Assignment Booklet

PS 541: Game Theory

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Spring 2010

Assignments are listed my the date of introduction.
They are due the *following* class period.

SHOW ALL WORK!

Name: ____

Jan 19: Ordinal Preferences

An actor has the following decision problem at a fast-food restaurant:

Hamburger (u)
Cheeseburger (v)
Fish fillet sandwich (w)
Chicken breast sandwich (x)
Tuna melt (y)
Chicken salad sandwich (z)

This actor has the following pairwise preferences:

uPw vPy wPy xPv zPu zPx

What is this actor's preference ordering over all six alternatives (or what information would you need to make that determination)? Which alternative would the actor be expected to choose on the basis of simple decision analysis?

Name: ______________________________________

<u>Jan 21: Game Matrices</u>

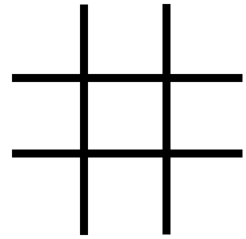
Solve the following game matrix.

	L	R
U	4 4	2 2
D	3 1	1 3

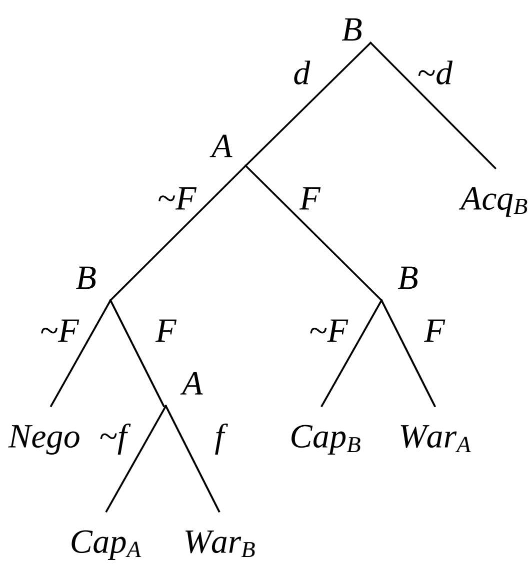
Name: ____

Jan 26: Game Trees

Draw the first two choices of the game Tic-Tac-Toe in game tree form. (Simplify the choices such that there are less than nine choices on the first move.)



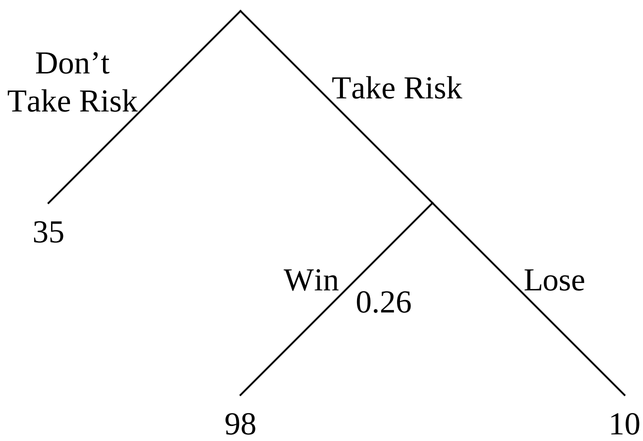
Find the subgame perfect equilibrium of the following game. Show your work by labeling the ordinal preference values (6 = best) for all outcomes and also identifying the expected behavior off the equilibrium path.



Preference Orderings:
A: $Nego > Acq_B > Cap_B > Cap_A > War_A > War_B$
B: $Cap_A > Nego > Acq_B > Cap_B > War_B > War_A$

Name: ____

Jan 28: Expected Utility



Calculate the expected utility of "taking the risk" in the above figure (where the probability of winning is 0.21). Show your work. Would a risk neutral rational actor be willing to take the risk?

Name: ______________________________________

Feb 2: Mixed Strategies

Consider the following zero-sum game matrix and find the Nash equilibrium.

	L	R
U	2	-1
D	-2	3

For the two-player game below, let p_1 and p_2 represent the row player's probabilities of playing A and B respectively, and let q_1 and q_2 represent the column player's probabilities of playing D and E respectively. Find the equilibrium values of p_1, p_2, q_1, and q_2.

