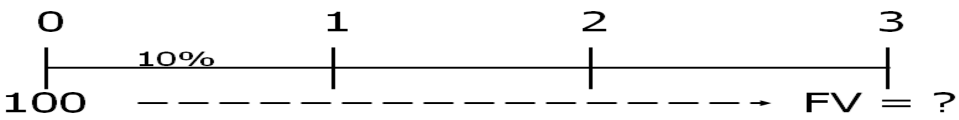
Math Workshop—Algebra
(Time Value of Money; TVM)



Finding FVs (moving to the right on a time line) is called compounding.

$FV_1 = PV + INT_1 = PV + PV*I = PV(1+I) = \$100(1+10\%) = \$110.00$
$FV_2 = FV_1(1+I) = PV(1+I)(1+I) = PV(1+I)^2 = \$100(1.10)^2 = \$121.00$
$FV_3 = FV_2(1+I) = PV(1 + I)^2(1+I) = PV(1+I)^3 = \$100(1.10)^3 = \$133.10$

In general, $\mathbf{FV_N = PV(1+I)^N}$

Financial calculator: TI BA II Plus

Set number of decimal places to display: 2nd FORMAT; use up and down arrows to display DEC=; press 9; press ENTER

Set AOS (Algebraic Operating System) calculation: 2nd FORMAT; down arrow 4 times until you see Chn (if you see AOS then just stop and hit CE/C); 2nd SET (AOS should display); hit CE/C.

Using **AOS**™ (algebraic operating system), the calculator solves problems according to the standard rules of algebraic hierarchy, computing multiplication and division operations before addition and subtraction operations. (Most scientific calculators use **AOS**.)

For example, when you enter 3 + 2 × 4 =, the **AOS** answer is 11 (2 × 4 = 8; 3 + 8 = 11).

Set END (for cash flows occurring at the end of the year): hit 2nd BGN; 2nd SET will toggle between cash flows at the beginning of the year (BGN) and end of the year (END). Leave it as END.

Set 1 payment per period: hit 2nd P/Y 1 ENTER.

Reset TVM calculations: 2nd CLR TVM

Financial calculators solve this equation: $\mathbf{FV_N + PV(1+I)^N = 0}$. There are 4 variables. If 3 variables are known, the calculator will solve for the 4th.

Return to $FV_3 = PV(1+I)^3 = \$100(1.10)^3 = \$133.10$ ➔

INPUTS	3	10	-100	0	
	N	I/Y	PV	PMT	FV
OUTPUT					133.10

➔**That is, press** 3 N 10 I/Y -100.......

Compute future value: CPT FV

Reset TVM calculations: 2nd CLR TVM

The following example is from TI BA II Plus Guidebook.

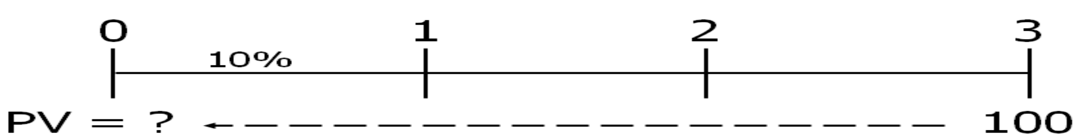
Example: If you open the account with $5,000, how much will you have after 20 years?

Enter number of payments.	**20** N	**N=**	**20.00**◃
Enter interest rate.	**.5** I/Y	**I/Y=**	**0.50**◃
Enter beginning balance.	**5000** +/– PV	**PV=**	**-5,000.00**◃
Compute future value.	CPT FV	**FV=**	**5,524.48***

In EXCEL, =FV(I, N, PMT, PV)

=FV(0.10, 3, 0, -100) = 133.10

Question: What's the PV of $100 due in 3 years if I/YR = 10%?



