Group Interactions Christopher K. Butler The University of New Mexico

August 17, 2009

Abstract: Predicting social outcomes on the basis of numerous individual-level interactions is an exhaustive task that only a computer can complete. Even a computer takes exponentially longer to compute all the interactions as the number of actors increases. In addition, the measurement of relevant information (minimally, preferences, but often power and salience as well) becomes increasingly difficult as the number of actors gets larger. I propose a different way of thinking about these large number of interactions for bargaining in a single issue space by re-conceptualizing the interactions as between probability distributions at opposite ends of the issue area rather than between individuals. The measurement of individual information is presumed to be a sample of the opposing distributions rather than a complete picture of all the relevant actors. This sampling can be used to describe each opposing distribution. Each distribution is thought of as a heterogeneous group of individuals. The density of the distribution at a given position represents the proportion of individuals desiring that position as their ideal position. This presumes that all individuals at a given position would be similarly affected by bargaining. The joint distribution of the two opposing distributions both summarizes the total "society" and provides the basis of a probability density function of prediction (rather than a point prediction). Once the framework of this type of analysis is established, a particular model of distributional interaction is put forward to demonstrate how the framework can be used to generate dynamic predictions over time.

My thanks to the students in PoliSciFi who pushed me to think even farther outside the box than I usually do.

1 Introduction

The nature of prediction and how it is arrived at and presented is of great importance to the study of international conflict. Narrowly defined, conflict is a violent contest between actors (whether states, nonstate actors, or even between a government and a rebel group). More broadly defined, conflict is the conflict of interest between actors that could lead to a violent contest but could be resolved in some other way.

Violent contests are the earthquakes of international relations. They are rare events with high visibility and the potential for great change. But their prediction is difficult at best, and the best predictions are confined to "zones" of greatest likelihood rather than anything more specific.

Conflicts of interest are the root of politics. There are various types of prediction concerning conflicts of interest. We are interested in knowing what kinds of conflict of interest are most likely to lead to violent contests and the circumstances in which they do so. We are interested in knowing which conflicts among all that exist are likely to push their way to the forefront. We are interested in knowing how a particular conflict is likely to be resolved or managed.

It is this last type of prediction that I address in this paper. Even here, there are different types of prediction. Will the conflict become a crisis or can it be resolved without (or before) such escalation? Will the conflict end through violence, compromise, or one side yielding, or is the conflict likely to remain unresolved? If unresolved, can the conflict be managed in such a way as to avoid violence? If resolved through negotiation, which side is likely to get the better end of the bargain and what is that bargain likely to look like?

The remainder of this paper addresses this last question. As in much international relations theory, the short answer is that the more powerful side is likely to get a better deal roughly in proportion to how much more powerful that side is.

2 A Brief History of a Subset of Social Prediction Models

While there are many models that offer some kind of predictive capability, I review three here that all share a unidimensional issue (or bargaining) space as their basic foundation. Thus, actors' preferences are linked to their position in the issue space, preferring outcomes closer to their position over outcomes farther away. All three models also explicitly incorporate multiple actors. Beyond these similarities, the models differ. The first two are static while the third is dynamic. Two of the models predict winning positions while the other predicts a probability of success for each side of the conflict. Finally, the first has the most minimal inputs (actors and their positions) while the last two require two additional actor characteristics (salience and capability).

Black's (1948) Median Voter Theorem (MVT) is the simplest of the three models. Given actors and their positions and assuming a simple-majority voting rule, the position of the median actor is able to defeat every other proposal brought against it in separate head-to-head contests. This provides the basis of a point prediction, namely, the position of the median voter. Based as it was on committee voting and otherwise devoid of what international-relations scholars would consider "conflict", the MVT has seen more application in American and comparative politics than in world politics (though we'll see an example below). While Black assumed that each voter has a single vote, his model easily handles the notion of weighted voting. An individual voter with five votes is logically equivalent to five voters who share the same position. This allows for a re-conceptualization of "votes" as a form of power or capabilities. This is essentially what the other two models do (implicitly or explicitly).

The so-called "Prince System" of Coplin and O'Leary (see, e.g., 1976; 1985) also works along a unidimensional issue space and assumes that actors have positions within that issue space. In addition, each actor has some power to affect the outcome on that issue (relative to the other actors) and a priority or salience regarding this issue (relative to other, unspecified issues). An actor's power times the same actor's salience is a measure of how much effort that actor will exert in changing or maintaining the outcome on that issue.