	D	E	F
A	1 1	0 0	0 0
B	0 0	1 0	0 1
C	0 0	0 1	1 0

Name: ____

Feb 4: Nash Equilibria and N-Player Games (2 pages)

Find all the pure-strategy Nash equilibria of the following three matrices. Show your work by correctly identifying the best replies of all actors involved.

	F	F	G	H	I
A	4 5	5 6	5 6	6 3	6 3
B	4 5	5 6	5 6	2 1	2 1
C	4 5	3 4	1 2	3 4	1 2
D	4 5	3 4	1 2	3 4	1 2

Find all the pure-strategy Nash equilibria of the following three matrices. Show your work by correctly identifying the best replies of all actors involved.

	C_2		D_2	
	E_3	F_3	E_3	F_3
A_1	4 24 6	11 9 1	23 15 2	16 17 13
B_1	14 20 7	12 3 22	19 5 21	18 8 10

U_3 U_2 U_1

Name: ____

Feb 4: Nash Equilibria (continued)

Find all the pure-strategy Nash equilibria of the following three matrices. Show your work by correctly identifying the best replies of all actors involved.

		E_3		F_3	
		G_4	H_4	G_4	H_4
A_1	C_2	56 47 62 9	11 14 34 30	59 32 35 15	1 22 23 49
	D_2	42 52 58 39	10 60 45 41	46 7 64 19	26 28 6 29
B_1	C_2	57 53 21 20	27 36 63 43	4 24 54 55	8 33 16 2
	D_2	38 25 12 13	31 3 51 37	50 48 17 40	44 18 5 61

U_4 U_3 U_2 U_1

Name: ____

Feb 9: Collective Action Problems (2 pages)

Assuming b > c, identify the pure-strategy Nash equilibria to this collective action game.

k = 3		C_3		D_3	
		C_4	D_4	C_4	D_4
C_1	C_2	b - c, b - c, b - c, b - c	b - c, b - c, b - c, b	b - c, b - c, b, b - c	- c, - c, 0, 0
	D_2	b - c, b, b - c, b - c	- c, 0, - c, 0	- c, 0, 0, - c	- c, 0, 0, 0
D_1	C_2	b, b - c, b - c, b - c	0, - c, - c, 0	0, - c, 0, - c	0, - c, 0, 0
	D_2	0, 0, - c, - c	0, 0, - c, 0	0, 0, 0, - c	0, 0, 0, 0

U_1, U_2, U_3, U_4

Name: ____

Feb 9: Collective Action Problems (continued)

Assume b > c > b/2. Under which set(s) of assumptions is the public good produced? Is it produced for certain in those cases?

Joint Production & Jointness of Supply

Cost Sharing

	C	D
C	b - c/2 b - c/2	0 - c
D	- c 0	0 0

Fixed Contributions

	C	D
C	b - c b - c	0 - c
D	- c 0	0 0

Joint Production & Diminishing Supply

Cost Sharing

	C	D
C	b/2 - c/2 b/2 - c/2	0 - c
D	- c 0	0 0

Fixed Contributions

	C	D
C	b/2 - c b/2 - c	0 - c
D	- c 0	0 0

Individual Production & Jointness of Supply

Cost Sharing

	C	D
C	b - c/2 b - c/2	b b - c
D	b - c b	0 0

Fixed Contributions

	C	D
C	b - c b - c	b b - c
D	b - c b	0 0

Individual Production & Diminishing Supply

Cost Sharing

	C	D
C	b/2 - c/2 b/2 - c/2	b/2 b/2 - c
D	b/2 - c b/2	0 0

Fixed Contributions

	C	D
C	b/2 - c b/2 - c	b/2 b/2 - c
D	b/2 - c b/2	0 0

Name: ______________________________________

<u>Feb 11: The Iterated Prisoner's Dilemma (2 pages)</u>

	C	D
C	0 9	10 -1
D	-2 10	-1 0

The matrix above is a Prisoners' Dilemma game. Find the critical value of the discount parameter for the each player.