$FV_N = PV(1+I)^N$ ➔**$PV = FV_N/(1+I)^N$** ➔$PV = 100/(1+10\%)^3 \approx \$75.13$ ➔

INPUTS	3	10		0	100
	N	I/YR	PV	PMT	FV
OUTPUT			-75.13		

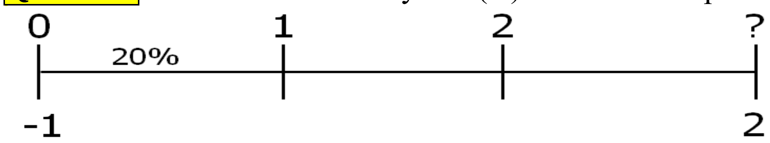
Compute present value: CPT PV

Reset TVM calculations: 2nd CLR TVM

In EXCEL, =PV(I, N, PMT, FV)

=PV(0.10, 3, 0, 100) ≈ -75.13

Question: Find the number of years (N) to double the present value.



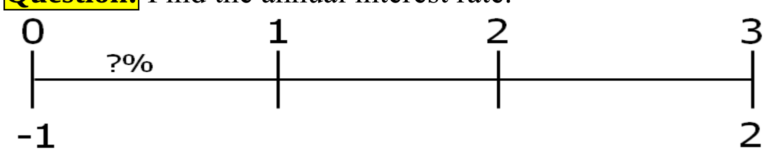
$$FV = PV(1 + I)^N$$

$2=1(1+0.20)^N \rightarrow (1.2)^N = 2/1 = 2 \rightarrow N*\ln(1.2)=\ln(2) \rightarrow N=\ln(2)/\ln(1.2) \approx 0.6931/0.1823 \approx$ **3.80**

Compute N: CPT N
Reset TVM calculations: 2^{nd} CLR TVM

In EXCEL, =NPER(I, PMT, PV, FV)
=NPER(0.20, 0, -1, 2) ≈ 3.801784017

Question: Find the annual interest rate.

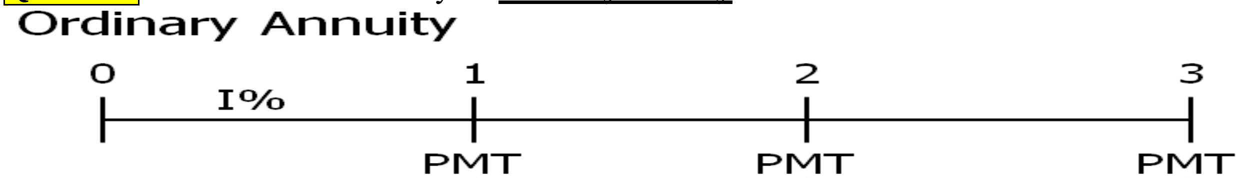


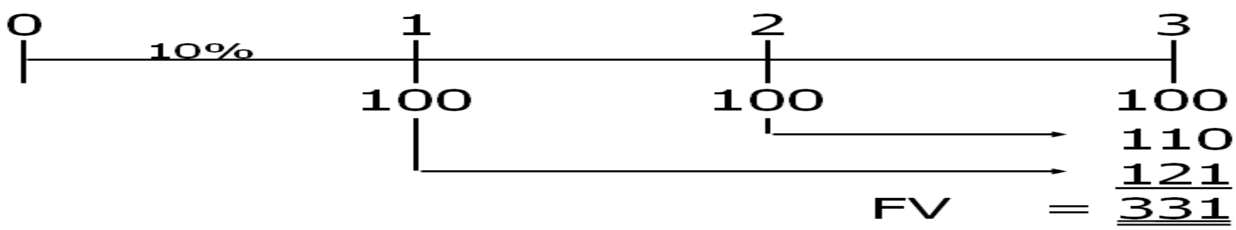
$2=1*(1+I)^3 \rightarrow (2)^{(1/3)} = (1+I) \rightarrow (1+I) \approx 1.2599 \rightarrow I \approx 0.2599 =$ **25.99%**

Compute interest rate: CPT I/Y
Reset TVM calculations: 2^{nd} CLR TVM

In EXCEL, =RATE(N, PMT, PV, FV)
=RATE(3, 0, -1, 2)=25.992105%

Question: What's the FV of a 3-year **ordinary annuity** of $100 at 10%?



