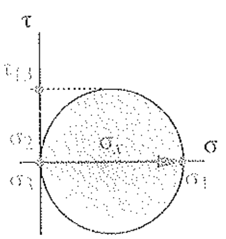
5.2 FAILURE OF BRITTLE MATERIALS UNDER STATIC LOADING

Brittle materials fracture rather than yield. **Brittle fracture in tension** *is considered to be due to the normal tensile stress alone* and thus the maximum normal-stress theory is applicable in this case. **Brittle fracture in compression** *is due to some combination of normal compressive stress and shear stress* and requires a different theory of failure. To account for all loading conditions a combination of theories is used.

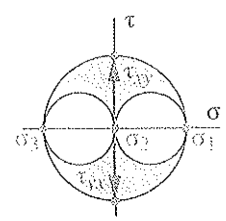
Even and Uneven Materials

Some wrought materials, such as fully hardened tool steel, can be brittle. These materials tend to have compressive strengths equal to their tensile strengths and so are called *even materials*. Many cast materials, such as gray cast iron, are brittle but have compressive strengths much greater than their tensile strengths. These are called *uneven materials*. Their low tensile strength is due to the presence of microscopic flaws in the casting, which, when subjected to tensile loading, serve as nuclei for crack formation. But when subjected to compressive stress, these flaws are pressed together, increasing the resistance to slippage from the shear stress. Gray cast irons typically have compressive strengths 3 to 4 times their tensile strengths, and ceramics have even larger ratios.

Another characteristic of **some cast, brittle materials** is that their *shear strength can be greater than their tensile strength*, falling between their compressive and tensile values. This is quite different than ductile materials, in which the shear strength is about one-half the tensile strength. The effects of the stronger shear strength in cast materials can be seen in their failure characteristics in the tension and torsion tests. Figure 2-3 (p. 61) shows a ductile-steel tensile specimen whose failure plane is at 45° to the applied tensile stress, indicating a shear failure occurred, which we also know to be true from the distortion-energy theory. Figure 2-5 (p. 62) shows a brittle cast-iron tensile specimen whose failure plane is normal to the applied tensile stress, indicating that a tensile failure occurred. The Mohr's circle for this stress state is shown in Figure 5-1*a*, repeated here, **and is the same for both specimens**. The different failure mode is due to the difference in relative shear and tensile strengths between the two materials.



(*a*)



(*b*)

FIGURE 5-1 Repeated
Mohr's Circles for Unidirectional Tensile Stress (*a*) and Pure Torsion (*b*)

Figure 2-8 (p. 64) shows two torsion-test specimens. The Mohr's circle for the stress state in both specimens is shown in Figure 5-1*b*, repeated here. The ductile-steel specimen fails on a plane normal to the axis of the applied torque. The applied stress here is pure shear acting in a plane normal to the axis. The applied shear stress is also the maximum shear stress, and the failure is along the maximum shear plane because the ductile material is weakest in shear. The brittle, cast-iron specimen fails in a spiral fashion along planes inclined 45° to the specimen axis. The failure is on the planes of maximum (principal) normal stress because this material is weakest in tension.

Figure 5-10 shows Mohr's circles for both compression and tensile tests of an *even material* and an *uneven material*. The lines tangent to these circles constitute failure lines for all combinations of applied stresses between the two circles. The area enclosed by the circles and the failure lines represents a safe zone. In the case of the even material, the failure lines are independent of the normal stress and are defined by the maximum shear strength of the material. This is consistent with the maximum shear-stress theory for ductile materials (which tend also to be even materials). For the uneven material, the failure lines are a function of both the normal stress σ and the shear stress τ.