This model provides a weather-like prediction: for example, "a 71 per cent chance of success". "Success" is defined as the "positive" end of the issue space, as determined by the researcher. To calculate this probability, the issue space must be divided in half, thereby identifying with which end of the issue space (positive or negative) each actor sympathizes. Those precisely at the midpoint are identified as neutrals. The probability is calculated by taking the sum of the product of the "positive" actors' positions, power, and salience plus one half position times power of the neutral actors all divided by the sum of the product of all the actors' positions, power, and salience. This gives a weighted voting ratio that incorporates the actors' intensity of preference through both salience and position.

Bueno de Mesquita's Expected Utility Model (EUM) can be described as Black's Median Voter Theorem

with coercive bargaining. The EUM (see, e.g., Bueno de Mesquita, Newman, and Rabushka 1985; Bueno de Mesquita 2002) has the same inputs as the previous model: actor's positions, power, and salience. The unique aspect of the EUM is that it is dynamic. Actors are assumed to interact in such a way as to influence their future positions (and/or their power or salience). As such, the EUM is a dynamic model. At the beginning of each period, the power-and-salience-weighted median position is the starting point for bargaining. If the strategic interactions of the actors would change this median, then the dynamics are allowed to continue.

3 Estimating Groups from Individuals

Many models of conflict start—and often stop—by assuming two actors. This usually because it is easier to analyze the interactions between two actors rather than the interactions among many actors. Additionally, we often collapse conflicts that involve many actors into a conflict between two groups of actors. I explicitly adopt this assumption here. Conflicts do indeed involve many actors. To understand how a conflict is likely to unfold, we need information about all actors relevant to the conflict, the stakeholders who have some influence on the final outcome.

To this point, my assumptions are not very different from the one's reviewed above. However, I contend that there are more actors behind the "relevant" actors typically identified for inclusion. The assumption underlying the model here is that individuals powerful enough to stand out for analysis represent larger groups of people. While these people may generally agree with their representative, they do not do so perfectly and are, in fact, heterogeneous. The included actors are presumed to be a sample that helps identify how much heterogeneity there is on each side. Collectively, this presumes that the two groups themselves can be characterized as probability distributions with one distribution representing the left side and the other representing the right side.

3.1 Reviewing the Inputs

For inputs to the model, I simply rely on actors' positions (T_i) , capabilities (C_i) , and salience (S_i) for the issue at hand. Because the model will be relying on a Beta probability distribution, the position scores need to be normalized to be between 0 and 1 (if they were not originally so). Capabilities are constrained to be positive numbers. Salience is conceived of as the percentage of effort an actor is willing to apply to this issue and, hence, is constrained to be between 0 and 1 (or normalized to be so).

3.2 Dividing the Issue Space

The assumption that there are two sides necessitates dividing the issue space in some arbitrary way. The most natural is to assume that those actors with $T_i \in [0, \frac{1}{2})$ are on the left side while those actors with $T_i \in (\frac{1}{2}, 1]$ are on the right side. This creates a problem of what to do with those actors at $T_i = \frac{1}{2}$. Dropping them altogether could drastically alter the overall distribution, essentially artificially eliminating the most moderate moderates. Creating a third distribution for them would add complications that this method is trying to get away from (interactions among many actors) while introducing a degenerate distribution in the process (a spike with no variance). I instead assume that these actors could be leaning left or leaning right with equal probability. Therefore, half of their weighted capabilities will be attributed to the left side while the other half will be attributed to the right side.

3.2.1 Power of Each Side

While each side is characterized as a probability distribution, the relative size of one distribution compared to the other is a function of the relative weighted capabilities of one side to the other. Thus, P_L represents the power of the left distribution and $P_R = 1 - P_L$ represents the power of the right distribution. P_L is estimated following equation 1 where $\alpha > 0$ and $\beta > 0$. This is the Prince probability of Coplin and O'Leary.

$$P_{L} = \frac{\sum_{i:T_{i} \in [0,\frac{1}{2})} C_{i}S_{i} + \frac{1}{2} \sum_{i:T_{i} = \frac{1}{2}} C_{i}S_{i}}{\sum_{i=1}^{N} C_{i}S_{i}}$$
(1)

3.2.2 The Group as a Beta Distribution

While any number of probability distributions might work to characterized the two sides, I rely on Beta distributions due to the flexibility of the beta distribution. The probability density function of a Beta distribution is given by equation 2.

$$f(x;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$
(2)

While a Beta distribution is characterized by parameters α and β , these parameters can be estimated from their sample mean (\overline{x}) and sample variance (v) such that $\alpha = \overline{x} \left(\frac{\overline{x}(1-\overline{x})}{v} - 1 \right)$ and $\beta = (1-\overline{x}) \left(\frac{\overline{x}(1-\overline{x})}{v} - 1 \right)$.

