Chapter 2. Units

2.1. Units of Measure

One of the foundations of physical science is measurement. What can be measured can be communicated, can (hopefully) be reproduced, and can be manipulated to derive further information.[1] **Measurable quantities** have two components, a *numerical value* and a *unit*. The unit is the basic amount and the numerical value tells how many basic units there are in the measured amount e.g., three dozen, 4.7 feet, –68 °F, 1.234×10^5 mi, etc. Note the use of an abbreviation in the units of the last example, and the use of *scientific notation* which displays all the *significant* (certain) digits (''figures'') by showing one digit before the decimal point and powers of ten to indicate the magnitude of the number.

In this chapter we will discuss problems for which the *amount* remains constant, but the *value* changes when the basic *unit* is changed. A key element is the proportionality factor between different units of the same type, called the **conversion factor** (sometimes called the unit factor). We will develop a units conversion heuristic from familiar examples, and eventually apply it to problems dealing with scientific measurements.

2.2. Metric Units

There are two common systems of measurement units, the *metric* system of units (inherited from the French) used throughout the world, except in the United States where the *British* system of units (inherited through the British Colonies) persists. The standard metric system is called the **SI** system (for ''Système International d'Unités''). Certain types of measurement are considered fundamental, or basic, with all others derived from the fundamental types. The basic metric units are the *kilogram* (kg) for mass, the *meter* (m) for length, the *second* (s) for time, the *kelvin* (K) for temperature, the *ampere* (A) for electric current, the

[1] This is not to say that unmeasurable properties don't exist. For example, just because creativity may not be measurable doesn't mean it doesn't exist. By definition it is that quality that generates new ideas. What it does mean is that until it can be quantified, creativity must lie outside the realm of physical science.

candela (cd) for luminosity, and the *mole* (mol) for amount of matter.[2] In order to use these quantities, we need to understand some things about them. Measurable quantities which are derived from the basic units include *area*, with units of length squared, *volume*, with units of length cubed,[3] *density*, defined as mass/volume ("mass per unit volume"), *velocity*, defined as length/time, *acceleration*, defined as velocity/time, *force*, defined as mass × acceleration, and *energy*, defined as force × distance. SI force and energy units have special names, the **Newton** (N), defined as one kg m s^{-2}, the **Joule** (J), defined as one kg m^2 s^{-2}, and the **Coulomb** (C), defined as 1 As.

Certain prefixes shift units into larger and smaller scales. Kilo means 1000 and nano means $1/10^9 = 10^{-9}$, so one kilogram is one thousand grams, and one nanosecond is one billionth of a second. The complete list of standard prefixes is given in Table 2.1.

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[2] For the record, here are the definitions of the basic SI units. The kilogram is the mass of a certain prototype platinum block. The meter is the distance of 1,650,763.73 wavelengths of radiation from the sp^{10} to $5d^5$ electronic transition of a krypton-86 atom. The second is the duration of 9,192,631,770 periods of radiation between the two hyperfine levels of the ground electronic state of a cesium-133 atom. The kelvin is 1/273.16 of the triple point of water. The ampere is the current of two straight parallel conductors separated by one meter required to produce a force of 2time10^{-7} newtons per meter. The candela is the perpendicular luminous intensity of 1/600,000 square meter of a totally absorbing body at the freezing temperature of platinum under a pressure of 101,325 newtons per square meter. The mole, or mol is the number of carbon-12 atoms in 0.012 kg, equivalent to 6.0221367×10^{23} (Avogadro's number, N_A).

[3] The measurement of area and volume depends on the shape of the object and the coordinate system used to measure it. Thus the area of a rectangle in Cartesian coordinates is length times width, whereas the area of a circle in polar coordinates is $24\pi r^2$, etc. The *units* of area, on the other hand do not depend on the shape, but rather on the units of the factors involved in the formula for the area of the shape under consideration.

