Manipulatives with Mathematics:
A Unit on Fractions

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Academic Setting

For it to better understood why I am structuring my unit the way I am, I need to explain the type of school that my students and I come from. Truman Middle School is on the West Side, 98th Street and I-40 to be exact. This is one of the poorer areas in the Albuquerque Public School district. Truman has 43% of its families making below $25,000 a year. This is almost half. 14% of the 43% make less than $10,000 a year. These statistics are astonishing to me.

A large number of the parents are not working at high paying jobs that give ample time to be with their child. These parents are working at low paying jobs that will not allow a lot of time off. This accounts for the low level of parent involvement in our school compared to others.

Truman also has 61.5% of our students receiving free meals and 15.2% with reduced cost meals. With both free and reduced lunch, they bring the Truman percentage up to 76.7% with free or reduced lunch. Over three/fourths of our population are at the poverty or very low-income level.

Truman's population is made up of mostly Hispanic students. Eighty percent are Hispanic. This is four/fifths of the total population. The White students account for 10.6% of the population. Then it lowers to 4.0% for Blacks and 4.0% for Indian. These are our largest ethnic groups. Asian is only 0.5% and "other" is 0.4%.

The last thing to mention is Truman's Terra Nova scores.

Test Type	Truman's Score 6th/8th	Albuquerque's Score 6th/8th	National Score
Reading	26/40	50/60	50
Language	28/33	50/52	50
Math	30/31	47/52	50
Composite	29/36	49/56	50

These scores are what have placed Truman on probation with the State. Our scores improve over the three years that we have the students, but there is a lot of work that we need to do to improve our scores to meet the rest of Albuquerque and the nation.

Class Setting

This unit is designed for a sixth grade general education class. I have only a few special education students in my class. The students in my classes are a good representation of Truman’s population. Their reading and math scores are at the lower end of the spectrum. I have to reinforce the reading for them to even understand what the math books are trying to tell them. I am looking for something that will hit home with them that they will remember. That is why I want to do activities with fractions, so that it uses their minds as well as their bodies. All my students are active, so I want them moving around and work with their hands as well as work problems out on paper. I spent a whole year trying just the paper idea, and it did not work. Now, I want to try what I learned in college, not how I was taught. The students did not understand math. I think that they felt that it had nothing to do with them, so they did not want to learn about fractions. Their words to me were always, "When will I ever use this outside of school? What does this have to do with me? This is too hard, I am never going to understand it so I am not going to even try." These are the attitudes that they need to overcome.

Goals

The goal of my unit is to increase my students knowledge of fractions. I would like them to come out with the understanding that fractions are a part of everybody’s lives, including their own. I want them to remember fractions and think of them as a fun topic.

Objectives

My unit objectives for students are:

1. to be able to reduce fractions to the simplest terms.
2. to be able to find the common denominator of two fractions.
3. to add two fractions with unlike denominators.
4. to subtract two fractions with unlike denominators.

These goals and objectives are going to be acheived in a variety of ways. The first way is with a cooking unit. I want the students to understand what the fractions look like and ways to use them. The next way is with manipulatives. I want the students to have hands on experience with fractions. I have learned the hard way that fractions can not be taught in just one way, and I want to demonstrate a variety of methods for working with fractions. This should also help the special education students. I want all the students to learn and remember fractions.

Context

This class is a blocked unit so that the same teacher teaches Math and Science. The unit that I have developed deals with and expends the math series that Truman has adopted. The series that I am working with is *Math in Context* by Britannica. The whole series deals with word problems. It is supposed to help the students improve their reading and writing abilities. I have noticed the series does not give good examples to introduce the unit. I want to improve and expand on this with the Fraction Unit. I want to give good examples for others like me who needed the extra help during the school year. I also want to have manipulatives for the students to work with as expansion activities, or even as a small unit on their own.

Last year was the first year that I had to teach sixth grade math. My previous experience was teaching kindergarten. When I started the fraction unit, I thought that my students had had some introduction to fractions. What I learned was either the introduction had not happened, or it was not a good experience for them. My students really did not remember anything about fractions and hated them already. I had to start all over from scratch. I did the only thing I could think of with them: I had them use paper and pencil and try to rationalize fractions in their heads. What was I thinking? I did the same thing that my teachers had done to me and it caused me to lose my students' confidence. I was very frustrated, and I know they were too. From that experience, I learned the hard way that my teachers in college were correct about using a concrete idea to solve an abstract idea. Using manipulatives helps to make the concept of a fraction more clear.

