

Money

Engineers frequently find themselves working as much with money and financing as they do with technical design. All engineering requires money. Money is needed to pay salaries, to buy materials for producing the product, and to buy the manufacturing machines.

This money may be borrowed from banks or investors and must be repaid. These loans have interest attached to them.

SIMPLE INTEREST

Simple interest applies the interest to the original loan. The interest is usually applied annually. We can illustrate this with an example.

You borrow \$5,000 from you uncle to pay for college. You promise to pay him back 1 year after graduation. You both agree on a simple interest rate of 8%. If you repay the loan in 5 years, how much will you pay?

F = the amount to be paid back

P = the principle originally borrowed

n = the number of periods the interests is applied (usually number of years).

i = the interest rate per year.

We can create a table showing equations for the amount to be paid back each year. The formulas show the year, the amount due at the beginning of the year, and the amount due at the end of the year.

Payback		
Year	Beginning	Ending
1	$F = P$	$F = P + P i$
2	$F = P + P i$	$F = P + P i + P i$
3	$F = P + P i + P i$	$F = P + P i + P i + P i$
...

Looking at the table, we can create a formula for any number of years. It is:

$$F = P + P i n \quad (1)$$

We can compute how much you need to repay your uncle.

$$F = 5000 + 5000 \times 0.08 \times 5 = \underline{\underline{\$7,000.00}}$$

You will owe him 7000 dollars at the end of 5 years.

COMPOUND INTEREST

Banks and investors usually look at compound interest rather than simple interest. With compound interest, the interest over the first year or period is added to the loan and the interest for the next year or period is computed using that new principle.

F = the amount to be paid back

P = the principle originally borrowed

n = the number of periods the interests is applied (usually number of years).

i = the interest rate per year.

We can create a table for this similar to the one created for simple interest.

Payback		
Year	Beginning	Ending
1	$F = P$	$F = P(1 + i)$
2	$F = P(1 + i)$	$F = P(1 + i)(1 + i)$
3	$F = P(1 + i)(1 + i)$	$F = P(1 + i)(1 + i)(1 + i)$
...

The formula for this can be written:

$$F = P(1 + i)^n \quad (2)$$

Problem 1

The bank says it will loan you the same \$5,000 at 7% compounded annually. Who has the best deal, the bank or your uncle?

$$F = 5000(1 + 0.07)^5 = 5000 \times 1.07^5 = \underline{\underline{\$7,012.76}}$$

The bank charges 1% less then your uncle but they compound the interest annually. The bank will charge you \$12.76 more for the loan.

Problem 2

In 10 years, you would like to have \$10,000 in the bank to put down on a house. How much do you have to put in the bank now to have \$10,000 in ten years. The bank will give you 6% interest compounded annually.

Starting with equation 2.

$$F = P(1 + i)^n \quad (2)$$

Solving for P

$$P = F / (1 + i)^n \quad (3)$$

$$P = 10,000 / (1 + .06)^{10} = 10,000 / 1.7908 = \underline{\underline{\$5,583.95}}$$

Another bank compounds the interest monthly. What would you have to invest for this bank?

We start with equation 3 but this time we do not use the annual interest rate because the interest is compounded monthly. Instead we use:

$$i = i_{\text{annual}} / 12 = 0.06 / 12 = 0.005$$

The interest is being compounded monthly so there are 12 compounding periods per year.

$$N = \text{interest periods} = 10 \text{ years} \times 12 = 120$$

Substituting this into equation (3) yields:

$$P = F / (1 + i)^n = 10,000 / 1.005^{120}$$

$$P = \underline{\underline{\$5,496.33}}$$

Problem 3

The bank gives you 10% interest compounded annually. How long will it take for your money to triple in value?

Starting with equation (2)

$$F = P(1 + i)^n \quad (2)$$

Divide both sides by P

$$F/P = (1 + i)^n$$

If the money triples in value then

$$F/P = (1 + i)^n = 3$$

We take the log of both sides of the equation and solve for **n**

$$\text{Log}(F/P) = \text{Log}(3) = n\text{Log}(1+i)$$

Or

$$n = \text{Log}(3) / \text{Log}(1+i) = \text{Log}(3) / \text{Log}(1.1) = 0.47712 / 0.04139 = \underline{\underline{11.5 \text{ years}}}$$