well known book under the nam de plume of "Fibonacci" called, *Liber Abaci.* This book was written about the arithmetic and algebra that he learned during his travels, and it was instrumental in introducing Arabic numerals into western culture.

Fibonacci enjoyed making up his own word problems, one of which is his famous rabbit problem that shows the Fibonacci sequence for which he is best remembered. This problem was in his book *Liber Abaci* but it was a French mathematician named Edouard Lucas who gave the name "Fibonacci numbers" to this sequence.

The Rabbit problem

The problem involves a pair of baby rabbits that take one month to grow to maturity and another month to produce offspring. This one pair can produce offspring every month after maturity. Each successive offspring produces its own offspring in the same manner. Assume that no rabbits die and they can reproduce without any problems.

Let: ab=baby rabbits AB=mature rabbits

Let: ab_{1}, ab_{2}, ab_{3}, etc... be the offspring of AB

Let: ab_{1a}, ab_{1b}, etc… be the offspring of AB_1 and so on with AB_2, etc.

1st month	ab	**1** pr rabbits
2nd month	AB	**1** pr rabbits
3rd month	AB¬ ab1	**2** pr rabbits
4th month	AB¬ ab2 AB_1	**3** pr rabbits
5th month	AB¬ ab3 AB_2 AB_1¬ ab1a	**5** pr rabbits

	AB¬ ab4	
6^{th} month	AB_3 AB_2¬ ab2a AB1¬ ab1b AB1a	**8** pr rabbits

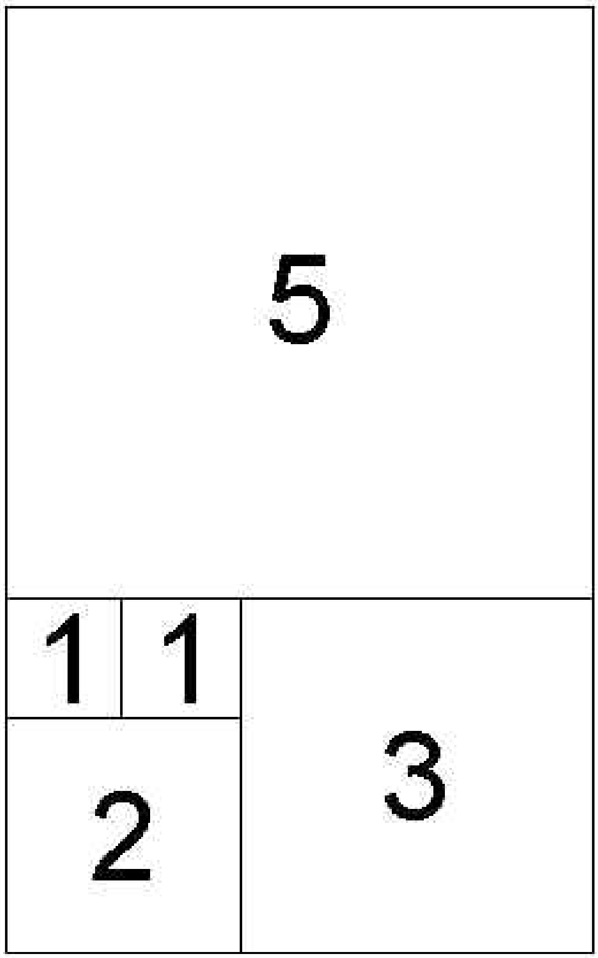
The 7^{th} month gives **13** pairs of rabbits:

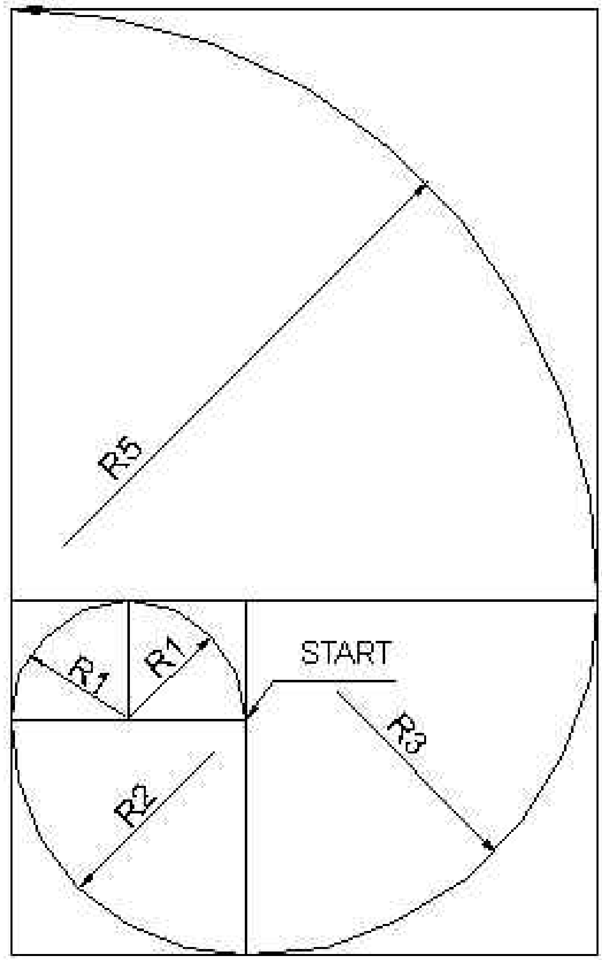
AB¬ ab5 AB4 AB_3¬ ab3a AB_2¬ ab2b AB2a AB_1¬ ab1c AB_{1b} AB_{1a}¬ ab_{1aa}