"The NCTM's Curriculum and Evaluation Standards for School Mathematics recommends that students solve mathematics problems that are in real-world contexts." (Celements). Keeping this in mind, I want to have the students make a recipe book with many different recipes from their families. Besides the use of manipulatives, a critical component of the use of manipulatives is making sure that students make the connection between the conceptual work done with manipulative and the procedural knowledge that such work is supposed to support. Again, as teachers we need to make sure that the students realize that the manipulatives are just examples of fractions.

Manipulatives that are used by students help them to better understand math, and help them to achieve more in math. As teachers we need to know that manipulatives (concrete) may help more students than just pencil and paper (abstract), but it does not guarantee success in mathematics. With this, we need to ask the question: "What is concrete?" First we need to remember that children think and remember things differently than do adults or teachers. In our mind's

eye, we all see different things than each other. Second, Physical actions with certain manipulatives may suggest mental actions different from those that teachers wish students to learn." (Clements). Students may just be following the steps and doing the work, but they may not be internalizing the information they need to understand. Many of my students just go through the motions to please me and get me off their case. Teachers need to make sure that the students understand and learn from the manipulatives, then reflect later on paper.

The different types of concrete knowledge are:

1. Sensory-Concrete
2. Integrated-Concrete

Sensory-concrete knowledge is when sensory material is used by students to make sense of what they need to learn in a problem. For example, the older students understand fractions when we hand them a candy bar and ask them to split it with four other students. Integrated-Concrete knowledge is expanded with knowledge and learning. This is the base of concrete: to grow together. This uses many different ideas to teach a new concept. It uses past knowledge to build on what they know or have learned.

Manipulatives include a wide variety of items that can be found in every classroom. Manipulatives could be trash that you have saved, knowing that it can be used later. Some examples are beans, old computer paper with the holes, or plastic bottle caps. In a math class, the common manipulatives are pattern blocks, Cuisenaire rods, boxes of tiles and cubes, tangrams, coins, dice, and geoboards. These are the items in our classrooms that we need to use, and explain to others that they are useful. We are not giving our students free time when they use manipulatives.

In response to some people thinking we are giving our students free time, Marilyn Burns has a few answers: "Manipulative materials give students a way to make abstract ideas concrete." Just as coaches have always said, if you do not try it you will never learn to do the activity. This is the same with math. If you do not work with it as a hands-on experience, you will never fully understand the activity. Burns continues, "Manipulative materials lift math off textbook pages and give students a way to get their hands on ideas." These ideas are in the students’ minds, not on a piece of paper. It is with the hands-on activities that the students can relate their ideas better on paper. Also "Manipulative materials build students’ mathematical confidence by giving them a way to test and confirm their reasoning." Manipulatives to the students are fun and enjoyable. As teachers, we need to keep up

the encouragement and show the students that math is not hard, it is fun. Manipulatives are useful tools for solving problems. Just as a math class will use hands-on activities to solve a math problem, a scientist or architects will use manipulatives to solve a problem that they are working on. Manipulatives make learning math interesting and enjoyable. Just ask any student who needs to use manipulatives to answer a question.

Marilyn Burns is an author who has written many books on how to teach math, and lessons for math concepts. She has some great ideas for the use of manipulatives. As a teacher, manipulatives were always scary for me. I could picture the materials being thrown across the room, things missing, and just general chaos. I had to set down the rules and make sure to give students time to freely explore everyday with any kind of manipulative. The first couple of days I had to keep reminding the students to stop playing with the materials and get back on task. It took the students almost three days to get over the new experience of using manipulatives. That was when I finally got their full attention. Make sure that you always give them five to ten minutes each day to explore with the manipulatives. If you remind the students of this, there does not seem to be as much of a struggle with the students. One major reminder: teaching demonstrations with the manipulatives are not free explore time with the materials.

Manipulative materials should be used as often as you need them. Do not feel that you are over or under using the manipulatives, as long as you are using them to help the students get the math concept you are working on. But if you are using them for the students to work with a certain concept, make sure that the manipulatives are available for the students to use at other times as well. Students do not have to be using the same manipulatives at the same time. Some students understand the concept faster than others do, so they do not feel that they need to be using the manipulatives any longer. This is okay, but make sure they understand the math concept before they move to just paper and pencil. There will be times that you will be working on a specific problem and the students will need to be using the same materials to do the work. Just make sure that this is told to the students at the introduction of the lesson.

A lot of teachers do not have the money for expensive manipulative materials and their school will not provide the items that the teacher is asking for. Manipulatives do not always need to be pricey items. They could be just paper and scissors. Many students enjoy making their own manipulative materials. Folding the paper or taping cut up pieces of paper is another way that the students get hands-on experience to help them understand. Making origami is an example of paper folding

that gives the students time with the work that they are trying to solve, or making tangram pieces to show that they do fit back together. The plastic manipulative materials are made to stand up to a lot of use, whereas paper manipulatives must be remade often.