Table 2.1 Unit Prefixes

Name	*Abbreviation*	*Definition*	*Name*	*Abbreviation*	*Definition*
deci	d	10^{-1}	deca	da	10^{1}
centi	c	10^{-2}	hecto	h	10^{2}
milli	m	10^{-3}	kilo	k	10^{3}
micro	μ	10^{-6}	mega	M	10^{6}
nano	n	10^{-9}	giga	G	10^{9}
pico	p	10^{-12}	tera	T	10^{12}
femto	f	10^{-15}	peta	P	10^{15}
atto	a	10^{-18}	exa	E	10^{18}

Conversions between SI and British units help one to gain a feeling for the magnitudes of the quantities involved. One pound = 0.45359237 kg, one yard = 0.9144 m, one quart = .94635925 liter. Temperatures are not directly proportional in the two systems, but are shifted as well, so that temperature conversions involve linear equations:

$$\boxed{\begin{aligned} {}^\circ F &= (9/5){}^\circ C + 32 \qquad (2.1) \\ {}^\circ C &= K - 273.15 \qquad (2.2) \end{aligned}}$$

2.3. Unit Conversions

Conversion between units changes the number of units of some quantity without changing the amount of the quantity. For example, the unaided human eye can resolve objects as small as 1/10 mm. From Table 2.1 we can see this is the same as 1/10,000 m, 100 microns (micrometers) 100,000 nanometers, etc. How namy yards would it be? To address problems like this we need to understand the conversion factor process. We'll proceed from familiar territory to the more general situation in steps.

Example 2.1 Suppose someone asks you, "How many dimes are in two dollars?" That should be fairly easy for anyone familiar with U.S. currency. Note that the word "in" in the question is used to mean "equivalent to."[4]

[4] In fact, there really aren't *any* dimes *in* dollars.

Example 2.2 Now try a slightly harder problem: How many quarters are in five dollars? You should get $5 \times 4 = 20$ quarters pretty quickly.

We have practiced money *conversion* problems so often they sometimes seems second-nature, or automatic. But are they really? Try to solve this one in five seconds or less:

Example 2.3 How many dollars are equivalent to 53 quarters?

Because of all the practice we have had dealing with money, it may be a little difficult to appreciate the fact that all the conversion problems we have discussed so far are solved by the *same process*.[5] If we can just discover this process, then it should be possible to solve every similar question with the same ease.

Consider Example 2.1 above, the number of dimes in two dollars. Obviously, twenty dimes have the same monetary value as two dollars. But how do we *arrive* at that answer? Before reading further, hide the next paragraph and see if you can describe each *detailed* step in your reasoning that led you to the value twenty dimes, and thereby deduce what the money conversion *process* is.

Perhaps you figured that since there are ten dimes in one dollar (by definition), there must be twice as many in two dollars, giving a total of twenty dimes. Perhaps you imagined writing \$2.00, and knowing that to convert to dimes, which are one-tenth of a dollar, you shift the decimal one place to the right. Perhaps you just guessed the answer and were lucky.[6] The claim is that you actually did a rather complicated calculation in your head, almost automatically. *Understanding precisely what this thought process is can be very useful to your future success in problem solving.*

One more example should convince you that money conversions are not necessarily automatic. Consider the following:

Example 2.4 How many *yen* are in \$10.00?

If you thought, "I need some more information before I can compute an answer," you are on the right track to success with conversion factors. It should be clear that an

[5] Supporting evidence comes from the fact that, unlike human calculators, digital calculators compute the answer to all such problems in about the same amount of time (after the input has been received).

[6] If this seems a little far-fetched, try presenting this problem to a three-year-old child and see what you get for an answer.

answer cannot be given until one knows *how many yen there are equal to one dollar* (or, equally useful, how many dollars are equal to one yen.) In the dollars and dimes example, you might have missed this essential component to the conversion process. Such an equivalence is termed a *conversion factor*, for reasons which will become apparent soon. Note that in the case of the dollars and yen, there may be several answers to the question, "how many yen to the dollar"? For example, are we considering Japanese Yen, or Chinese Yen? In a changing economy, do we want *today's* rate of exchange, or will we settle for yesterday's?