As you may now see, this problem becomes very confusing because the number of rabbits grow so quickly and it is difficult to keep track of all the rabbits and where they are in the reproductive cycle set forth in this problem. In all of the research, I did not see an excellent or easy example to follow for this particular problem.

Equiangular Spiral

Another example of Fibonacci numbers that can be seen in nature is the equiangular spiral, also known as a logarithmic spiral. It is a long, slow spiral and it can be constructed by drawing squares of sides 1,1,2,3,5….alongside each other and connecting opposite corners with quarter-circle arcs so that the arcs connect.





Fibonacci Numbers and the Golden Ratio

The golden ratio leads to one of the most famous and pleasing shapes in mathematics. The golden rectangle has been used by architects and artists since the time of the ancient Egyptians. If ratios of two successive Fibonacci numbers are taken, with the larger number in the numerator, and the decimal equivalent is calculated you will find the following numbers: 1/1=1, 2/1=2, 3/2=1.5, 5/3=1.666…, 8/5=1.6, 13/8=1.625, 21/13=1.61538… As you continue to take the decimal equivalents of Fibonacci numbers it becomes closer and closer to converging, to one value. This value is known as the Golden Ratio and the value is approximately 1.618034. It is also called the golden section, golden number, golden proportion or golden mean. It is represented in mathematics with the Greek letter **Phi** Φ . This ratio is seen in golden rectangles, a shape that is described as very pleasing to the eye. An example in which the golden ratio in a rectangle is seen is in three by five and five by eight note cards. Other items are light switch covers, mirrors and other manufactured goods. It is also seen in the human body. There are golden proportions of the head, face, the hand and the body.

Fibonacci in Nature. Why?

Scientists have some ideas about why Fibonacci numbers are so prevalent in nature. They think this number sequence is found in plants and animals because it offers the optimal growth or optimal packing arrangement. In plants, while growing, leaves will grow so that leaves above will not block sunlight to the leaves below. The leaves grow in a spiral of Fibonacci numbers.

Fibonacci numbers in plants

Fibonacci numbers are found in branching plants as they grow. There is one stem which branches into two. Then one of the new stems branches into two while the other one waits. This pattern of one branching while the other waits is repeated for each of the new stems. An example would be the sneezewort although some trees, root systems and algae exhibit this type of branching pattern .

The branches of a tree may spiral upward in a Fibonacci ratio. Find a starting point at the bottom of the tree and count how many branches and how many turns are needed to get to a branch that is directly above your starting point. The ratio will be spirals/branches.

They are found on flowers in the number of petals. Some examples are lilies, irises, buttercups, delphiniums, corn marigolds, asters, and daisies. They are also found in the seed heads of flowers. The number of spirals coming from the center both left and right are Fibonacci numbers. They can be seen in a sunflower or a daisy.

Pine cones are another example wherein Fibonacci numbers show up. Soak pine cones in water so that they close up and it will be easier to count the spirals. Count the number of spirals seen in both directions starting from the base of the pine cone. You can do this with artichokes and pineapples as well.

Fibonacci numbers can be found in animals in a few ways. First there are the pentagonal shapes (five is a Fibonacci number) found in some animals. Examples would be starfish, sand dollars and sea urchins.

The equiangular spiral is the next example of Fibonacci in animals. Examples are the shell of a chambered nautilus, snail shells, sea horses, the tusks of elephants, and horns of some animals.

Follow the family tree of a male bee and Fibonacci numbers will appear much like the rabbit problem. The reason is that male bees only have one parent, a female bee, but female bees have two parents, both a male and a female.

Fibonacci numbers are seen in the proportions of the human body. For example the measurement from the navel to the floor and the top of the head to the navel is the golden ratio. Another example is found when you measure the length of the middle bone in a finger and compare it to the shortest bone of that same finger; the golden ratio is found again.