3.2.3 Average Weighted Position of Each Side

For the inputs of the model, the sample mean of each side needs to be weighted by the capabilities and salience of the actors. The average weighted position of the left side is given by equation 3. The average weighted position of the right is similarly constructed.

$$T_{L} = \frac{\sum_{i:T_{i} \in [0, \frac{1}{2})} T_{i}C_{i}S_{i} + \frac{1}{2} \sum_{i:T_{i} = \frac{1}{2}} T_{i}C_{i}S_{i}}{\sum_{i:T_{i} \in [0, \frac{1}{2})} C_{i}S_{i} + \frac{1}{2} \sum_{i:T_{i} = \frac{1}{2}} C_{i}S_{i}}$$
(3)

3.2.4 Cohesion of Each Side

The weighted sample variances are also calculated on the basis of $T_i C_i S_i$ for the left and right. While variance itself needs only be non-negative, to satisfy the conditions that $\alpha > 0$ and $\beta > 0$, it is the case that $v \in (0, \overline{x}(1-\overline{x}))$. For the dynamic aspects of the model, the distributions will be allowed to change over time. Rather than constantly checking that the group variance is in the appropriate range, I introduce a concept of group cohesion (H_g , where $g = \{L, R\}$) that is a unit measure of how much variance within the allowed range a group has. Thus, $H_g = 1$ corresponds to v = 0 and $H_g = 0$ corresponds to $v = \overline{x}(1-\overline{x})$. Given this mapping, initial estimated cohesion is given by equation 4 and subsequent variance (in the dynamic model) is given by equation 5.

$$H_g = \frac{T_g(1 - T_g) - v_g}{T_g(1 - T_g)}$$
(4)

$$v_g = T_g (1 - T_g)(1 - H_g) \tag{5}$$

3.3 Prediction as the Joint Probability Distribution between Groups

Now having the power and an estimated probability distribution for each side, it is possible to put the pieces together to generate a joint probability distribution that represents a prediction for the likely outcome in the policy space assuming no dynamic interaction. The joint probability distribution is given by equation 6 where $\alpha_g = T_g \left(\frac{T_g(1-T_g)}{T_g(1-T_g)(1-H_g)} - 1 \right)$ and $\beta_g = (1-T_g) \left(\frac{T_g(1-T_g)}{T_g(1-T_g)(1-H_g)} - 1 \right)$.

$$\varphi(x; P_L, T_L, T_R, H_L, H_R) = P_L f_L(x; \alpha_L, \beta_L) + (1 - P_L) f_R(x; \alpha_R, \beta_R)$$
(6)

3.3.1 Why not just a Beta distribution reflecting all actors?

A single Beta distribution is very flexible. It can be used to represent a tight distribution, a uniform distribution, or even a U-shaped distribution. In this flexibility, it can, therefore, represent one particular kind of bimodal distribution. Additionally, it often represents an asymmetric distribution which, in the model, reflects one side being more powerful than the other.

While a single Beta distribution can be bimodal, the modes are always at the extremes of the issue space. Intuitively, one can think of situations where one side is concentrated at a moderate position while the other is concentrated at an extreme position (though maybe not the most extreme one). In addition, a single Beta distribution that is bimodal has a common variance. This implies that the cohesion of each mode (i.e., group) is the same. Again, one can easily conjure situations where one side has high cohesion compared to the other. Having two distributions allows the researcher to capture these situations (and many others).

4 Static Examples

Here, I offer some examples of the static but distributional predictions that can be generated with this conceptualization of group politics. In these examples, I contrast the distribution made by a single Beta estimation with the joint distribution of equation 6. The first example is from Bueno de Mesquita's (1998) ex post analysis of the emergence of the Cold War using only data from 1948. The actors are the most powerful 36 states in the international system in 1948 according to CINC score (which was also used to measure power in the model). The actors' positions are a function of their τ_B scores with the "system leader"—namely the United States. Bueno de Mesquita assumed initial salience of 1 which was then randomly altered after the first round. I simply assume salience of 1 to reflect that first round. Figure 1 shows the static distributional prediction.