To proceed, suppose there are 641 Japanese Yen to one dollar. How can you use this information? One way would be to reason that since purchasing an item of given value would require more Yen than dollars (641 times as much, to be exact), and since the purchase value in dollars is \$10, it will take 10 times as many Yen as it would take to purchase an item worth \$1, yielding 6410 Yen. However, a more systematic way to solve the problem exists, which may be used not only for converting dollars to Yen, and Yen to dollars, but many other problems as well. Again, once the *method* is understood, it can be applied to any conversion problem that involves fixed ratios between the units. Further, the same approach may be extended to entire classes of problems frequently encountered in science.

The key to the general method is to understand how *ratios* relate to *proportions*. Consider the dollars-to-dimes example above. The mathematical way to express the equivalence between ten dimes and one dollar is

$$10 \text{ dimes} = 1 \text{ dollar}. \tag{2.3}$$

Since "equals divided by equals are still equal," one can divide both sides of the equation by the same quantity, say 1 dollar. In this way, one arrives at the following *ratio*:

$$\frac{10 \text{ dimes}}{1\ \$} = \frac{1\ \$}{1\ \$} = 1 \tag{2.4}$$

Note that the *units* are carried along with the *numbers* in this process. This sometimes seems strange to students who are used to working only with numbers in mathematics. In the conversion factor business it is quite legitimate to "mix apples and oranges," or different units. If this weren't so, Eq. (2.3) would imply that 10 equals 1, which obviously doesn't hold for just the numbers!

Now that we have a ratio, the next thing to recognize is that mathematically speaking, this particular kind of ratio is identically equal to one (or unity). Just look at the last equality in Eq. (2.4).[7] Given this fact, the last idea we need is to understand that *multiplying a measured quantity by a conversion ratio equal to unity doesn't change its value, just its units*. Thus in order to convert \$2.00 to dimes, we simply multiply the quantity in dollars by the ratio of dimes to dollars:

$$\$2.00 = \$2 \times \left(\frac{10 \text{ dimes}}{1 \text{ \$}}\right) = 2.00 \times 10 \text{ dimes} = 20 \text{ dimes} \tag{2.5}$$

Note that the *units* of dollars (\$) *cancel*. (This is true even when two different symbols, \$ and "dollars", stand for the same thing.) This is not only *necessary* to get the right units for the answer, this is the clue *how* to get the right answer.

2.4. Unit Conversion Factors

The procedure which is emerging is: multiply the given quantity by the proper ratio to get an answer with the proper units. *Ratios equal to unity with numbers and units are called* **conversion factors**. Conversion because they convert between units and factors because they do so by *multiplication*. Conversion factor ratios are commonly derived from definitions, expressions stating the equality between two sets of units.[8] Success in converting units is based on the appreciation that the *units are as important as the numbers*, and that *units conversion ratios (conversion factors) are used to cancel one set of units and introduce another set of units*.

Note that the *inverse* of Eq. (2.4) is just as true as Eq. (2.4) itself (since the inverse of unity is still unity).[9] The difference between the two is simply which units appear in the numerator, and which appear in the denominator. This suggests that not only can dollars be converted to dimes through the dollars-to-dimes conversion factor, but that dimes can be

[7] This reflects the equal purchasing power between ten dimes and one dollar. It doesn't make any difference to the store owner who asks, "how would you like your change?"

[8] What difference is there between stating that an object is 1 foot long and stating that it is 12 inches long?

[9] Inverse in the context of fractions means "to turn upside down"; thus the inverse of 3 (which can be thought of as $\frac{3}{1}$) is 1/3, or 3^{-1}.

converted to dollars as well. Which form of the conversion factor ratio to use depends on which units need to be canceled from the given information, and which need to be introduced to get the desired result.

The conversion process can be represented graphically in the form of a "map" which takes one from the origin (given units) to the destination (desired units). Fig. 2.1 depicts the process. The two-headed arrow suggests conversion in either direction uses the same conversion factor, and each dashed arc represents one of the conversion paths. (Lines will be used in place of arrows when it is not necessary to specify the direction of conversion.)

