Money

Engineers frequently find themselves working as much with money and financing as they do with technical design. All engineering requires money. Money is needed to pay salaries, to buy materials for producing the product, and to buy the manufacturing machines.

This money may be borrowed from banks or investors and must be repaid. These loans have interest attached to them.

SIMPLE INTEREST

Simple interest applies the interest to the original loan. The interest is usually applied annually. We can illustrate this with an example.

You borrow $5,000 from you uncle to pay for college. You promise to pay him back 1 year after graduation. You both agree on a simple interest rate of 8%. If you repay the loan in 5 years, how much will you pay?

F = the amount to be paid back
P = the principle originally borrowed
n = the number of periods the interests is applied (usually number of years).
i = the interest rate per year.

We can create a table showing equations for the amount to be paid back each year. The formulas show the year, the amount due at the beginning of the year, and the amount due at the end of the year.

Payback		
Year	Beginning	Ending
1	$F = P$	$F = P + P\,i$
2	$F = P + Pi$	$F = P + Pi + Pi$
3	$F = P + Pi + Pi$	$F = P + Pi + Pi + Pi$
...	...	...

Looking at the table, we can create a formula for any number of years. It is:

$$F = P + Pin \qquad (1)$$

We can compute how much you need to repay your uncle.

$$F = 5000 + 5000 \times 0.08 \times 5 = \underline{\mathbf{\$7,000.00}}$$

You will owe him 7000 dollars at the end of 5 years.

COMPOUND INTEREST

Banks and investors usually look at compound interest rather than simple interest. With compound interest, the interest over the first year or period is added to the loan and the interest for the next year or period is computed using that new principle.

F = the amount to be paid back
P = the principle originally borrowed
n = the number of periods the interests is applied (usually number of years).
i = the interest rate per year.

We can create a table for this similar to the one created for simple interest.

Payback		
Year	Beginning	Ending
1	F = P	F = P(1 + i)
2	F = P(1 + i)	F = P(1 + i)(1 + i)
3	F = P(1 + i)(1 + i)	F = P(1 + i)(1 + i)(1 + i)
...	...	...

The formula for this can be written:

$$F = P(1 + i)^n \qquad (2)$$

Example 1

The bank says it will loan you the same $5,000 at 7% compounded annually. Who has the best deal, the bank or your uncle?

$$F = 5000(1 + 0.07)^5 = 5000 \text{ x } 1.07^5 = \underline{\textbf{\$7,012.76}}$$

The bank charges 1% less then your uncle but they compound the interest annually. The bank will charge you $12.76 more for the loan.

Example 2

In 10 years, you would like to have $10,000 in the bank to put down on a house. How much do you have to put in the bank now to have $10,000 in ten years. The bank will give you 6% interest compounded annually.
Starting with equation 2 we see.

$$F = P(1 + i)^n \qquad (2)$$

Solving for P

$$P = F / (1 + i)^n \qquad (3)$$

$P = 10{,}000 / (1 + .06)^{10} = 10{,}000 / 1.7908 = \underline{\textbf{\$5,583.95}}$

Another bank compounds the interest monthly. What would you have to invest for this bank?

We start with equation 3 but this time we do not use the annual interest rate because the interest is compounded monthly. Instead we use:

$i = i_{annual} / 12 = 0.06 / 12 = 0.005$

The interest is being compounded monthly so there are 12 compounding periods per year.

$N = \text{interest periods} = 10 \text{ years} \times 12 = 120$

Substituting this into equation (3) yields:

$P = F / (1 + i)^n = 10{,}000 / 1.005^{120}$

$P = \underline{\textbf{\$5,496.33}}$

Example 3

The bank gives you 10% interest compounded annually. How long will it take for your money to triple in value?

Starting with equation (2)

$$F = P(1 + i)^n \qquad (2)$$

Divide both sides by P

$$F/P = (1 + i)^n$$

If the money triples in value then

$$F/P = (1 + i)^n = 3$$

We take the log of both sides of the equation and solve for **n**

$$\text{Log}(F/P) = \text{Log}(3) = n\text{Log}(1+i)$$

Or

$$n = \text{Log}(3) / \text{Log}(1+i) = \text{Log}(3) / \text{Log}(1.1) = 0.47712 / 0.04139 = \underline{\textbf{11.5 years}}$$

A Uniform Series of Compound Interest

Define: A = the end of the period disbursement or receipt from a uniform sum
of money.

