

6.0 Impact Loads

A common type of structural analysis problem results from an impact load. The impact could be caused by a weight falling on the design object or possibly from the design object falling and striking a hard surface. In both cases, the loads are not obvious but can be easily derived from our knowledge of mechanics.

We will start by looking at a weight dropped from a height onto our design object. We know that the weight will deform the design object when it strikes it. The design object will behave like a spring being compressed. The kinetic energy of the falling weight will be transferred to the design object and stored as compression.

Figure 1 shows both a weight suspended above a spring and the same weight and spring after the weight has fallen on the spring and stopped its downward motion. We will assume the weight is at rest before it starts to fall. All of its kinetic energy is derived from the fall. We can write an energy balance as:

$$U = W(H + D) = \int_0^D kx dx \quad (6.1)$$

Integrating

$$U = W(H + D) = \frac{1}{2} kD^2 \quad (6.2)$$

where

- U the total energy of the system
- W the weight of the design object or the weight being dropped onto the design object.
- H the height of fall not including deformation
- D the deformation of the design object due to the impact
- k the spring constant for the design object

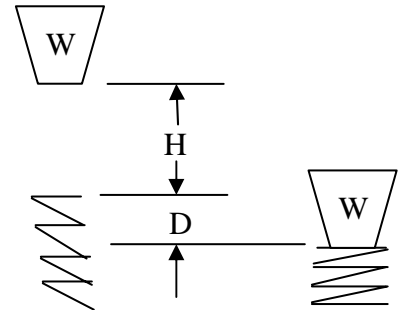


Figure 1 - Spring being compressed by a falling weight.

If we assume the deformation of the spring is small compared to the height the weight is dropped then the energy acquired by the weight dropping D is negligible. Our equation becomes

$$U = WH = \frac{1}{2} kD^2 \quad (6.3)$$

or

$$D = \sqrt{\frac{2WH}{k}} \quad (6.4)$$

This equation can be used for the problem where a weight is dropping onto the design object or the design object is dropping onto a hard surface. The spring represents

the deformation of the design object when it strikes a hard object or an object is dropped onto it.

Looking at equation 6.4, the problem statement will usually tell us W and H but we do not know k . We can determine k by using Hook's law. It says that the force applied

$$F = kD \quad (6.5)$$

Solving for k yields

$$k = F/D \quad (6.6)$$

We can apply a small force to the design object at the impact area and compute the displacement using Mechanics or any finite element program. Once we have the force and deflection, we can easily compute k .

6.1 Example

Calculators are usually housed in a plastic case. What are the stresses they will experience if the calculator is dropped on a hard, non-yielding surface.

We will create the calculator case with Pro/Engineer and analyze it with Mechanics. We will assume the case is a polycarbonate plastic or similar material. The case can be idealized as a 3D shell. We apply a 10 lb load at one corner and constrain the case at the opposite end. This is shown in Figure 1 below.