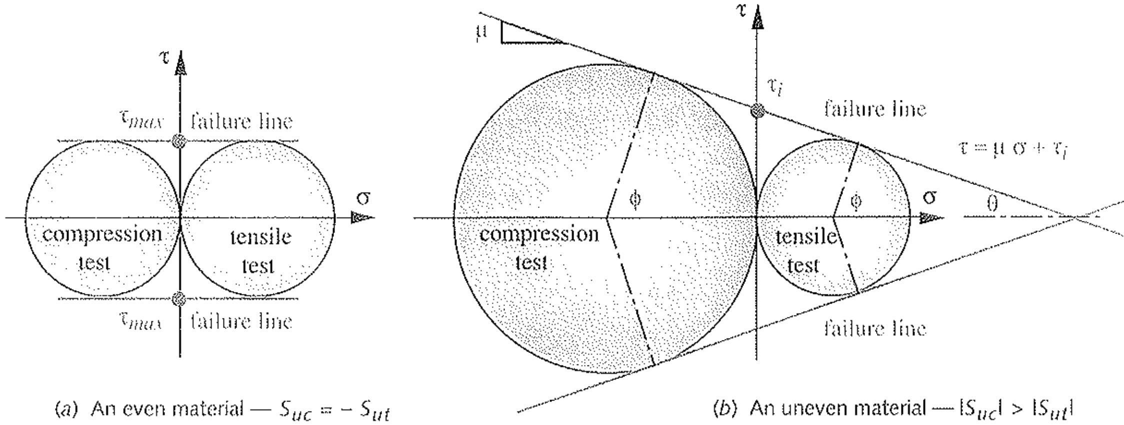


FIGURE 5-10

Mohr's Circles for Both Compression and Tensile Tests Showing the Failure Envelopes for (*a*) *Even* and (*b*) *Uneven* Materials

For the compressive regime, as the normal compressive stress component becomes increasingly negative (i.e., more compression), the material's resistance to shear stress increases. This is consistent with the idea expressed above that compression makes it more difficult for shear slippage to occur along fault lines within the material's internal flaws. The equation of the failure line can be found for any material from the test data shown in Figure 5-10. The slope μ and the intercept τ_i can be found from geometry using only the radii of the Mohr's circles from the tensile and compression tests.

The interdependence between shear and normal stress shown in Figure 5-10*b* is confirmed by experiment for cases where the compressive stress is dominant, specifically where the principal stress having the largest absolute value is compressive. However, experiments also show that, in tensile-stress-dominated situations with uneven, brittle materials, failure is due to tensile stress alone. The shear stress appears not to be a factor in uneven materials if the principal stress with the largest absolute value is tensile.

The Coulomb-Mohr Theory

These observations lead to the Coulomb-Mohr theory of brittle failure, which is an adaptation of the maximum normal-stress theory. Figure 5-11 shows the two-dimensional case plotted on the σ_1, σ_3 axes and normalized to the ultimate tensile strength, S_{ut}. The maximum normal-stress theory is shown for an *even material* as the dotted square of half-dimensions $\pm S_{ut}$. This could be used as the failure criterion for a brittle material in static loading if its compressive and tensile strengths were equal (an even material).

The maximum normal-stress theory envelope is also shown (gray-shaded) for an *uneven material* as the asymmetric square of half-dimensions S_{ut}, $-S_{uc}$. This failure envelope is only valid in the first and third quadrants as it does not account for the interdependence of the normal and shear stresses shown in Figure 5-10, which affects the second and fourth quadrants. The Coulomb-Mohr envelope (light-color shaded area) attempts to account for the interdependence by connecting opposite corners of these two quadrants with diagonals. Note the similarity of the shape of the Coulomb-Mohr hexagon to the maximum-shear-theory hexagon for ductile materials in Figure 5-5 (p. 280).