Name: ____

<u>Feb 11: The Iterated Prisoner's Dilemma (continued)</u>

	C	D
C	4, 4	1, 7
D	7, 1	2, 2

The matrix above is a Prisoners' Dilemma game. Consider the following repeated-game strategies:

> TF2T: Tit-for-two-tats; cooperates on the first two moves; defects on the next move if its opponent has defected on both of the previous two moves; cooperates otherwise.
> TFTd: Tester tit-for-tat; defects on the first move; mimics the opponent's previous move for all further play.

Calculate numeric expected utilities for the following matrix (i.e., using the payoffs from the matrix above and the discount parameter, δ), letting $\delta = 0.65$. Show your work on a separate page. Then find the pure-strategy Nash equilibria for the game.

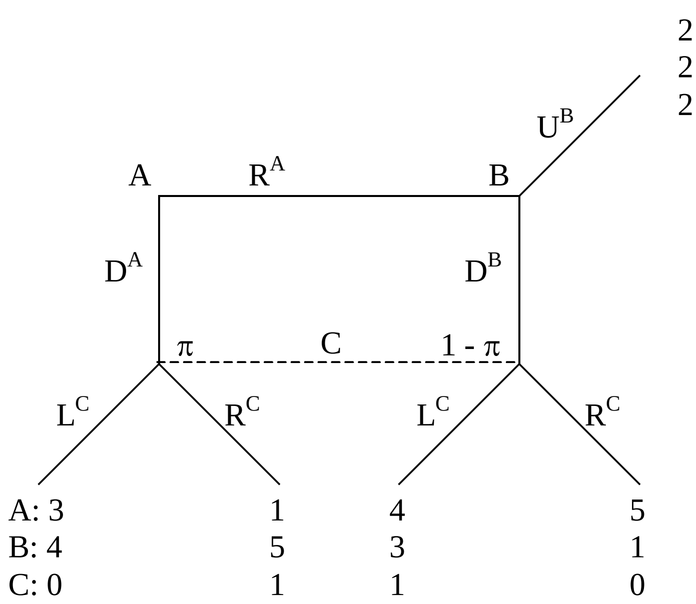
	All C	TF2T	TFTd	All D
All C				
TF2T				
TFTd				
All D				

Name: ____

<u>Feb 16: Game Trees and Matrices</u>

(1) Draw the following game matrix as a game tree. (2) If this game were strictly sequential, under what circumstances would the SPE outcome be different from the NE of the simultaneous game?

	L	R
U	1 1	2 3
D	3 4	4 2



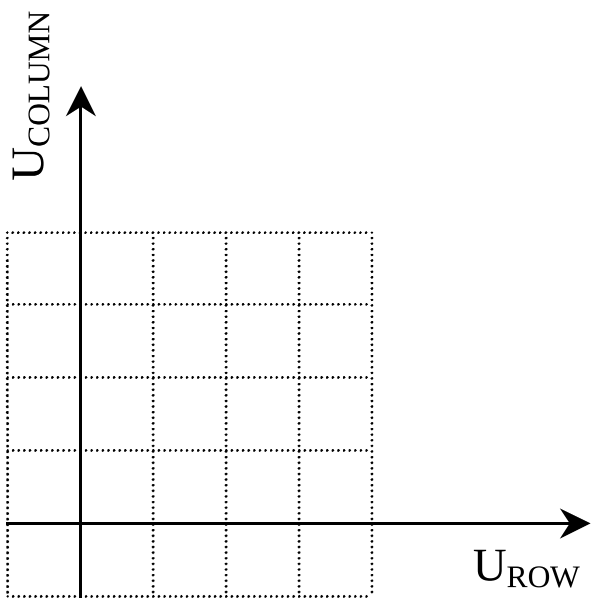
In the figure above, actor C gets to make a decision only if actor A or B chose “down”; however, C does not know for certain whether A or B gave her the opportunity to make a decision. Redraw this game tree as a three-player game matrix and find its Nash equilibria.

Name: ____

Feb 23: Nash Bargaining

Map the following non-zero-sum game matrix onto the grid provided. Identify (i) the bargaining space, (ii) the Pareto frontier, (iii) a reasonable disagreement point, and then (iv) find the Nash Bargaining Solution of the bargaining space among the definitive outcomes of the game. (The outcomes are labeled for convenience.)