When you are just first starting with manipulatives it is always a good idea to become familiar with one or two to start with. Do not overwhelm yourself with trying to learn how to use all the different manipulative materials out there at once.

If it is not the first time that the students have used a certain manipulative, then move on to others. The students will then have a variety of items to work with. Students will be able to think of the same problem with different ways to solve it. Nothing is rigid about life, why should the way that the students learn be? Let them have some freedom to learn the way that is best for them. One way does not have to be the only way. We all learn differently, so we know students will solve problems differently.

Manipulative materials are for all students. The slower students and the advanced students can benefit from the materials. It is the teacher who needs to remember that the challenge is to make the lesson accessible and challenging to all students. Have the student’s work on a problem, then if you have some students finish early have an extension for them to work on.

Manipulative materials are for all ages. I have used manipulatives with sixth through eighth graders, and I have not had any student complain that this is "babyish." All my students, when introduced to colored tiles, build towers, pyramids, checker boards, dominos, and designs. They are upset to take down what they had build with the colored tiles. I am asked every day when they can again have free explore time with the colored tiles.

This unit will cover the following:

1. Equivalent Fractions: A lot of the students do not understand, and do not want to understand, equivalent fractions. Teachers need to remind students that equivalent means

equal. Students need to know that 1/2 is the same as 2/4 or 3/6. In lessons that follow, I will give examples on how to do this with different manipulatives.

2. Reducing Fractions: You can divide 2/4 by two, or 3/6 by three, to get the beginning number of 1/2. Students need to see that if the same number can be used to divide both the top and the bottom number, then the fraction needs to be reduced.
3. Common Denominators: Students have a hard time understanding that 1/4 + 1/2 is not 2/6. Students need to understand that you can not add or subtract fractions when the denominator is not the same number. The word "common" means that they need to find a number that both of the denominators can

go into.

4. Addition of Fractions: This is where we will put all the other information together to be able to add fractions with unlike and like denominators.
5. Subtraction of Fractions: The same rule applies to subtraction. Again, the common denominator is used. The students will need to be able to add unlike and like denominators.

The unit is on fractions, emphasizing the use of manipulative materials. Each lesson will hit a target area that all sixth graders need to know about fractions. I want to make sure that the students understand fully the concepts that lie beneath manipulative materials. The unit should be fun and meaningful at the same time.

The unit length will be two weeks. Since I have a block period, the time will be twice what other lessons are. This unit can be used with *the Math in Context Series* by Britannica, or it can be used alone to help students understand addition and subtraction of fractions.

Each lesson has an introduction, and an extension at the end of the lesson. The introduction should be used with the lesson as the base for where they are going that day. The extension does not need to be used every day, but it can be used as part of the lesson. Some times I have students who will finish early. The extension is a game that the students may play to reinforce the concept that was taught that day. The game that is suggested may be used, or the students may play other games that were introduced before.

Implementation

Lesson 1: Introduction to fractions

Grade: 6th
Timeline: One block or Two - 45 minute class periods
Objectives: To have student's talk about fractional parts, work with concrete materials, and relate their experiences to the standard mathematical notation.
Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II 11, 12
New Mexico State Content Standard: #5-A, C #7-A, B, C
Materials: Six pack of soda
Five pack of gum
Seven bananas
A box of crackers with three separately wrapped packages
12 two-color counters for each student
A Hershey's milk chocolate bar for each student
Anticipatory Set: *The Hershey's Milk Chocolate Fractions Book*

Hand out a Hershey bar to each student. They are not to eat the chocolate bar until we are finished for the day. Read the book and have the students follow the fractions in the book. Talk about the terms as they are brought up.

Procedure: Introducing fractions as parts of sets.
Have a six pack of soda for this whole class introduction. Take off one can of soda, and tell the students that they can write a fraction that shows what part of the six pack of soda you just took away. Write on the board "1/6." Tell the students to read this as one-sixth. This means that I have one soda can out of six parts. Then ask these questions:

1. What does the six refers to?
2. What does the one stands for?
3. Why does this mathematical notation make sense?
4. What could I write to show the fractional part of the rest of the six-pack, the part I did not drink?

Remove another can and ask the same kind of questions. Continue, with three, four, five and six cans with their related fractions. Make

sure that you point out that 6/6 is equal to one whole or one six pack. Do the same with other examples if needed: (five pack of gum, seven bananas, and a box of crackers with three separately wrapped packages (Burns).

Fractions with two-color counters:

Hand out 12 two-color counters to each student. Give the students the directions and questions. Have them discuss the questions in pairs and then present their answers to the rest of the class. They need to explain the reason for each answer. Students will write their answers in a spiral math notebook.