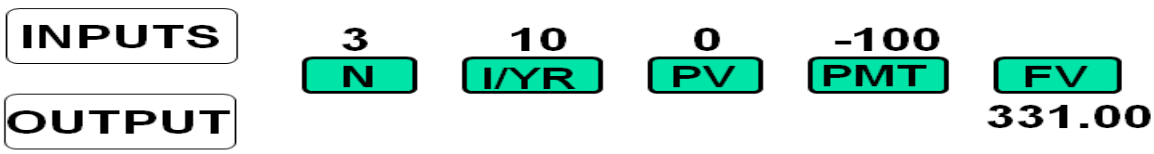


The future value of an ordinary annuity with N periods and an interest rate of I =
$FVA_N = PMT(1+I)^{(N-1)} + PMT(1+I)^{(N-2)} + PMT(1+I)^{(N-3)} + \ldots + PMT(1+I)^0$

$= (PMT/I)[(1+I)^N - 1]$

$= (100/0.1)[1.1^3 - 1] = 331$

Financial calculators solve this equation: **$FV_N + PV(1+I)^N + (PMT/I)[(1+I)^N - 1] = 0$**. There are 5 variables. If 4 variables are known, the calculator will solve for the 5th.

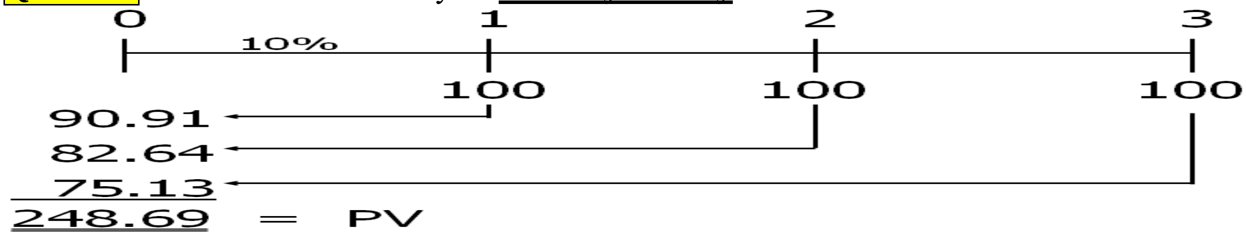


Compute the future value of ordinary annuity: CPT FV
Reset TVM calculations: 2nd CLR TVM

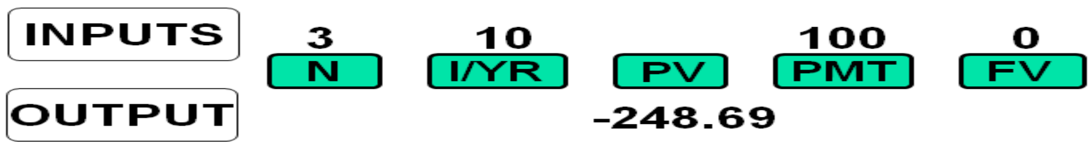
In EXCEL, =FV(I, N, PMT, PV)
=FV(0.10, 3, -100, 0) = 331.00

Math: $FVA_N = PMT(1+I)^{(N-1)} + PMT(1+I)^{(N-2)} + PMT(1+I)^{(N-3)} + \ldots + PMT(1+I)^0$……(1)
$(1+I)^*FVA_N = PMT(1+I)^N + PMT(1+I)^{(N-1)} + PMT(1+I)^{(N-2)} + \ldots + PMT(1+I)^1$…….. (2)
(2)-(1) ➔ $I^*FVA_N = PMT(1+I)^N - PMT$
➔ $FVA_N = (PMT/I)[(1+I)^N - 1]$

Question: What's the PV of a 3-year **ordinary annuity** of $100 at 10%?



$$PVA_N = PMT/(1+I)^1 + PMT/(1+I)^2 + PMT/(1+I)^3 + \ldots + PMT/(1+I)^N$$
$$= PMT\{(1/I) - 1/[I(1+I)^N]\}$$
$$= (100)\{(1/0.1) - 1/(0.1*1.1\text{^}3)\} \approx 248.69$$

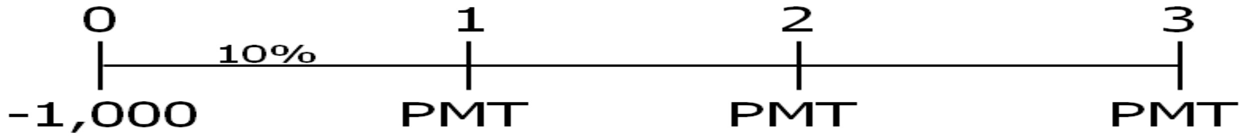


Compute the present value of ordinary annuity: CPT PV
Reset TVM calculations: 2nd CLR TVM