[Figure 1 about here]

With p_L (i.e., the Soviet side) only 0.253, $T_L = 0.010$, and $T_R = 0.729$, $\varphi(x)$ (i.e., the joint distribution) shows the much greater weighted power of the U.S. side compared to the single Beta distribution. The joint distribution also shows the greater cohesion of the Soviet side, which has fewer actors who are more tightly concentrated at the extreme left end. Part of the reason for showing this example first is that the joint distribution can be compared with Bueno de Mesquita's simulation results. Bueno de Mesquita randomly varied actors' salience across rounds in 100 separate simulations. He found that 22% of the simulations ended with a Soviet "win" while 78% of the simulations ended with an American "win" (where a "win" means a final median position on that actor's side and convergence of all actors to that position). Further, he gives a "distribution of policy outcomes" that are even more comparable with the joint distribution in Figure 1 (1998, 144). This distribution is broken down as follows for round 15 (approximating 1978) of the simulation. Pro-Soviet: 24%; Weakly Pro-Soviet: 5%; Weakly Pro-U.S.: 20%; Pro-U.S.: 51%. Estimating the definite integrals for each quartile of $\varphi(x)$ yields the following comparison. Pro-Soviet: 29%; Weakly Pro-Soviet: 10%; Weakly Pro-U.S.: 19%; Pro-U.S.: 42%. This compares reasonably well with Bueno de Mesquita's results.

There are (at least) two drawbacks of using $\varphi(x)$ as the prediction in this case. First, the EUM provides actor-level information regarding predictions. For example, China shifts position in some of the simulations, mirroring the changes in Beijing's foreign policy (despite real-world changes in regime). Second, the EUM includes "continuation" as an outcome (e.g., a case where there is no convergence among actors). Even so, the static joint distribution is picking up much of the distribution of policy outcomes without the dynamic programming.

As a second example, I took the top 40 countries by CINC score for the year 2000 and used the CINC score to measure power. Using the τ_B measure with the system leader—still the United States—and again assuming that all actors' salience is 100%, I was able to construct an initial data set comparable to the previous example.¹ Table 1 shows the individual-level data. Note that rather than normalizing the τ_B measure to cover the [0,1] range, I opted map the [-1,1] τ_B range to the [0,1] Beta distribution range. This choice is justified by the fact that the τ_B scores are not as polarized in 2000 as they were in 1948. Figure 2 shows the static distributional prediction.

[Figure 2 about here]

This joint distribution is much more pronounced in its differences from the single Beta distribution. It reflects the relatively high cohesion of the left as well as the dominance of the United States on the right. The single Beta distribution also reflects the strength of the U.S. and its Great-Power allies, but suggests

¹The raw data was generated by EUGene (Bennett and Stam 2000).

a greater likelihood for compromise outcomes and a relatively low likelihood for outcomes close to the U.S. position.

5 Dynamic Interactions between Groups

The underlying logic of the model is that the groups are in fact composed of individuals who interact with one another. As such, one can think of the interactions among these individuals as a large N-player game in which the N_L individuals of the left are interacting each other and interacting with the N_R individuals of the right (who are also interacting with each other). In this way, there are two types of interactions: in-group and out-group. (One such model is presented by Fearon and Laitin 1996.) The groups here, however, are explicitly heterogeneous in that individuals hold different positions (that is, location x_i) while being part of the same group. Within this heterogeneity, individuals have differing numbers of other individuals who share their position (represented by the probability density at that location $f_G(x_i)$).

The overall model being based on the presumption that single individuals are not predictable, I rely on mixed-strategy solutions among these many interactions to gain predictive power over the actions of many individuals. These mixed-strategy solutions are affected and weighted by the probability density of individuals who share characteristics. Even here, however, it is not that a set of individuals who share characteristics are assumed to behave identically, but that are all drawing upon the same mixed-strategy solution (i.e., $\sigma_{G,i}(x_i, f_G(x_i))$) for players of their type. Their behavior, therefore, can be varied while their aggregate behavior should approximate their mixed-strategy.