1. Divide the 12 counters into three equal groups with all yellow sides showing. What fractional part of the whole set does each group represent? How many counters are in 1/3?
2. Flip the counters in one group. What fractional part of the whole set is red? Yellow? How many are in 1/3? In 2/3?
3. Rearrange the counters into six equal groups with all yellow sides up. Flip the counters in one group. What fractional part of the whole set is red? Yellow? Flip another group. What fractional part of the whole set is red? Yellow? Continue until all groups have been flipped.
4. Arrange the counters so 1/4 of them have red sides showing. What fractional part of the whole set is yellow? Show another set with

fewer than 12 counters that also has 1/4 of the set with red sides showing. See how many different solutions you can find.

5. Show a set of counters that has 2/6 with their yellow sides showing. Find as many different solutions as you can that use 12 or fewer counters. Continue for other fractions: 2/8, 5/6, 3/5, etc… (Burns)

Assessment: These are assessments that could be used:
Observation
Notebook
Presentation

Extension: Fraction five-in-a-row:
Materials: Small group, regular die labeled one to six, pencils, and game board for each player (worksheet 1).
Roll the die to see who begins. The player who rolls a three, or the number closest to 3, begins. Play proceeds in a clockwise direction. Take turns rolling the die. Use the number rolled to write either the numerator or the denominator of one of the fractions shown on the game board. The numerator is the shaded part of each region, or the dot on the number line. Check each other's thinking. If you cannot use a number rolled, your turn ends. The person who correctly writes five fractions in a row in any direction is the winner. Play again; include equivalent fractions to name pictures (Noble).

Lesson 2: Introductions to Fractions II/ Equivalent Fractions
Grade: 6th
Timeline: One block or Two-45 minute class periods
Objectives: To introduce students to fractions as part of the whole.
Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II - 11, 12.
New Mexico Content Standards: #5 A, C #7- A, B, C
Materials: five strips of construction paper in five different colors, three - 18 inches scissors
envelopes

fraction dice
one per pair: dice

Anticipatory Set: The Fraction Kit
This activity introduces students to fractions as parts of a whole. Pair off the students. Directions: 1) Light Blue: Fold it in half, and cut it into two pieces. Label each piece 1/2. (review the rationale for the notation by explaining that the whole has been divided into two pieces of the same size, that each piece is one of the two pieces, and that the 1/2 notation means one of two equal pieces. 2) Light Green: Students need to fold and cut it into four equal parts. Talk about each piece being one of four, or one-fourth, and ask students to label each piece 1/4. 3. Yellow: Fold, cut, and label it into eighths. 4) Pink: Fold, cut, and label into sixteenths. 5) Red: Whole, label it 1, or 1/1. Make sure they do not fold or cut this color. Measure and draw a line on each strip at six inches and 12 inches. Then cut each strip into three pieces. 6) Label each dark blue piece 1/3. 7) Dark Green: Fold in half and crease. Unfold and cut each one on the crease. Label each of the six pieces 1/6. 7. White: Fold each sixth in half and crease. Unfold and cut each one on the crease. Label each of the 12 pieces 1/12.

Procedure: Cover Up
This is a game for two or more players. Each player starts with a whole strip. The goal is to be the first to cover the whole strip completely with other pieces of the fraction kit. No overlapping pieces are allowed.

Rules:

1. Students take turns rolling the cube labeled with fractions.
2. The fraction face up on the cube tells what size piece to place on the whole strip.
3. When the game nears the end and a student needs only a small piece, such as 1/8 or 1/16, rolling 1/2 or 1/4 will not do it. The students must roll exactly what is needed.
4. The dice has matching fractions: 1=1/6, 2=1/12, 3=1/16, 4=1/16, 5=1/8, 6=1/4.

Have the students write down in their spiral notebook the fractions that they used to cover up. After they have finished one game have the students continue with another game.

Race to Two Wholes: Place the two wholes end to end and between players. Each player rolls the die. The higher roll is Player One. Player One rolls the die and reads the matching fraction of the die. Player One places that fraction beside the first whole. Player Two

rolls the die and reads the matching fraction. Player Two places that fraction on the other side of the first whole. Continue play by adding fraction pieces in a race to reach two wholes. When a player is near two wholes, she can freeze. The winner is the player who is closer to two wholes without going over. Dice: 1=1/16, 2=1/6, 3=1/4, 4=1/8, 5=1/12, 6=1/2. Have the students write down in their spiral notebook the fractions that they used to cover up (MailBox, 1997)

Assessment: Notebook
Observation
Extension: Using Your Fraction Tool Kit
Layer By Layer. The object of this game is to be the first player to stack all eight layers of the fraction pieces on top of the whole. To Play: Place the two wholes between players. Each player rolls the die. The higher roll is Player 1. Player 1 rolls the die. Player 1 rolls the die and reads the matching fractions to the die. Player 1 chooses one of the two fractions. Player 1 places that fraction pieces on top of his whole strip, lined up at the end. Player 2 repeats. Play continues alternately until a player has made all eight layers. Die: 1=1/2 or 1/12, 2=1/4 or 1/16, 3=1/12 or 1/6, 4=1/16 or 1/8, 5=1/8 or 1/6, 6=1/3 or 1/16. (Crump).