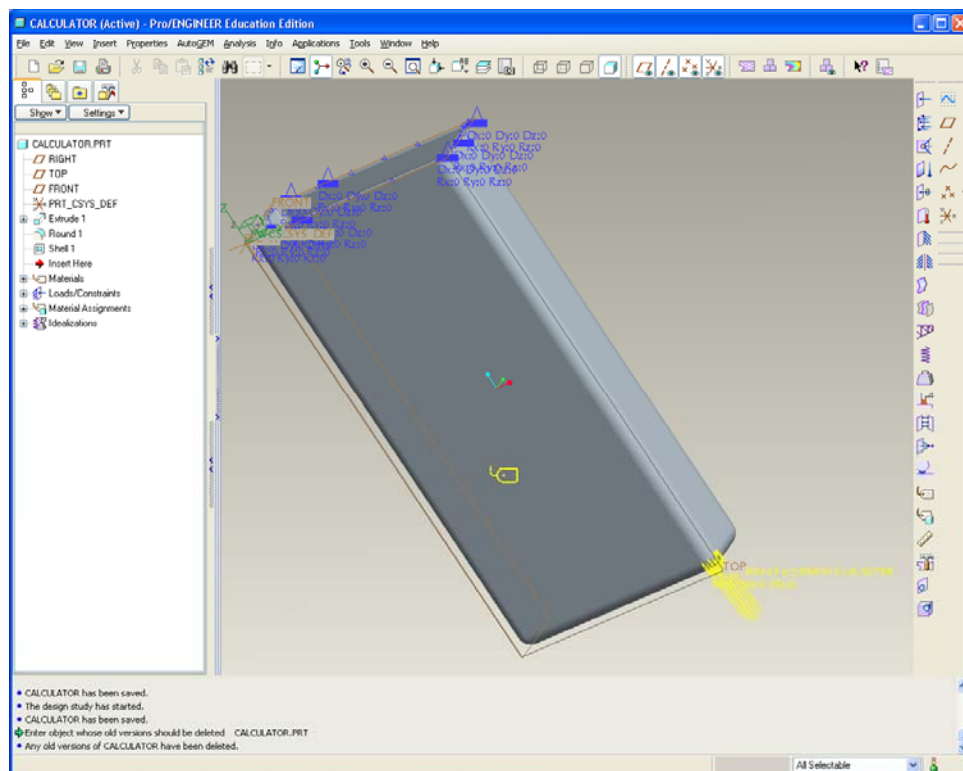


Figure 1 - Calculator case being analyzed.

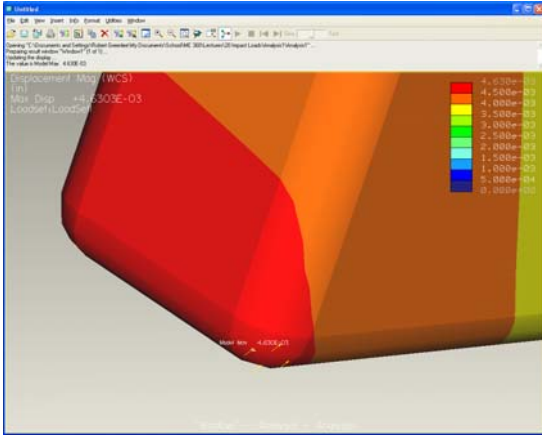


Figure 2 - Displacements at the load point.

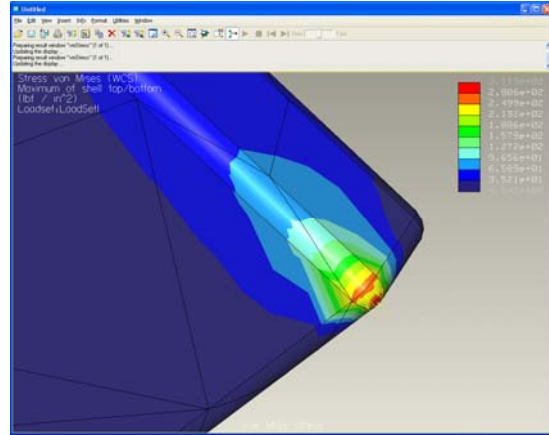


Figure 3 - Von Mises stresses at the load point.

Constraining the far end of the case is probably unrealistic but we are mainly interested in the stresses at the point of contact. This will be this highest stress area. The constraints are far enough away from the point of contact that they will make very little difference in our computations.

The results are shown in Figures 2 and 3. Figure 2 shows the maximum displacement of 0.00463 inches and Figure 4 shows the von Mises stresses at the corner where the stress is applied.

The results of the analysis show a force of 10 lbs produces a deformation of 0.00463 inches. We can now use equation 6.6 to compute k .

$$k = F / D = 10 / 0.00463 = 2160 \quad (6.7)$$

We will assume the calculator weighs .75 lbs and is dropped from a height of 5 feet or 60 inches. With this we can compute the displacement at the point of impact with equation 6.4. It is:

$$D = \sqrt{\frac{2WH}{k}} = \sqrt{\frac{2(0.75)(60)}{2160}} = 0.2041 \text{ Inches} \quad (6.8)$$

We can now use equation 6.5 to compute the stresses imposed by this displacement.

$$F = kD = 2160 \times 0.2041 = 441 \text{ lbs} \quad (6.9)$$

We can now solve the problem again with a load of 441 pounds. As a check, we can examine the displacement. It should be very close to 0.204 inches.

