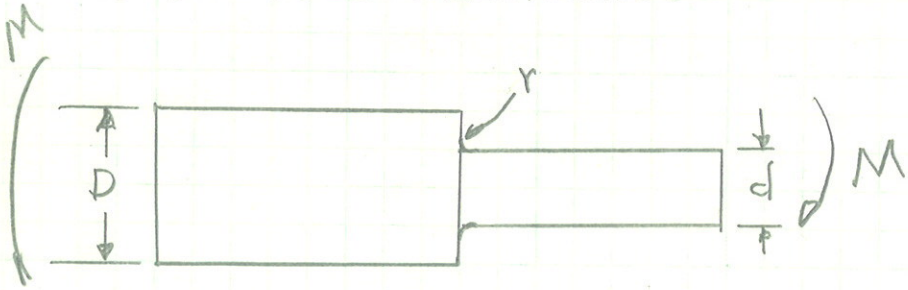
TCH STRESS CONCENTRATION (STATIC)

An abrupt change in the geometry of a part can cause a stress concentration where the stresses increase dramatically.

For example, look at a round shaft. Frequently, the shaft has a change in diameter to help locate a bearing, gear, or sheave.

These changes in diameter can cause a stress concentration



The stress at the change in size is greater than the nominal stress.

$$\sigma_{nom} = \frac{Mc}{I}$$

computed at the outer fiber using the smaller d for the calculation

The maximum stress is computed with

$$\sigma_{max} = K_t \, \sigma_{nom}$$

where K_t is computed with

$$K_t = A \left(\frac{r}{d}\right)^b$$

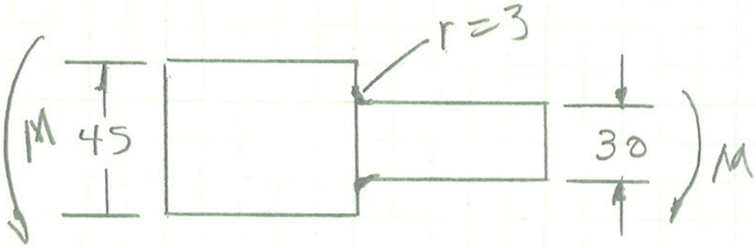
r = the radius of the fillet

d = The smaller size of the shaft

A = A value from a graph in appendix A-15

Note that K_t is dimensionless.

EXAMPLE



$M = 1000 \text{ N-M}$

$\sigma_{nom} = \dfrac{Mc}{I}$

$d = 30 \text{ mm}$

$C = d/2 = 15 \text{ mm} = .015 \text{ m}$

$D = 45 \text{ mm} = .045 \text{ m}$

$d = 30 \text{ mm} = .030 \text{ m}$

$$I = \frac{\pi d^4}{64} = \frac{\pi (.03)^4}{64} = 3.97 \times 10^{-8}$$

$$\sigma_{nom} = \frac{Mc}{I} = \frac{1000(.015)}{3.97 \times 10^{-8}} = 378 \text{ MPa}$$

$K_t = A \left(\dfrac{r}{d}\right)^b \qquad r = 3 \text{ mm} = .003 \text{ m}$

The ratio $D/d = \dfrac{45}{30} = 1.5$

From table in figure C-1

$A = 0.938\,36 \qquad b = -0.257\,59$

$$K_t = .938 \left(\frac{.003}{.03}\right)^{-.258} = 1.70$$

$$\sigma_{max} = K_t \sigma_{nom} = 1.70(378)$$

$$\boxed{\sigma_{max} = 642 \text{ MPa}}$$

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TORQUE

The same approach can be applied to torque.

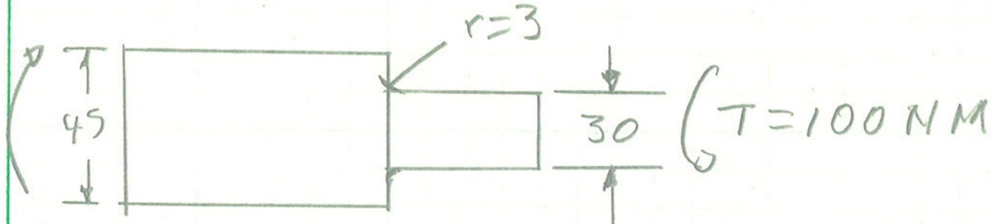
$$\tau_{nom} = \frac{Tc}{J}$$

$$K_{ts} = A\left(\frac{r}{d}\right)^b$$

$$\tau_{max} = K_{ts}\,\tau_{nom}$$

EXAMPLE

We will add a torque of 100 NM to the shaft in the previous problem.



$$J = \frac{\pi d^4}{32} = \frac{\pi (.03)^4}{32} = 7.95\,E{-8}$$

$$c = d/2 = 15\,mm = .015\,m$$

$$\tau_{nom} = \frac{100(.015)}{7.95\,E{-8}} = 18.9\ MPa$$

$$K_{ts} = A\left(\frac{r}{d}\right)^b$$

The ratio $D/d = \frac{45}{30} = 1.5$

Interpolating table C-3 in handout

$$\frac{1.5 - 1.33}{2.0 - 1.33} = .254$$

$$A = .254(.863 - .849) + .849 = .853$$

$$b = .254(-.239 + .232) - .232 = -.233$$

$$K_{ts} = A\left(\frac{r}{d}\right)^{b} = .853\left(\frac{.003}{.030}\right)^{-.233}$$

$$K_{ts} = 1.46$$

$$\tau_{max} = K_{ts}\,\tau_{nom} = 1.46(18.9)$$

$$\tau_{max} = 27.6 \text{ MPa}$$

COMPUTING THE PRINCIPAL STRESSES

$$\sigma_1 = \frac{\sigma_x + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$\sigma_1 = \frac{642 + 0}{2} + \sqrt{\left(\frac{642 - 0}{2}\right)^2 + 27.6^2}$$

$$\boxed{\sigma_1 = 321. + 322.2 = 643.2 \text{ MPa}}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$\sigma_2 = \frac{642 + 0}{2} - \sqrt{\left(\frac{642 - 0}{2}\right)^2 + 27.6^2}$$

$$\boxed{\sigma_2 = 321 - 322.2 = -1.2 \text{ MPa}}$$