Lesson 3: Equivalent Fractions: Rectangular and Circular Parts

Grade: 6^{th}
Timeline: One block or two-45 class periods

Objectives: To be able to show different names for the same area. To be able to show fractions equivalent to the given fractions.

Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12.
New Mexico State Content Standards# 5-A, B #7 A, B, C

Materials: Three sheets of paper: One - with 1/2 shaded, One - with 1/3 shaded, One - with 1/4 shaded
circular fraction pieces
numberlines
fraction bars

Anticipatory Set: Rectangular Parts
Have the students bring out the 1/2-shaded sheet of paper. Ask the following questions regarding the sheet of paper.

1. How many parts are represented on your sheet? How many are shaded? Fold your sheet down the center. How many parts are defined now? How many are shaded? Is 1/2 the same as 2/4?

2. By repeated folding show that the shaded region becomes 2/4, 4/8, 8/16, 16/32. If you could fold it again, would 1/2 have still another name? Use 1/4 and 1/3 sheets in the same manner.
3. Use your ruler to draw a rectangle. Divide it into tow congruent parts by drawing a horizontal line across the center with a red pencil. What part of your rectangle is above the red line? Now draw a green line down the center. What are two names for the part above the red line?
4. Draw another rectangle and divide it into two parts as before. This time make two green lines from top to bottom to divide your region into three parts. Now what is the name of the region above the red line? Now there are three times as many parts above the red line. 1/2=3/6, right. This procedure may be continued with other fractions.

Procedure: Circular Parts

Hand out the circular fraction parts to each student. Ask the following question regarding the fraction pieces:

1. How many halves will it take to cover the whole circle?
2. How many thirds will it take to cover the circle? Fourths? Sixths? Eighths?
3. Now take one half. In how many ways can you exactly cover 1/2? Are there any other ways that you can cover 1/2 exactly with the parts that you have? What other ways could you cover it if you had smaller parts?
4. Use the same procedure for 1/3, 1/4. (Howell)

Assessment: Observation: Look to see if the students are understanding the concept.

Extension: Equivalent Fraction Four-Square

Small groups, each need a fraction die (labeled 1/2, 1/3, 1/4, 1/6, 1/8, 1/12) ten counters for each player, and one game board (worksheet 2). The player who lives nearest the school rolls the die and calls out the fraction rolled for each round of the game. After each roll of the die, the fraction is called out. All players try to cover an equivalent fraction on their game boards. Try to make a four-square. The first player to make a four-square is the winner. The winner rolls the die and calls the fractions for the next game. Play again, be the first to make a nine-square to win. For two players, use counters in two colors and one game board. Try to block each other to keep the other player from making a four-square. Play until all the spaces are covered. The player having the most four-squares is the winner.

Lesson 4: Equivalent Fractions
Grade: 6th
Timeline: One block or two-45 class periods
Objectives: To be able to change one fraction into an equivalent fraction.
Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7 A, B, C.
Materials: Equivalent fraction strips
Anticipatory Set: Equivalent Fractions

Remind the students what 1/3, and 2/6 look alike. Draw this on the board. Show that 1/3 and 2/6 are equal to each other, but they are written with different numbers. But we do not what to draw the rectangles all the time. A different way to see this is to multiple the numerator and denominator by the same number. 1/3 x 2/2 = 2/6. Do the same with 3/4=6/8 and 5/10=1/2.

Procedure: Fraction Strips

Give each student a copy of the fraction strips (worksheet 3) and an envelope for storing the strips. Have them color each section of the strips a different color. Example, have the students color the whole blue, both 1/2 parts red, and etc... Then have the students cut out the fraction strips. Ask the class to find another fraction equivalent to 1/4. Write their answers on the board, and ask what number they multiplied to get the answer.

Divide the class into pairs and have the students look for all the equivalent fractions that they can find with the fraction strips. Have the students write down their answers with the original fraction that the others are equivalent to in their spiral notebooks. Then have the students work on the following three worksheets.