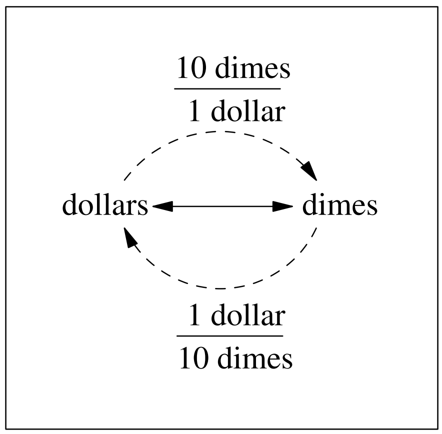

Fig. 2.1 The Dollars to Dimes Conversion Map

At this point it would be useful for you to go back and work Example 2.2 using a conversion factor between quarters and dollars. Be sure to set the problem up in just the same format as Eq. (2.5). Why? Because this is basically the way *all* conversion factor problems look before they are solved.

These examples suggest a common pattern that becomes a conversion factor strategy or *heuristic* for a general method to convert measurements to different units.

Units Conversion Heuristic

Purpose: To convert from one set of units to another.

Procedure:

1. Identify the given quantity and the requested quantity (number and units).
2. Identify a conversion path with a set of conversion equalities between the given units and the requested units, possibly converting to intermediate units if direct conversions are not known.
3. Multiply the given quantity by one or more conversion factors which cancel units that are not wanted in the result, and introduce units that are, until the *units* of the requested quantity are obtained.

Note that in order to apply the conversion factor process, three things must be known: the units of the given quantity (input), the units of the requested quantity (output), and the factor(s) which convert the given units to the desired units (we will shortly see examples which require more than one conversion factor). It may be useful to think of the process in terms of following a path, or crossing a bridge. On one side is the given units, on the other the desired units. The conversion factor is the path or bridge which connects the given side to the desired side. Fig. 2.2 illustrates graphically the general conversion factor process analogous to a road map. Reminiscent of Fig. 1.1, when more than one conversion is involved, there may be more than one route between the given input and the desired output. For example, one may convert directly between inches and yards, or indirectly through feet.

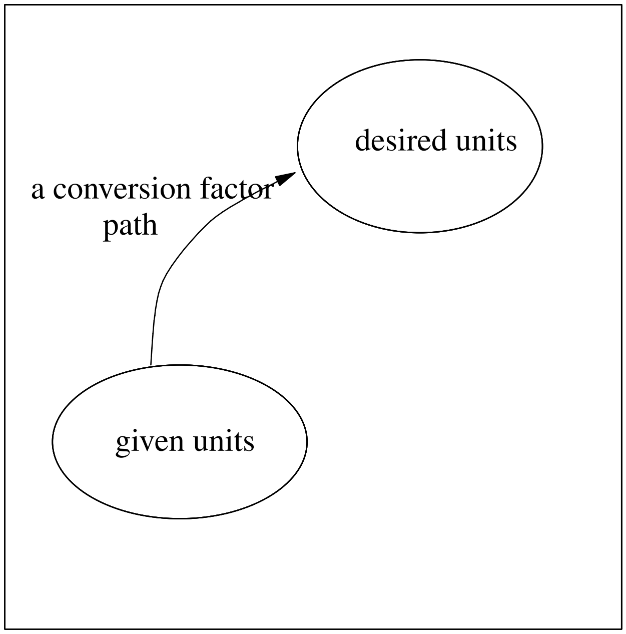

Fig. 2.2 A General Conversion Factor Map

Because English construction *inverts* the order of objects in questions, it may require some practice to distinguish the *given* quantity from the *requested* quantity. Thus we tend to say, "How many dollars are in (equivalent to) 53 quarters?", rather than "53 quarters is equivalent to how many dollars?". In this case, the place one *begins* the conversion factor process is where the statement of the question *ends*. You needn't let the English confuse you, however. A clue to the given quantity is that it usually has a particular, but arbitrary, numerical value (e.g. 53 quarters). On the other hand, conversion factors (which may also be given in the statement of the problem, particularly if they are unfamiliar), are *universal*; they remain the same numerically for any conversion for which they are needed (e.g. 4 quarters/$1).

Now let's apply the Units Conversion Heuristic to the question of how many dollars are equivalent to 53 quarters. We will number the steps of the process just as they are numbered in the Heuristic.