But what game should they be playing? If the actors are playing explicit games, it is difficult to specify what those games are. Each actor will be engaging in many, potentially different actions across as many as N-1 interactions.

5.1 Internal and External Tug of War

Rather than playing a game, how much leverage can be gained by assuming that the parameters of one group affects the other and vice versa? For example, the left "pulling" on the right will be more or less successful (i.e., moving T_R toward T_L) the greater its group capabilities (P_L) and the greater its cohesion (H_L). However, the right will itself become more cohesive (increasing H_R) as they "circle the wagons" in response to this pull. Because the right is also pulling on the left, this increase in their cohesion will also increase their ability to pull the left (or at least resist being pulled by the left). At the same time, the tug of war within a given group implies that T_G shifts according to how much relative weight there is to the left and right of that point while group cohesion decreases as a result of "in fighting".

For the in-group interactions, let them be modeled as a tug of war or gravity model such that the relative weight of a set of individuals at a given location $(f_G(x_i))$ and their distance from the group position (i.e., $|T_G - x_i|$) are both factors of greater pull. This makes two assumptions. First, the greater the density of individuals at a location, the more they pull. Second, the farther they are from the group position, the more they desire change and, hence, the more they pull. The net effect of all of these pulls across the locations of group members ($\Delta_G T_G$) can be represented as in equation 7. The cohesion of the group will decrease the larger the absolute net change in position.

$$\Delta_G T_G = \int_0^1 f_G(x) \left(T_G - x \right) \partial x \tag{7}$$

The pull from one group on another can also result in a change in a group's position. This pull is proportional to its capabilities relative to the other group, increasing in its cohesion, and greater for individuals within the group that are *closer* to the other group's position. The idea regarding this last assumption is that individuals within one group who are closer to to the other group's position care more about changing the position of the other group relative to individuals who are farther away from the other group's position. This is akin to assuming that one group's extremists more interested in changing their own group's position than they are in changing the position of another group. However, this can be accomplished in a variety of ways. It is possible that the "leaders" of a group (i.e., those at and/or near T_G) are the one's who care most about changing the position of the other group. The "moderates" of this group (perhaps oddly defined as those between their leaders and the leaders of the other group) may care more about moving their own group's position than that of the other group. The logic of the other assumption (that the pull of one group on another is greater for individuals within the group that are closer to the other group's position) is one of affinity; the closer $x_{i,L}$ is to T_R , the more they want the other group to further resemble their position within their own group. Staying with this assumption, the leadership would exert the next most effort to shift the other group's position. This makes sense in that the leadership should be more concerned with its own position than the position of the other group. However, there are no firm lines delineating "moderate", "leadership", and "extremist". I instead assume that individuals at each position exert slightly different pull in the tug of war regarding the other group's position. This portion of the between-group tug of war can be represented by equation 8, where $T_{\bar{G}}$ is the position of the group being pulled. This representation indirectly incorporates the "increasing in cohesion" assumption in that a group with greater cohesion will, by definition, have greater density around T_{G} . Under most conditions, a group with more cohesion will exert greater pull following equation 8 than a group with the same position but lower cohesion.

$$\int_{0}^{1} f_{G}(x)(1 - |x - T_{\bar{G}}|)\partial x$$
(8)

This pull is offset by the other group's resistance. The pull of one group on another, however, presents an opportunity and a dilemma to the other group's moderates. On the one hand, the other group's moderates want to pull their own group's position in the same direction as the other group is pulling. On the other hand, the other group's moderates don't want to be seen as relying on outside interference to achieve their political objectives. Modeling this tradeoff, I assume that almost everyone in the group offers resistance but that the resistance is increasing in distance from the pulling group's position. In particular, individuals who are at or to the extreme side of the position of the pulling group's position offer no resistance. (This explicitly assumes that they also do not help the other side in pulling.) Thus, if the left is pulling on the right, the resistance of the right is given by equation 9. (The resistance of the left when the right is pulling it is given by equation 10.)

$$\int_{T_L}^{1} f_R(x) |x - T_L| \partial x \tag{9}$$

$$\int_0^{T_R} f_L(x) |x - T_R| \partial x \tag{10}$$

The net effect of group L pulling on group R ($\Delta_{LR}T_R$) can be represented by equation 11. Similarly, the net effect of group R pulling on group L ($\Delta_{RL}T_L$) can be represented by equation 12.