Other areas where Fibonacci numbers are found

My unit is on Fibonacci numbers and nature, but they are found in many areas of life. They are found in art and architecture. There is a shape that is unconsciously favored by most people. It is known as the "golden rectangle." They are found in music, most notably on a piano keyboard. An octave is made up of eight white keys and five black keys. Also seen in stock market analysis is the Elliot Wave Principle. The Elliot Wave Principle is a graph of the up and downward trends of the stock market. It shows five upward waves and three downward waves forming a complete cycle of eight waves. All of these numbers are Fibonacci numbers. In the human body, the golden ratio can be found in the proportions of the entire body. For example the head, face, and hands contain the golden ratio. Crosses, clocks, game boards and musical instruments are just a few examples of where Fibonacci numbers turn up.

Implementation

My unit will be implemented at the beginning of the year when I am using the math book, from our math series, which deals with patterns. I may spread the activities throughout the year depending on the availability of plants needed, and when I am teaching about plants and animals during science. These lessons are planned for an 88 minute block of time and should take approximately seven to eight days.

Assessment

Assessment will be on going throughout the unit and will be based on observation on a daily basis. A math journal will be kept during this unit on Fibonacci numbers. It will include graphs, calculations and reflections.

State Standard 1: Unifying concepts and processes

Students will understand and use mathematics in problem solving. **B1:** Find examples of numerical and geometric concepts to interpret the environment and culture of their community or state.

E1: Use appropriate tools (e.g., manipulatives, calculators, and computers) to observe and explore mathematical properties and relationships from numeric, algebraic and geometric perspectives.

State Standard 5: Number and operation concepts

Students will understand and use numbers and number relationships.

A1: Translate among equivalent forms of numbers including integers

, fractions, decimals, percents, exponents, and scientific notation as appropriate for a given situation.

D1: Explore one- and two-dimensional graphs of actual situations and describe the numerical relationships they illustrate.

State Standard 12: Functions and Algebra Concepts

Students will understand and use patterns and functions.

A1: Given the first six Fibonacci numbers, describe the pattern and extend it to the next number.

A2: Create similar patterns.

State Standard 13: Represent situations and number patterns with tables, graphs, rules and equations.

Students will understand and apply algebraic concepts.

B1: Generalize number

patterns to model observed physical patterns.

*State Standards will be noted by (**12-A1**). This means Content Standard 12, performance standard A1.

Lesson 1—**Objective:** Students will discuss what they already know about patterns. They will learn about repeating patterns and growing patterns.

Materials/Preparation: Examples of each type of pattern. Math tiles. Plain paper. -Start the class with a rhythmic pattern such as: Clap, clap, snap, clap, clap…

-Class discussion on what they know about patterns.
-Go over repeating patterns and then growing patterns. Give several examples of each.

Guided Practice: (1-E1), (12-A2)
-Have students build repeating and growing patterns with tiles.
-Have pairs of students build and draw a pattern for other students to analyze. To do this, fold a

piece of plain paper into fourths with the paper turned so that the length is turned horizontally (the students know this as the "hamburger way").
-Have them draw the first three pictures of their pattern on the first three quarters of the paper. Have them put the fourth pattern on the back of the fourth section.

Independent Practice -Students will write about patterns found at home for homework.

Lesson 2—**Objective:** Students will analyze each others patterns to see if they can build the fourth term from their analysis of the first three.

Materials/Preparation: Previous days patterns from the students. Math tiles.

Guided Practice: (13-B1)

Have students

build the first three pictures of the pattern and try to build the fourth. They will check to see if they have the correct answer by looking on the back of the fourth section. Students should write how they came to their conclusion for the fourth term and make a drawing of the fifth term. They also give some feedback on whether it was easy, hard, etc…

Lesson 3—**Objective:** Students will analyze the Fibonacci pattern.

Materials/Preparation: Some basic history on Fibonacci the mathematician and a transparency of the rabbit problem. The rabbit problem is available in this paper as well as in any information on Fibonacci numbers.

Guided Practice: (12-A1)
-Have students trace the family tree for a male bee on their own or with a partner.
-Have students work on Fibonacci puzzles found in the web site http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html. Go to the home page and click on "easy puzzles."

Independent Practice:
Have students trace their

family tree and write the number pattern that goes with it.

Lesson 4—**Objective:** Students will learn about Fibonacci numbers found in nature.

Materials/Preparation: Pinecones(soaked in water), pineapples, apples and artichokes daisies and a giant sunflower. Also, a knife for the teacher, pins with colored heads, magnifying glass and markers. A transparency of the spirals that may be found will be helpful, and you can get that from books or the above web site.