Rerunning the program shows a displacement of 0.204 inches and a maximum von Mises stress of 13,726 psi. The ultimate strength this type of plastic is approximately 10,000 psi so it appears the case will break if dropped from 5 feet. A fringe plot of the von Mises stresses is shown in Figure 4. The legend shows everything above 10,000 psi in red. As you might expect, most of the damage is done on the corner where the impact occurred.

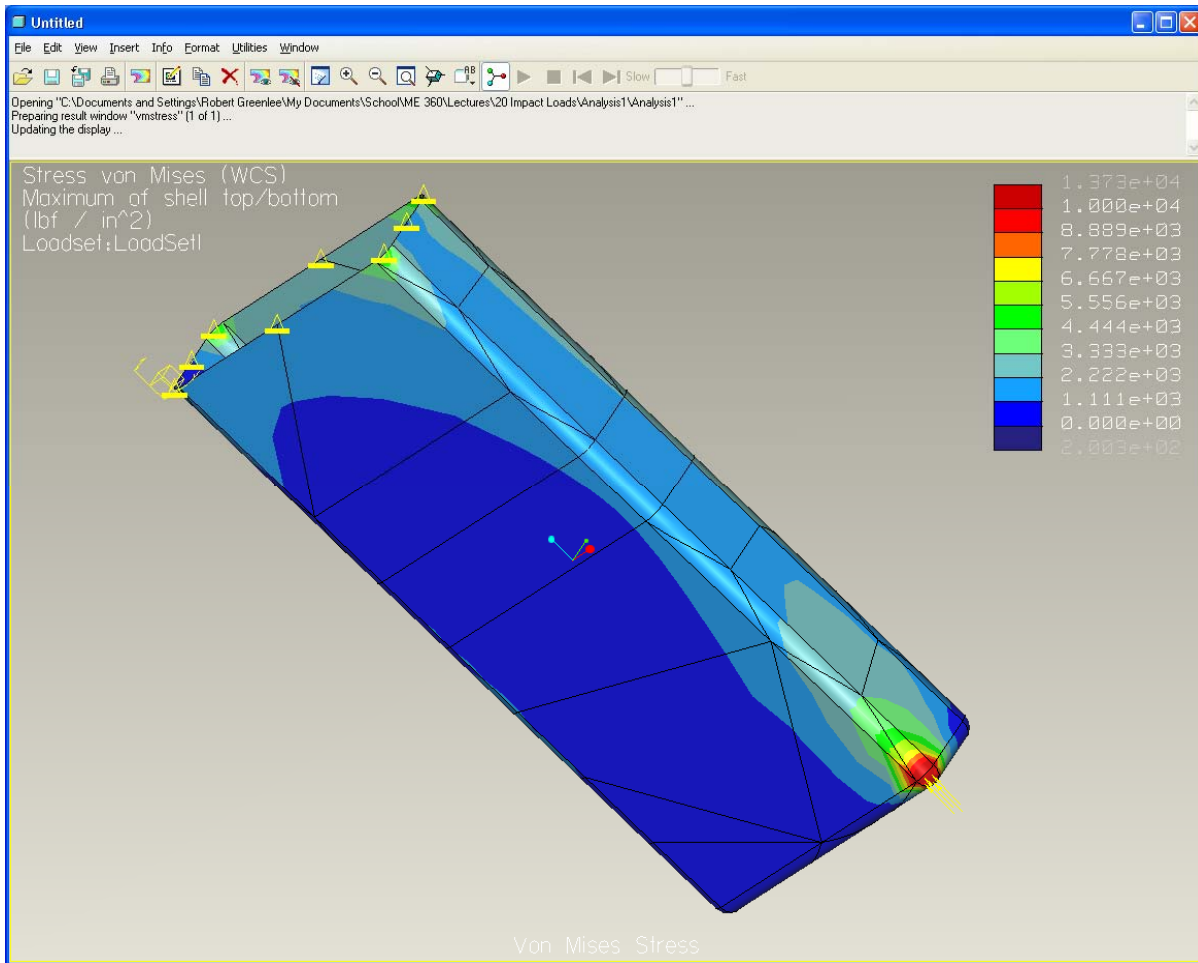


Figure 4 - von Mises stresses on the case when dropped from 5 feet.