1. What is the given information? What are the *values* and what are the *units* of the given information? (53 quarters.) Write that down:

   53 quarters

   What are the units of the requested quantity? (Dollars.) Write that down, leaving room for the factor which can convert the given units into those requested.

$$53 \text{ quarters} \times (\qquad\quad) = \underline{\quad} \text{ dollars} \tag{2.6}$$

2. Now to "build a bridge" between the two units, we need a conversion factor. But wait! There are always *two* possible conversion factors between two quantities. We know that there are four quarters to the dollar, so

$$\frac{4 \text{ quarters}}{1 \text{ \$}} = 1. \tag{2.7}$$

   But it is equally true that

$$\frac{1 \text{ \$}}{4 \text{ quarters}} = 1. \tag{2.8}$$

   Both are equal to unity. The only difference is which quantity is reduced to unity when the defining equality is divided by an identity (see Eq. (2.4)). In the first case, 4 quarters = 1 \$, both sides are divided by 1 dollar; in the second, both sides are divided by 4 quarters. Which one should we use? The key is to focus on the *units* in Eq. (2.6), which brings us to step 3.

3. We want to eliminate the units of quarters, and introduce the units of dollars. The way to do this is to *divide by something involving quarters*. The secret, then is to *multiply* the given quantity by Eq. (2.8).

$$53 \text{ quarters} \times \left(\frac{1 \text{ \$}}{4 \text{ quarters}}\right) = (53/4) \text{ dollars} = 13.25 \text{ dollars} \tag{2.9}$$

   Note that multiplying by Eq. (2.7) would introduce the units of quarters squared. Worse, dollars would end up in the denominator. What would it mean to have a certain number of *inverse* dollars?

What should we have done, if we had been asked for the number of quarters in 53 dollars (really quite a different question)? Applying the same *process*, you should be led to the following set up:

$$53 \text{ dollars} \times \left(\frac{4 \text{ quarters}}{1\ \$}\right) = (53 \times 4) \text{ quarters} = 212 \text{ quarters} \tag{2.10}$$

Note how the conversion factor between quarters and dollars has been inverted to handle the input and output units properly.

The lesson which we hope you have learned by now is, *in using the conversion factor process, focus your attention on the units*, and leave the numbers to your calculator!

2.5. Sets of Units

Some quantities have more than one unit, such as *miles per hour*. (Note "per" means "divided by".) No problem. Just apply as many conversion factors as needed to convert *each* of the units in the given quantity to those of the requested quantity.

Example 2.5 Suppose you are driving down the road at 60 mi/hr and you wonder how many *ft/sec* that is equal to. A double conversion factor is required:

$$60 \frac{\text{mi}}{\text{hr}} \times \left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) \times \left(\frac{1 \text{ hr}}{3600 \text{ sec}}\right) = \underline{\quad\quad} \text{ ft/sec} \tag{2.11}$$

If you are curious, you will fill in the answer. If not, that's ok too, because we have been emphasizing *units*, not numbers. However, you should note how units in the denominator (hr) are converted by the *same process* as units in the numerator (mi).

2.6. The Itinerary

Suppose you want to take a trip. You have a starting point and a destination. But what about the route which leads from the starting point to the destination? It could involve a number of intermediate points, like an airplane trip with connecting flights, or layovers. The same thing can happen using the conversion factor process. Suppose that we don't have access to the conversion factor to get from a given set of units to the desired set in one step. That could have happened with the mi/hr to ft/sec of Example 2.4 if we didn't know how many feet are in a mile, but if we did know that there are 60 seconds in a minute and 60 minutes in an hour. The way to proceed is to multiply by all the conversion factors needed, arranging them so that unwanted units cancel, thus heading *in the direction of* the desired destination units.[10]

$$60\ \frac{\text{mi}}{\text{hr}} \times \left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right) \times \left(\frac{1\ \text{hr}}{60\ \text{min}}\right) \times \left(\frac{1\ \text{min}}{60\ \text{sec}}\right) = \underline{\quad\quad}\ \text{ft/sec} \tag{2.12}$$

Note that it doesn't matter which units are converted first, mi or hr, since both must be eventually converted.