Assessment: Worksheets
Observation
Extension: Equivalent Fraction tic-tac-toe

Two players each need a fraction die labeled 1/2, 2/3, 3/4, 5/6, 7/8, 11/12, 20 counters, ten each of different colors, and gameboard (worksheet 4). The player who has the shorter first name begins. Each player takes counters of one color. Roll the die and cover an equivalent fraction on the game board with a counter. Take turns and keep playing in the same way. The first player to cover three fractions in a row in any direction is the winner. Play again. To win, you must be first to cover four fractions in a row in any direction (Noble).

Lesson 5: Reducing Fractions
Grade: 6th
Timeline: One-block or two-45 minute class periods
Objectives: To understand that fractions will be in a simpler form when divided by the same number.

Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7 -A, B, C.

Materials: equivalent fraction strips
fraction bingo cards
counters
fraction Bingo calling cards

Anticipatory Set:
Tell students that they are to find fractions that are in a simpler form of equivalent fractions. Then show that instead of multiplying they will divide by the same number. Use equivalent fraction strips to help demonstrate reducing fractions. Sometimes a fraction has a numerator and denominator that are large and hard to work with. We find a smaller fraction that is equivalent and that is called reducing. For example, look at the halves and the fourths fraction strips. What is another fraction that is equal to 2/4? It is 1/2. We can change 2/4 to 1/2 by dividing the two and the four by two. Give the students time to look at the fractions to prove to themselves that this is correct.

In the fraction 5/10, what number can divide evenly into both the five and ten? It is five. Show with the fraction strip that 5/10=1/2. When a number will divide evenly into two numbers, it is called the common divisor. The common divisor for 5/10 is five. The common divisor for 2/4 is two.

What is the common divisor for 8/10? Two. When you reduce, what do you get? 4/5 Check with the fraction strips. If needed, continue with other examples.

Procedure: Fraction Bingo
Hand out copies of the fraction bingo cards (worksheet 5). Each student should have his or her own card. Have the students write FREE in a space of their choice. Write the fractions on the board. 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 4/7, 3/8, 5/7, 0, 6/7, 2/3, 3/4, 3/5, 4/5, 3/7, 2/5, 5/6, 2/7, 1, 5/8, 7/8, 1/9, 4/9, 5/9, 7/9, 8/9, 3/10, 7/10, 9/10. They will not be able to use all fractions. Tell them to put fractions in random spaces so they will not all bingo at the same time.

When the students play bingo, they will put counters on the top of the fraction that was called. Call out the fraction from the fraction calling cards. Have the students use scratch paper if necessary to reduce the fractions. Example: If the teacher calls 2/8, the students will mark 1/4.

For additional practice, have some worksheets that the students may work on.
Assessment: Observation
Extension: Fraction Slap-Jack
Two-Players use a standard deck of playing cards. Remove face cards, aces and sevens. Deal out remaining cards into two piles facedown, one pile to each player. Players turn the tip card face-up at the same time. If the two numbers can be made into a fraction that can be reduced, call out the reduced fraction and slap the table before the other player can. If someone calls out the wrong reduced fraction and slaps, the other player gets the cards. Use the smaller number as the numerator to form the fraction. If the fraction can not be reduced, turn over the next pair of cards. Continue until a pair of cards can be reduced. The winner of that pair gets all the cards face-up at that time.

One person can play this game to practice reducing fractions alone. Also, two people can play it and take turns giving answers instead of competing for speed and slapping cards (Hein)

Lesson 6: Simple Addition and Subtraction
Grade: 6^{th}
Timeline: One - block or two- 45 minute class periods

Objectives: To understand that when adding or subtracting fractions you add or subtract the numerators and the denominators are left alone.

Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7-A, B, C.

Materials: rectangle fraction pieces
circular fraction pieces

Anticipatory Set: Rules for Addition and Subtraction of Fractions.
To introduce the addition and subtraction of fractions always use real life examples. Use pies and cakes to show circles. Use boards, candy bars, and bars to show rectangles.

Example: This can be shown on the board or with hands-on examples. Dan has 1/3 of his pizza to warm up in the microwave. His sister Amanda has 1/3 of her pizza to warm up, also. If they put both pieces

together on a plate, how much of the pizza is on that plate?

Example: Use a rectangle or drawing of a rectangle on the board. Rachel cut her banana bread into five pieces. She ate 1/5 and gave 1/5 to Hannah to eat. How much of the bread have the two girls eaten?

Ask the class if they can make up a rule to help them add fractions.

> Rule 1: When adding, add the two top numbers (numerators), and leave alone the bottom numbers (Denominator).
> Rule 2: When subtracting, subtract the two top numbers, keep the common denominator.