In EXCEL, =PV(I, N, PMT, FV)
=PV(0.10, 3, 100, 0) ≈ -248.69

Practice: Calculate $PVA_N = PMT/(1+I)^1 + PMT/(1+I)^2 + PMT/(1+I)^3 + \ldots + PMT/(1+I)^N$
$= PMT\{(1/I) - 1/[I(1+I)^N]\}$

Question: Find the annual payment for a \$1,000, 10% annual rate loan with 3 equal payments.



Math: $PVA=PMT/(1+i)+ PMT/(1+i)^2+\ldots+ PMT/(1+i)^N$……(a)

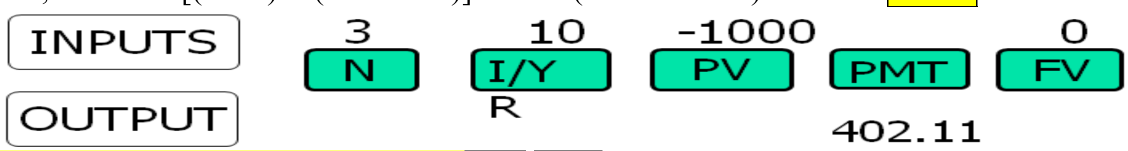
➔ $PVA=PMT[(1/i)-1/(i(1+i)^N)]$

Note: $(1+i)PVA=PMT+ PMT/(1+i)+\ldots+ PMT/(1+i)^{N-1}$……(b)

(b)-(a) ➔ $i*PVA=PMT-PMT/(1+i)^N$

➔ $PVA=PMT[(1/i)-1/(i(1+i)^N)]$

➔ $1,000=PMT[(1/0.1)-1/(0.1*1.1\text{^}3)]=PMT*(2.486851991)$ ➔ PMT≈402.11



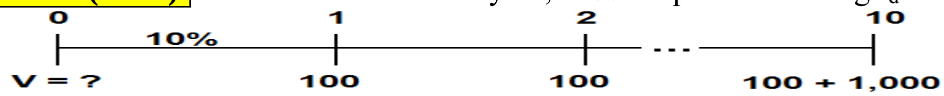
Compute the payment per period: CPT PMT

Reset TVM calculations: 2nd CLR TVM

In EXCEL, =PMT(I, N, PV, FV)

=PMT(0.10, 3, 1000, 0) ≈ -402.11

Question (bond): Find the value of a 10-year, 10% coupon bond using r_d = 10%.



$$V_B = \frac{\$100}{(1+r_d)^1} + \ldots + \frac{\$100}{(1+r_d)^N} + \frac{\$1{,}000}{(1+r_d)^N}$$

$$= \$90.91 + \ldots + \$38.55 + \$385.54$$
$$= \$1{,}000.$$

Note: $PVA = PMT/(1+i) + PMT/(1+i)^2 + \ldots + PMT/(1+i)^N$……(from the previous page)

➔ $PVA = PMT[(1/i) - 1/(i(1+i)^N)]$

➔Bond value V_b = $PMT[(1/i)-1/(i(1+i)^N)]$ + $M/(1+i)^N$; find the following two components!

PV annuity	=	$ 614.46
PV maturity value	=	385.54
Value of bond	=	$1,000.00

	N	I/YR	PV	PMT	FV
INPUTS	10	10		100	1000
OUTPUT			-1,000		

Compute the present value of bond: CPT PV
Reset TVM calculations: 2^{nd} CLR TVM

In EXCEL, =PV(I, N, PMT, FV)
=PV(0.10, 10, 100, 1,000) = -1,000.00

Question (r_d change): What would happen if expected inflation rose by 3%, causing r_d = 13%?

	N	I/YR	PV	PMT	FV
INPUTS	10	13		100	1000
OUTPUT			-837.21		

In EXCEL, =PV(I, N, PMT, FV)
=PV(0.13, 10, 100, 1,000) = -837.21 ➔The bond value would fall.

Question (r_d change): What would happen if r_d = 7%?