$$\Delta_{LR}T_R = P_L\left(\int_0^1 f_L(x)(1-|x-T_R|)\partial x\right) - (1-P_L)\left(\int_{T_L}^1 f_R(x)|x-T_L|\partial x\right)$$
(11)

$$\Delta_{RL}T_L = (1 - P_L) \left(\int_0^1 f_R(x) (1 - |x - T_L|) \partial x \right) - P_L \left(\int_0^{T_R} f_L(x) |x - T_R| \partial x \right)$$
(12)

In the next round of interactions, the new positions of the left and right are given by 13 and 14.

$$\hat{T}_L = T_L + \Delta_L T_L + \Delta_{RL} T_L \tag{13}$$

$$\hat{T}_R = T_R + \Delta_R T_R - \Delta_{LR} T_R \tag{14}$$

6 References

- Bennett, D. Scott, and Allan Stam. 2000. "EUGene: A Conceptual Manual." International Interactions 26:179-204.
- Black, Duncan. 1948. "On the Rationale of Group Decision-Making." The Journal of Political Economy 56(1): 23-34.
- Bueno de Mesquita, Bruce. 1998. "The End of the Cold War: Predicting an Emergent Property." Journal of Conflict Resolution 42(2): 131-55.
- Bueno de Mesquita, Bruce. 2002. Predicting Politics. The Ohio State University Press.
- Bueno de Mesquita, Bruce, David Newman, and Alvin Rabushka. 1985. Forecasting Political Events: The Future of Hong Kong. New Haven: Yale University Press.
- Coplin, William D. and Michael K. O'Leary. 1976. Everyman's Prince: A Guide to Understanding Your Political Problems, Revised Edition. North Scituate, MA: Duxbury Press.
- Coplin, William D. and Michael K. O'Leary with Carole Gould. 1985. Power Persuasion: A Surefire System to Get Ahead in Business. Reading, MA: Addison-Wesley Publishing Company.
- Fearon, James D., and David D. Laitin. 1996. "Explaining Interethnic Cooperation." The American Political Science Review 90(4): 715-35.
- Putnam, Robert D. 1988. "Diplomacy and Domestic Politics: The Logic of Two-Level Games." International Organization 42(3): 427-60.
- Siegfried, Tom. 2006. A Beautiful Math: John Nash, Game Theory, and the Modern Quest for a Code of Nature. Washington, DC: Joseph Henry Press.



Figure 1. The U.S.-Soviet Issue Space, 1948

Actor information from Bueno de Mesquita (1998, 141).



Figure 2. The U.S.-Rest of World Issue Space, 2000

Actor positions measured by tau-b scores with the United States.

Table 1. World Data, 2000

Country	Position	Power
Saudi Arabia	0.392	1.17%
Egypt	0.392	1.12%
Iraq	0.392	0.76%
Nigeria	0.404	0.72%
Ukraine	0.418	1.87%
Russia	0.424	6.14%
China	0.443	15.31%
North Korea	0.460	1.32%
Ethiopia	0.467	0.58%
South Africa	0.467	0.82%
Pakistan	0.467	1.63%
India	0.467	7.97%
Thailand	0.477	0.84%
Iran	0.477	1.51%
Viet Nam	0.477	0.91%
Myanmar	0.477	0.59%
Bangladesh	0.477	0.79%
Taiwan	0.477	1.39%
Romania	0.477	0.56%

T_L:

H_L:

p_L: 45.99%

0.447

0.305

The "Right" Side				
Country	Position	Power		
Indonesia	0.524	1.93%		
Australia	0.554	0.87%		
South Korea	0.565	2.82%		
Japan	0.581	6.18%		
Philippines	0.581	0.67%		
France	0.742	2.43%		
Netherlands	0.762	0.63%		
Italy	0.762	2.10%		
United Kingdom	0.762	2.77%		
Belgium	0.762	0.55%		
Poland	0.762	0.99%		
Spain	0.762	1.24%		
Germany	0.762	3.26%		
Turkey	0.762	1.82%		
Venezuela	0.867	0.55%		
Mexico	0.867	1.76%		
Brazil	0.867	3.09%		
Colombia	0.867	0.67%		
Argentina	0.867	0.73%		
Canada	0.976	1.44%		
United States	1.000	17.48%		
	T R:	0.812		
		54.01%		

H_R:

0.027