Procedure: Simple addition and subtraction
Have the students do the three worksheets (worksheet 6) that follow. Walk around the class and help the students who need help (Hein).
Assessment: Worksheets
Observation
Extension: Sum It Summit
Have Ready: Small groups, two fraction dice for each (labeled 1/6, 2/6, 3/6, 4/6, 5/6, 6/6), Colored markers, and game board (worksheet 7) for each player.

The player whose birthday is closest to January 1 begins. Write all the sums listed below in the triangles on your game board. Write each fraction three or four times to fill all the triangles: 2/6, 3/6, 4/6, 5/6, 7/6, 8/6, 9/6, 10/6, 11/6, 12/6. Take turns rolling the dice. Add the fractions you roll and color a triangle space that shows the sum. The first player to make a path of touching triangles from the bottom to the summit of the mountain is the winner. The triangles in the path must touch sides.

Play Again. Write these sums in the triangles on your game board and play again in the same way. 1/3, 1/2, 2/3, 5/6, 1 1/6, 1 1/3, 1 1/2, 1 2/3, 1 5/6, 2 (Noble).

Lesson 7: Common Denominators
Grade: 6th
Timeline: One-block or two-45 minute class periods
Objectives: To add and subtract unlike denominators.
Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7 A, B, C.
Materials: multiples of numbers page

Anticipatory Set: Rules
1. See if the larger denominator is a multiple of the smaller one. If it is, that is the common denominator. The 1/4 and 1/2 examples show this. If the students do not remember multiples, go over them.
2. Multiply the two denominators to get a common denominator. For 1/4 + 1/5 you can not use the larger denominator because five is not a multiple of four: 4 x 5 =20 and 20 will work. Find the equivalent fraction for both of them and add.

Have the students look for a common denominator for four and five. Have them look at the list of multiples and find the smallest number which is in both lists. Give them a few number pairs for practice: (Hein).

Procedure: Fraction Action
Use standard playing cards. Deal one card face-down and one card face-up to each player. Players may then ask for another card face-up or pass and get no more cards if they wish.

For each card dealt, think of the fraction using the number on the card as the denominator. Example: If you have two, it is worth 1/2. For a three, the value is 1/3. Aces and face cards are each worth ten.

The object of the game is to get the total of the fractions closest to one with out going over. Example: If you get two 2's, 1/2 + 1/2 =1 and you win. If you get three 3's, 1/3+1/3+1/3=1 and you win. If you get a 2 and a 4, 1/2+1/4=3/4 and if no one gets more than 3/4, you win. If you get a 2, a 2 and a 4, 1/2 +1/2 +1/4=1 1/4, you lose.

The winner for each hand gets one point. It is possible to have more than one winner for each hand. First person to get ten points wins (Hein).

Assessment: Observation
Extension: High or Low?
Materials: small groups make a die (labeled 1, 2, 2, 3, 4, 12), pencils, game board for each player, and cut along the dotted line to make two boards (worksheet 8). The player who has the shortest hair begins and gets to roll the die and call out he numbers for the first game. The first player chooses to play a round of Highest Sum (H) or Lowest Sum (L). All players circle H or L. After each roll of the die, all players write the number rolled as a numerator or a denominator in one of the four boxes. Once you write a number, it cannot be changed. Keep playing for three more rolls until all four boxes are filled in to make an addition sentence. Find the sum of your addition sentence. The player having the winning sum (highest or lowest) is the winner for that game. That player gets to roll the die, call the numbers, and

choose to play a round of highest or lowest sum for the next game. The player who wins the most games is the grand winner. Play again: Use a die labeled with six of these numbers: 2, 3, 4, 5, 6, 8, 10, 12 (Noble).

Lesson 8: Addition and Subtraction
Grade: 6th
Timeline: One block or two-45 minutes class periods
Objectives: To be able to add and subtract using rectangular and circular fractional parts. To be able to find the difference between two fractions.
Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7 -A, B, C.

Materials: Set of fractional parts for each student.
Set of overhead fractional parts.

Anticipatory Set: Using Rectangular and Circular Parts
Give each student a set of rectangular and circular fractional parts. The teacher will then ask the students to do the following steps. We want to use these fractional parts to find the sum of two fractions, 1/2 and 1/3. Place the circular part representing ½ beside the circular fractional part representing 1/3. Find fractional parts, all the same size, which will exactly fit over the 1/2 and 1/3. What fractional parts did you use? How may did you use? Now write this result in this way: 1/2 + 1/3= 5/6. Do the same addition problem using rectangular parts.