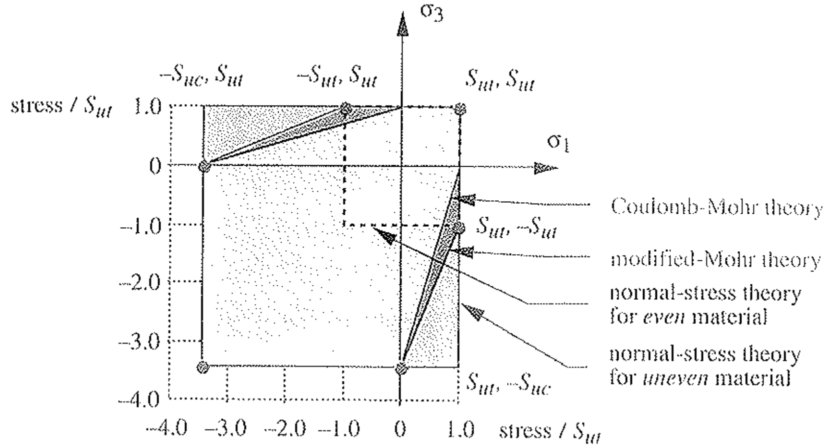


FIGURE 5-11

Coulomb-Mohr, Modified-Mohr, and Maximum Normal-Stress Theories for Uneven Brittle Materials

The only differences are the Coulomb Mohr's asymmetry due to the uneven material properties and its use of ultimate (fracture) strengths instead of yield strengths.

Figure 5-12 shows some gray cast-iron experimental test data superposed on the theoretical failure envelopes. Note that the failures in the first quadrant fit the maximum normal-stress theory line (which is coincident with the other theories). The failures in the fourth quadrant fall **inside** the maximum normal-stress line (indicating its unsuitability) and also fall well outside the Coulomb-Mohr line (indicating its conservative nature). This observation leads to a modification of the Coulomb-Mohr theory to make it better fit the observed data.

The Modified-Mohr Theory

The actual failure data in Figure 5-12 follow the even materials' maximum normal-stress theory envelope down to a point $S_{ut}, -S_{ut}$ below the σ_1 axis and then follow a straight line to $0, -S_{uc}$. This set of lines, shown as the combined light- and dark-color shaded portions of Figure 5-11 (also marked by colored dots), is the **modified-Mohr failure-theory envelope**. *It is the preferred failure theory for uneven, brittle materials in static loading.*

If the 2-D principal stresses are ordered $\sigma_1 > \sigma_3$, $\sigma_2 = 0$, then only the first and fourth quadrants of Figure 5-12 need to be drawn, as shown in Figure 5-13, which plots the stresses normalized by N/S_{ut} where N is the safety factor. Figure 5-13 also depicts three plane-stress conditions labeled A, B, and C. Point A represents any stress state in which the two nonzero principal stresses, σ_1, σ_3 are positive. Failure will occur when the load line OA crosses the failure envelope at A'. The safety factor for this situation can be expressed as

$$N = \frac{S_{ut}}{\sigma_1} \tag{5.12a}$$

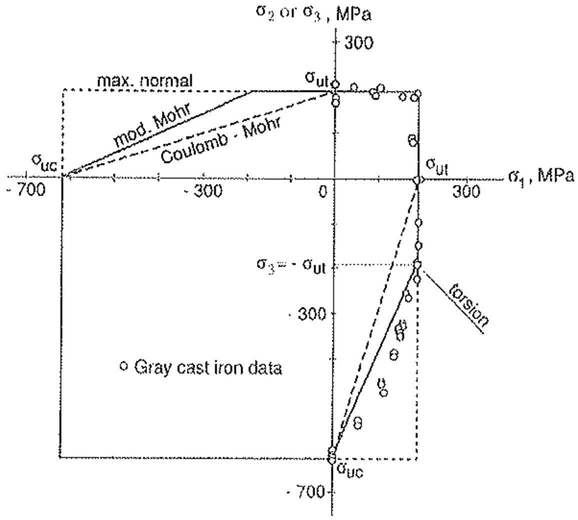


FIGURE 5-12

Biaxial Fracture Data of Gray Cast Iron Compared to Various Failure Criteria *(From Fig 7.13, p. 255, in Mechanical Behavior of Materials by N. E. Dowling, Prentice-Hall, Englewood Cliffs, NJ, 1993. Data from R. C. Grassi and I. Cornet, "Fracture of Gray Cast Iron Tubes under Biaxial Stresses," J. App Mech, v. 16, p.178, 1949)*

If the two nonzero principal stresses have opposite sign, then two possibilities exist for failure, as depicted by points B and C in Figure 5-13. The only difference between these two points is the relative values of their two stress components σ_1, σ_3. The load line OB exits the failure envelope at B' above the point $S_{ut}, -S_{ut}$, and the safety factor for this case is given by equation 5.12a above.