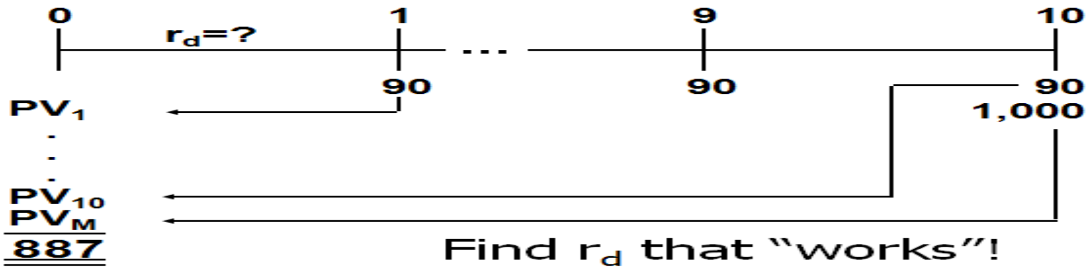
	N	I/YR	PV	PMT	FV
INPUTS	10	7		100	1000
OUTPUT			-1,210.71		

Compute the present value of bond: CPT PV
Reset TVM calculations: 2^{nd} CLR TVM

In EXCEL, =PV(I, N, PMT, FV)
=PV(0.07, 10, 100, 1,000) = -1,210.71 ➔The bond value would increase.

Yield to maturity (YTM) is **the rate of return** earned on a bond held to maturity.

Question (YTM): Find YTM on a 10-year, 9% annual coupon, $1,000 par value bond selling for $887.



$$V_B = \frac{INT}{(1 + r_d)^1} + \cdots + \frac{INT}{(1 + r_d)^N} + \frac{M}{(1 + r_d)^N}$$

$$887 = \frac{90}{(1 + r_d)^1} + \cdots + \frac{90}{(1 + r_d)^N} + \frac{1,000}{(1 + r_d)^N}$$

	N	I/YR	PV	PMT	FV
INPUTS	10		-887	90	1000
OUTPUT		10.91			

Compute YTM: CPT I/Y
Reset TVM calculations: 2nd CLR TVM

In EXCEL, =RATE(N, PMT, PV, FV)
=RATE(10, 90, -887, 1000)≈0.1091=10.91%

Question (YTM): Find YTM on a 10-year, 9% annual coupon, $1,000 par value bond selling for $1134.20.

	N	I/YR	PV	PMT	FV
INPUTS	10		-1134.2	90	1000
OUTPUT		7.08			

In EXCEL, =RATE(N, PMT, PV, FV)
=RATE(10, 90, -1134.20, 1000)≈0.0708=7.08%

Stock valuation—constant growth model

If you buy a stock for price P_0, hold it for one year, receive a dividend of D_1, then sell it for price P_1, your stock return, k, would be:

$$k = \frac{D_1 + (P_1 - P_0)}{P_0}$$

or

$$k = \underbrace{\frac{D_1}{P_0}}_{\text{dividend yield}} + \underbrace{\frac{(P_1 - P_0)}{P_0}}_{\text{capital gains yield}}$$

➔ $P_0 = (D_1+P_1)/(1+k)$; **a form of present value**

$$P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \ldots + \frac{D_n}{(1+k)^n} + \frac{P_n}{(1+k)^n}$$

➔The formula is called the **dividend discount model** (DDM).

If dividends are assumed to be growing at a constant rate forever and the last dividend paid is, D_0, then the model is:

$$P_0 = \frac{D_0(1+g)}{(1+k)} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \ldots \infty$$

Practice: Show the following result (**constant growth DDM; k>g**).

$$P_0 = \frac{D_1}{k - g}$$

Hint: Let r=(1+g)/(1+k).

To	Press	Display
Square 6.3^2	**6.3** [x²]	**39.69**
Find square root: $\sqrt{15.5}$	**15.5** [√x]	**3.94**
Find reciprocal: 1/3.2	**3.2** [1/x]	**0.31**
Find factorial: 5!	**5** [2nd] [x!]	**120.00**
Find natural logarithm: ln 203.45	**203.45** [LN]	**5.32**
Find natural antilogarithm: $e^{.69315}$	**.69315** [2nd] [e^x]	**2.00**