Take your rectangular fractional parts and find a part that represents 1/2. Find a fractional part that represents 1/4. Compare these two fractional parts. Which is the larger? How did you make this comparison? Try placing the smaller one on top of the larger one. Now find a part which can be placed next to the smaller part, which can be added to the smaller part to make a part exactly equal to the larger part. What fractional part did you use? We represent this in a sentence as follows: 1/2- 1/4= 1/4. Try anther one. Find fractional parts, which represent 1/2 and 1/3. Which is larger? Find out by placing the smaller one on top of the larger one. Find a fractional part or fractional parts the exact same size which will make the smaller one equal to the larger one. What fractional part did you use? How many? Use the same procedure to find the difference between 2/3 and 5/6, 3/4 and 1/3. Use the same procedure with the circular parts to find the differences.

Procedure:
Find the sum of the following fractional numbers using either the

circular or rectangular parts: a. 1/2+ 1/2= b. 1/3+ 1/6= c. 1/4+ 5/8= d. 3/4+ 1/6= e. 1/3+ 1/4= f. 2/3+ 1/4=

Using either circular or rectangular parts, find the answers to he following exercises: a. 1/2- 1/5= b. 2/3- 1/6= c. 5/8- 1/2= d. 4/5- 3/10= e. 3/4- 2/3= f. 5/6- 2/3= (Howell)

Assessment: Observation
Worksheets: Have the students draw every piece and label them to show how they got the answer.
Extension: Sum-Difference

Have ready: small groups, two fraction dice (labeled 1/2, 1/3, 14, 1/6, 1/8, 1/12 and 1/2, 2/3, 3/4, 5/6, 7/8, 11/12), and one game board (worksheet 9) for each player. The player who sits closest to the door begins. Play proceeds in a clockwise direction. Take turns. Roll both dice. Write the two fractions you roll on your game board. Find their sum, then find their difference. After each round, compare your sums and differences with the other player. After each round, find which player has the largest sum and circle it. Then find which player has the smallest difference and circle it. There can be two winners.

Play again: Use two dice labeled 1/2, 2/3, 3/4, 5/6, 7/8, and 11/12 and play in the same way.

Lesson 9: Cook Book
Grade: 6th
Timeline: one-three blocks
Objectives: To be able to use addition and subtraction to make a cookbook in which the recipes have to be changed for the number of people.
Benchmarks: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7 A, B, C.
Materials: recipes from home
construction paper
Anticipatory Set: Recipes From Home
Have the students bring in their favorite recipes from home. This should be done a week before this lesson.

Procedure: CookBook
Photo-copy each recipe for the students. Have the students make the recipes for from four servings to eight. (or have the students make the recipe from four to two, I want the students to use addition and subtraction of fractions). The students are to be paired off and they are then to pick a recipe that they are going to make double of for the

class. Off to the ovensis then to go make the recipes for the class.

Assessment: Observation
Conversations
Cook Book

Lesson 10: Beast of Burden
Grade: 6th
Time: One block
Objectives: To be able to work with both literature and fractions together.

Standards: Albuquerque Public Schools Content and Performance Standards: Strand II- 11, 12. New Mexico State Content Standards: #5-A, B #7 A, B, C.

Materials: Beasts of Burden
Anticipatory Set:
Read the book *Beast of Burden*.

Procedure:
Using the book Math and Literature talk about the book *Beast of Burden*. Discuss the splitting of fractions. The book has some great commentary on what to say and do with the students. This is an extension activity.

Standards and Benchmarks

Albuquerque Public Schools Content and Performance Standards

Strand II: Number Sense and Operations
Content Standard: The student demonstrates number sense through experiences with meaningful mathematical problems that focus on number meaning, number relationships, place value concepts, relative effects of operations, and multiple representations to communicate sound mathematical thinking.
6-8 Benchmark: The student understands problems involving fractions, decimals and percents and develops, analyzes, and explains a variety of algorithms and methods to solve problems.
11: Finds greatest common factor and least common multiple using a variety of strategies.
12: Develops and tests strategies for adding and subtraction fractions with like and unlike denominators.

New Mexico State Content Standards
Content Standard 5: Number and operation Concepts

Students will understand and use numbers and number relationships.

5-8: Students will:
A: Represent and use numbers in a variety of equivalent forms including integers, fractions, decimals, percents, exponents, and scientific notation.
C: Apply the relationships among fractions, decimals, and percents to ratios and proportion.

Content Standard 7: Number and Operation Concepts
Students will understand and use computation and estimation.
5-8: Students will:
A: Solve problems through computation with whole numbers, fractions, decimals, rational and irrational numbers.
B: Develop, analyze, and explain methods for solving problems.
C: Select and use an appropriate method for computing from various processes including mental arithmetic, paper and pencil, calculators, and technology.

Documentation

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