If the stress state is as depicted by point C, then the intersection of the load line OC and the failure envelope occurs at C' below point $S_{ut}, -S_{ut}$. The safety factor can be found by solving for the intersection between the load line OC and the failure line. Write the equations for these lines and solve simultaneously to get the **modified Mohr equation**.

$$N = \frac{S_{ut}\left|S_{uc}\right|}{\left|S_{uc}\right|\sigma_1 - S_{ut}\left(\sigma_1 + \sigma_3\right)} \tag{5.12b}$$

If the stress state is in the fourth quadrant, both equations 5.12a and 5.12b should be checked and the smaller resulting safety factor used.

Compare equation 5.12b to the less-accurate equation for the unmodified **Coulomb-Mohr theory** (which is not recommended for use).

$$N = \frac{S_{ut}\left|S_{uc}\right|}{\left|S_{uc}\right|\sigma_1 - S_{ut}\sigma_3}$$

To use the preferred modified Mohr theory of equation 5.12b, it would be convenient to have expressions for an **effective stress** that would account for all the applied stresses and allow direct comparison to a material-strength property, as was done for ductile materials with the von Mises stress. Dowling[5] develops a set of expressions for this effective stress involving the three principal stresses:*

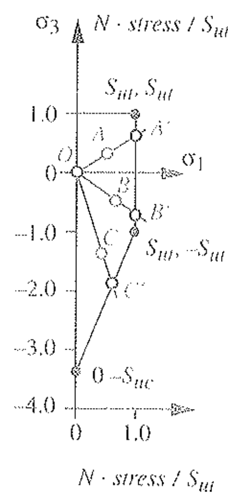


FIGURE 5-13

Modified-Mohr Failure Theory for Brittle Material

* See reference 5 for a complete derivation for both two- and three-dimensional Coulomb-Mohr and modified-Mohr theories and the effective stress.

$$C_1 = \frac{1}{2}\left[\left|\sigma_1 - \sigma_2\right| + \frac{2S_{ut} - \left|S_{uc}\right|}{-\left|S_{uc}\right|}\left(\sigma_1 + \sigma_2\right)\right]$$

$$C_2 = \frac{1}{2}\left[\left|\sigma_2 - \sigma_3\right| + \frac{2S_{ut} - \left|S_{uc}\right|}{-\left|S_{uc}\right|}\left(\sigma_2 + \sigma_3\right)\right] \qquad (5.12c)$$

$$C_3 = \frac{1}{2}\left[\left|\sigma_3 - \sigma_1\right| + \frac{2S_{ut} - \left|S_{uc}\right|}{-\left|S_{uc}\right|}\left(\sigma_3 + \sigma_1\right)\right]$$

The largest of the set of six values (C_1, C_2, C_3, plus the three principal stresses) is the desired effective stress as suggested by Dowling.

5

$$\tilde{\sigma} = \text{MAX}\left(C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3\right)$$

$$\tilde{\sigma} = 0 \quad \textit{if } \text{MAX} < 0 \qquad (5.12d)$$

where the signed function MAX denotes the algebraically largest of the six supplied arguments. If all of the arguments are negative, then the effective stress is zero.

This *modified-Mohr effective stress* can now be compared to the ultimate tensile strength of the material to determine a safety factor.

$$N = \frac{S_{ut}}{\tilde{\sigma}} \qquad (5.12e)$$

This approach allows easy computerization of the process.