At the end of each year, you are going to deposit a fixed amount of money into an account paying i percent interest. How much will you have after n year?

$$F = A(1+i)^{n-1} + \ldots + A(1+i)^3 + A(1+i)^2 + A(1+i) + A \qquad (2.1)$$

Multiply both sides by $(1+i)$

$$F(1+i) = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^3 + A(1+i)^2 + A(1+i)$$

Or

$$F + iF = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^3 + A(1+i)^2 + A(1+i)$$

Subtracting F from both sides

$$iF = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^3 + A(1+i)^2 + A(1+i) - F$$

Now substituting equation (2.1) for F on the right hand side of the equation yields.

$$iF = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^3 + A(1+i)^2 + A(1+i) - A(1+i)^{n-1} - \ldots - A(1+i)^3 - A(1+i)^2 - A(1+i) - A$$

Subtracting the terms results in:

$$iF = A(1 + i)^n - A$$

or

$$F = A\,[(1+i)^n - 1]/i \qquad (2.2)$$

Example 4

You deposit $500 in the bank at the end of each year for 5 years. How much money will you have after the last deposit if the bank pays 5% interest compounded annually?

$$F = 500(1.05^5 - 1) / 0.05 = \underline{\mathbf{\$2,762.82}}$$

F in equation (2.2) is the value of the money at some future time at the end of the investment period. How much is that money worth now? We can compute this by

substituting the compound interest formula we derived in the last lecture into equation (2.2). The compound interest formula was:

$$F = P(1 + i)^n$$

Substituting yields:

$$F = P(1 + i)^n = A [(1+i)^n - 1]/i$$

Solving for A

$$A = P[i(1+i)^n]/[(1+i)^n - 1] \qquad\qquad (2.3)$$

Example 5

You borrow $200,000 from the bank. How much must you pay monthly to repay the loan in 10 years? The interest is 6% compounded monthly.

$i = 0.06/12 = 0.005$

n = 10 years x 12 months/year = 120

$A = 200,000 [.005(1.005)^{120}]/[(1005)^{120} - 1]$

__A = $2220.41__

Example 6

Your company needs to buy a lathe for manufacturing parts. The lathe costs $35,000 but the company will allow you to pay $400 per month for 10 years. You know you could invest the money at 6 % interest. Which is the best deal?

To solve this, we will compare the present value of the loan to the $35,000. Both are in present dollars so we can compare them directly. You cannot compare a future value with a present value.

$$A = P[i(1+i)^n] / [(1+i)^n - 1]$$

Solving for P

$$P = A[(1+i)^n - 1]/[i(1+i)^n] \qquad\qquad (2.4)$$

$i = 6\% / 12 = 0.005$

A = $400 per month

$$n = 10 \times 12 = 120$$

Substituting

$$P = 400[(1.005)^{120} - 1] / [0.005(1.005)^{120}] = \underline{\mathbf{\$36,029.38}}$$

So what should you do? The present value of the payment plan is \$36,029.38 and the cost of buying the lathe outright is \$35,000. It appears that buying the lathe outright is the best arrangement.

Example 7

In Example 6 we looked at purchasing a lathe by comparing the present value of purchasing the lathe outright and making monthly payments for 5 years. We could have done the comparison by looking at the future value of both. We can compare present values or future values but we cannot compare a present value to a future value.

The future value of the \$35, 000 used to purchase the lathe outright is:

$$F = P(1 + i)^n$$

$$F = 35,000 \, (\, 1 + 0.06)^5 = \underline{\mathbf{\$46,837.90}}$$

The future value of the \$400 monthly payment is:

$$F = A \, [(1+i)^n - 1]/i$$
$$F = 400[(1.005)^{120} - 1]/0.005 = \underline{\mathbf{\$65,551.74}}$$

The difference is considerably larger when we compare future values but the results are the same. It will be to our advantage to purchase the lathe outright than to pay it off in monthly payments.