	C_1	C_2	C_3
R_1	3 r 1	4 s -1	2 t 3
R_2	2 u 2	2 v -1	0 w 4
R_3	-1 x 3	1 y 0	0 z 0



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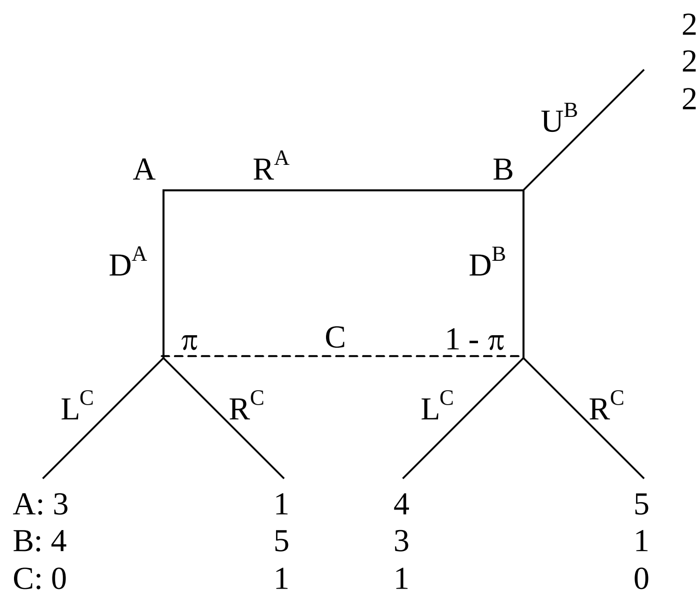
Feb 25: Sequential Games with Uncertainty



In the figure above, actor C gets to make a decision only if actor A or B chose "down"; however, C does not know for certain whether A or B gave her the opportunity to make a decision. With probability π, C thinks that A gave her the opportunity to make a decision between L and R; with probability $1 - \pi$, C thinks that B gave her this opportunity. Find the Nash and Bayesian equilibria of this game. What is C's equilibrium evaluation of π for the Perfect Bayesian equilibrium?

Name: ____

Mar 2: Sequential Bargaining

In this bargaining problem, A is proposing a division, $x \in [0, 1]$, of some unknown sized pie, $\pi > 0$. If B rejects the proposed division and A forces change, A gets 100 units with probability p and 0 units with probability $(1 – p)$ while B gets 0 units with probability p and 100 units with probability $(1 – p)$. If B rejects the proposed division and A accepts the status quo, A gets 25 units and B gets 75 units.

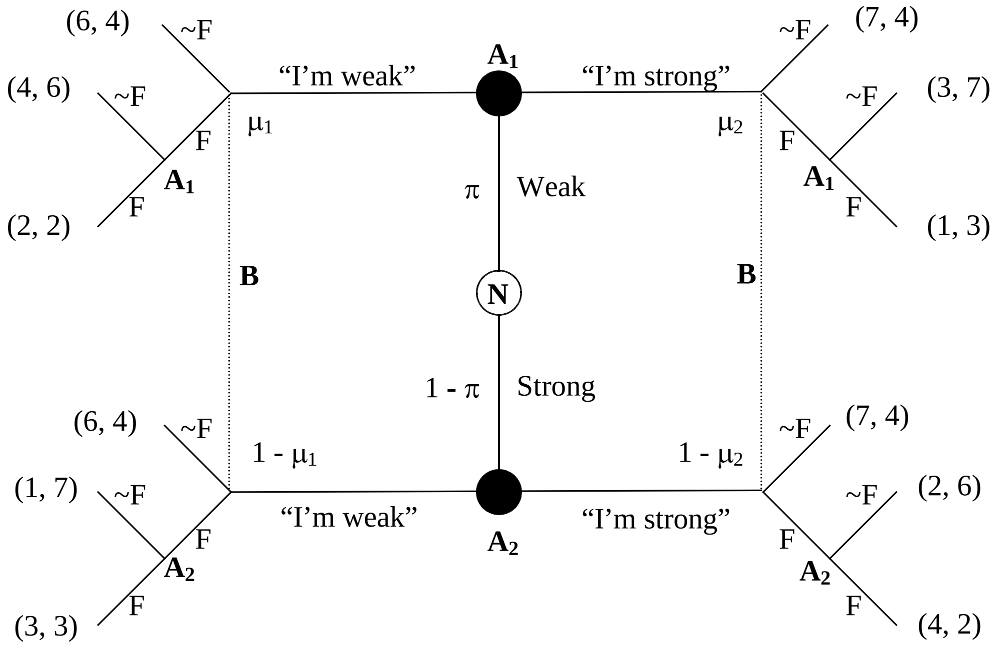
Draw a game tree representing this bargaining problem:

If $p = 0.4$, what is the smallest sized pie, π^*, in which A makes a proposal that (1) B accepts and (2) is Pareto improving for A? What is the proposed division, x^*, under π^*?

Still assuming that $p = 0.4$, how large would the pie need to be for the equilibrium division to be a 50-50 split?

Name: ____________________________________

Mar 4: Signaling Games



In the signaling game above, Nature first chooses what type of A that B is facing. Actor A is the Weak type A_1 with probability π or the Strong type A_2 with probability $1 - \pi$. Either type of actor A then signals its type by saying “I’m weak” or “I’m strong”. If B receives the signal “I’m weak”, he believes he is facing the weak type A with probability μ_1 or that the signal came from the strong type A with probability $1 - \mu_1$. If B receives the signal “I’m strong”, he believes he is actually facing the weak type A with probability μ_2 or that the signal came from the strong type A with probability $1 - \mu_2$. Uncertain of which type he is facing, B must choose between actions F and ~F. ~F always results in his middle preference (4 points). If B chooses F, A then chooses between F and ~F. The weak type A prefers the outcome associated with ~F while the strong type A prefers the outcome associated with F. The points in the parentheses are all in the order: (u_A, u_B).

Find μ_1^* and μ_2^*. Find one pooling equilibrium and one separating equilibrium.

Name: ____

Mar 9: Strategic Voting (2 pages)

Given the following three voters, their preferences, and the accompanying agenda, what is the predicted outcome if you assume that each voter votes sincerely? Now show that if **one** of the voters votes strategically while the others still vote sincerely, the voter voting strategically can improve the outcome for herself. Be clear about which voter is voting strategically.

	D
D_1:	a b z c
D_2:	b z c a
D_3:	c a z b

```
Assume sincere voting:             Assume strategic voting by one
                                   voter:

a vs. b                            a vs. b
   |                                  |

 ___ vs. c                          ___ vs. c
      |                                  |

     ___ vs. z                          ___ vs. z
          |                                  |

         ___                                ___
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Name: ____

Mar 9: Agenda Setting and Strategic Voting (continued)

For the following preference profile, voter 1 is the chair of a seven member committee. As chair, he gets to set an agenda to select one of the alternatives, {a, b, c, d, e}. Design an agenda that gets the best possible outcome for the chair. Identify what strategic voting is necessary to support this outcome. Is the agenda immune to additional strategic voting by the other members of the committee?

	D
D_1:	a b c d e
D_2:	b c e d a
D_3:	c b e d a
D_4:	c e b d a
D_5:	d a e c b
D_6:	d e a c b
D_7:	e c b d a

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___ vs. ___
     |

    ___ vs. ___
         |

        ___ vs. ___
             |

            ___ vs. ___
                 |

                ___
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Name: ____

Mar 11: Spatial Models and the Median Voter Theorem

Examine the following preference profile. Is there a Condorcet winner among the alternatives? Can all of the preferences be said to be “single-peaked” for some underlying political dimension? If yes, what would this political dimension look like? Show its utility plot. If no, show a utility plot using the alphabetical listing of the alternatives as the political dimension.

	D
D_1:	a d e c b
D_2:	b c e d a
D_3:	c b e d a
D_4:	c e b d a
D_5:	d a e c b
D_6:	d e a c b
D_7:	e c b d a