Guided Practice: (1-B1), (1-E1) -Group students so that they can share the materials. Once they are in groups have them examine the pinecones and pineapples to see if they can recognize the spirals they will be counting. Then have them mark on the pineapple with the pins, the course of one of the spirals. Using that as a reference point they can count all the remaining spirals, and it most likely will be a Fibonacci number. This technique with the pins will work well with the artichoke. Then they can do the same thing with a closed pinecone only using a marker. Cut up an apple, cross wise, and notice the seed pattern. It will be a Fibonacci number - a pentagon shape. -Have students look at the daisies and the sunflower with a magnifying glass. It is much too hard to actually count the spirals; they should just recognize them and know that they are Fibonacci numbers. They can count the petals of the daisies to search for the existence of a Fibonacci number. -They should be making a chart of all the Fibonacci numbers they find.

-Let them discuss why they think this happens in nature and then explain what scientists know about it. You can then eat the fruit.

Lesson 5—**Objective:** Students will learn about the equiangular spiral.

Materials/Preparation: Transparency of the equiangular spiral and how it is formed, graph paper, string, tape and snail shells.

Show the transparency and describe how it is formed. Then tell the students where the equiangular spiral is seen in nature. Snail shells will work well as a visual since it would be hard to get a rams horn or an elephants tusk.

Guided Practice: (1-E1), (5-D1)

-Students will draw the Fibonacci

numbers as squares on graph paper until they can’t make anymore squares on their paper.

-They can then free hand in the equiangular spiral.

-In groups, have them continue the equiangular spiral by taping squares together as needed. Use the string to draw in the curve for the larger squares. See how big they can make it and how quickly it grows.

Computer Experience: -Students will get on the Internet to search for examples in animals of the equiangular spiral.

Lesson 6—**Objective:** Students will compute and graph the *golden ratio*.

Material/Preparation: One inch graph paper, calculators, and a mini lesson in graphing and rounding to the hundreds place if needed.

Guided Practice: (1-E1), (5-A1), (5-D1) -Have students calculate the golden ratio of Fibonacci numbers and place them in a T-chart. Calculate the decimal equivalent of: 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, 89/55, 144/89, 233/144, 377/233 and round to the hundredths place.

Term	Ratio	Decimal	Rounded value

1	1/1	1	1.00
2	2/1	2	2.00
3	3/2	1.5	1.50
4	5/3	1.6666…	1.67
etc.	etc.	etc.	etc.
X-values			*Y-values*

-Graph. The Y-axis will be numbered by tenths from 1 to 2 and the X-axis will numbered 1 to 13. Divide each

block on the Y-axis into ten sections for the hundredths.

-Have students calculate and graph Fibonacci ratios for the inverse(reciprocal): 1/1, 1 /2, 2/3, 3/5… Students should investigate whether the graph of these ratios shows anything similar to the first graph.

Lesson 7—**Objective:** Students will investigate where Fibonacci numbers and the golden number are found on the human body.

Materials/Preparation: Meter sticks and rulers in metric.

Guided Practice: (1-E1), (5-A1)

-Have students work in pairs, of the same sex, to help with measuring.
-First make a list of all body parts (remind them nothing vulgar is acceptable) which are a Fibonacci number.
-Have students make another chart and measure and calculate the following:

Measurement 1	Measurement 2	Ratio 1/2
1. Mid neck to navel:	Top of head to mid neck:	
2. Navel to floor:	Top of head to navel:	
3. Knee to navel:	Knee to floor:	

4. Bottom of nose to mid eyes:	Bottom of nose to mid mouth:
5. Bottom of nose to chin:	Mid eyes to bottom of nose:
6. The length of the middle bone of any finger:	The length of the end bone of that same finger:
7. The length of the longest bone of any finger:	The length of the middle bone of that same finger:

-Have students compare results to see if everyone is proportioned to the golden ratio (Johnson).

This lesson is considered to be optional as finding suitable examples may be difficult.

Lesson 8—**Objective:** Students will learn why plants branch and spiral in Fibonacci numbers and be able to recognize this growth pattern.

Materials/Preparation: Transparency of branching plants and the growth spiral. Find examples at the web site mentioned in lesson 3. Permission slips so that students may go outside to examine trees and whatever plants that are around, may be needed.

Guided Practice:

-Have students work in pairs or teams and examine trees that may show the branch spiraling in Fibonacci numbers.
-Have them record this information. For example, how many branches and how many spirals to get to the point where there is a branch directly above the starting point?
-Have students look at all plant life around the

area and look for any examples of Fibonacci numbers. Record the information and include a drawing if applicable. Bring in the whole plant if possible.

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