Example Problems

Problem 1

If you wish to have $1000 in a saving account at the end of 5 years and the interest rate is 6% paid annually, how much should you put in the saving account now?

We know from the lecture:

$$F = P(1 + i)^n$$

Solving this for P results in:

$$P = F / (1 + i)^n$$

F = the future value = $1000
i = the interest rate = 6% = 0.06
n = the number of years = 5

$$\underline{P = 1000 / (1 + 0.06)^5 = \$747.26}$$

Problem 2

Joe read that a 1 acre parcel of land could be purchased for $1000 in cash. He decided to save a uniform amount at the end of each month so he would have the required $1000 at the end of one year. The bank pays 6% interest compounded monthly. How much should Joe save each month?

$$F = A [(1+i)^n - 1]/i \qquad\qquad (2.2)$$

Solving this for A results in:

$A = iF /[(1+i)^n - 1]$
$i = 0.06 / 12 = 0.005$
$n = 1 \times 12 = 12$
$F = \$1000$

$$\underline{A = (0.005 \times 1000) / [(1.005)^{12} - 1] = \$81.07}$$

Problem 3

On January 1 a man deposits $5000 in a credit union that pays 8% interest, compounded annually. He wished to withdraw all the money in 5 equal end-of-year sums, beginning December 31st of the first year. How much should he withdraw each year?

$$A = P[i(1+i)^n] / [(1+i)^n - 1]$$

$P = \$5000 \qquad n = 5 \qquad\qquad i = 8\% \qquad\qquad A = \text{unknown}$

$$A = 5000 \, [0.08 \, (1.08)^5] / (1.08^5 - 1)$$

A = $1252.28

Problem 4

A $5000 loan was to be repaid with 8% simple annual interest. A total of $5800 was paid. How long was the loan outstanding?

$$F = P + Pin$$

Solving for n

$n = (F - P)/Pi \qquad\qquad \text{where } i = 8\% \qquad\qquad P = \$5000 \qquad F = \$5800$
$n = (5800 - 5000)/(5000 \times 0.08)$
n = 2 years

Problem 5

A sum of money invested at 4% interest, compounded semi-annually, will double in amount in approximately how many years?

$$F = P(1 + i)^n$$

Solving for n yields

$F / P = (1 + i)^n$
$\text{Log } (F/P) = n\text{Log}(1 + i)$
Or
$n = \text{Log } (F/P) / \text{Log}(1 + i)$
where
$F/P = 2 \qquad\qquad i = 0.04 / 2 = 0.02$

$n = \text{Log } (2) / \text{Log } (1.02) = 35 \text{ periods or } \underline{\textbf{17.5 years}}$

MONEY Homework

1. A woman borrowed $2000 and agreed to repay it at the end of three years together with 10% simple interest. How much will she pay at the end of the three years?

    $2662 $3000 $2600 $2200 $2400

2. You put $6000 in the bank. How long will it have to stay there to double in value if the bank pays 8% interest compounded annually?

    6 Years   4 Years   9 years   8 years   20 Years

3. You invest $1000 in the bank each year for 3 years. The bank pays 7% interest compounded annually. How much money will you have in 5 years?

    $4125.36 $3822.57 $3938.39 $2655.37 $3600.04

4. You put $5000 in the bank and keep it there for 20 years. The bank pays 6.5% interest, compounded monthly. How much money will you have after 20 years?

    $18,282.23 $17,618.23 $11,500.00 $15,356.74 $12,721.56

5. Each year, your grandmother gives you $1000. You put the money in the bank at 6% annual interest compounded annually. How much money will you have after your 10th deposit?

    $10,600 $7,360 $13,181 $10,000   not listed

6. What is the present value of the money your grandmother will give you?

    $10,600 $7,360 $13,181 $10,000   not listed

7. You borrow $18,000 from the bank. The bank charges 8% annual interest compounded monthly. What will be your monthly payment if you repay the loan in 5 years?

    $300 $426 $331 $365   not listed

8. You win $10,000,000 (future value) in the lottery and choose to take the money as an annuity paid out over 50 years. The interest rate on the annuity is 6% annually which is compounded monthly. What will be the monthly payment to you?

    $2,640 $16,667 $41,800 $3,264   not listed