The two solutions, Eqns. (2.11) and (2.12) are reminiscent of taking a direct flight to some point versus taking an indirect flight to the same point (see Fig 2.3). The important thing to understand is that in using conversion factors, *one arrives at the same destination regardless of the number of intermediate steps*; it could involve many steps (layovers) or just one step (nonstop flight).

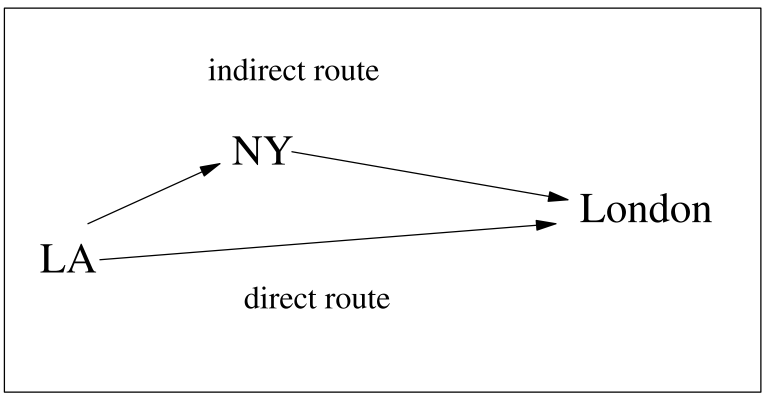

Fig. 2.3 Direct *vs* Indirect Routes

[10] Eq. (2.11) and Eq. (2.12) had better give the same result or you will want to check your calculator batteries.

You should now have all the equipment you need to travel comfortably around between the various types of units you might encounter. You can even visit "foreign lands" where the "landscape" (units) may be unfamiliar, with confidence, if you understand the conversion factor *process*. The conversion factor process is the "lingua franca" (common language) used by the inhabitants of all the different cultures (units) you may encounter.

To illustrate that the conversion factor process can treat general units, we will consider a favorite exercise.

Example 2.6 How many furlongs per fortnight is 60 mi/hr? We need to know some conversion factors involving furlongs and fortnights. A furlong is 10 chains (not too useful), 40 rods (not close to miles in conversion factor solution space), 220 yards, 660 feet (pretty close to miles), 7920 inches (a little farther away from where we want to be). A fortnight is 2 weeks or 14 days. Let's take a route that converts from miles to feet to furlongs, and from hours to days to fortnights:

$$60\,\frac{\text{mi}}{\text{hr}} \times (\frac{5280\text{ ft}}{1\text{ mi}}) \times (\frac{1\text{ furlong}}{660\text{ ft}}) \times (\frac{24\text{ hr}}{1\text{ day}}) \times (\frac{14\text{ days}}{1\text{ fortnight}}) = 161,280\text{ furlongs/fortnight} \tag{2.13}$$

The next example begins to expand the range of conversion applications. Food is usually sold according to some *unit* amount. Eggs are sold by the dozen, milk by the gallon, potatoes by the pound, etc.

Example 2.7 What is the cost of a glass of milk, assumed to be ½ pint, if milk is sold at $2.00/gal?

If we know how many pints there are in a gallon (there are eight), the answer can be obtained in two steps, first convert pints to gallons, then gallons to dollars (then to cents for convenience):

$$\tfrac{1}{2}\text{ pint} \times (\frac{1\text{ gallon}}{8\text{ pints}}) \times (\frac{2.00\text{ dollars}}{1\text{ gallon}}) \times (\frac{100\text{ cents}}{1\text{ dollar}}) = 12.5\text{ cents} \tag{2.14}$$

Note how the conversions are strung together. This type of set-up with *multiple* conversion factors saves the work of storing intermediate results in calculators and avoids transcribing errors. Suppose we know only that there are four quarts in a gallon and two pints in a quart. We can get the same result as before using more conversion factors:

$$\tfrac{1}{2}\text{ pint} \times (\frac{1\text{ quart}}{2\text{ pints}}) \times (\frac{1\text{ gallon}}{4\text{ quarts}}) \times (\frac{2.00\text{ dollars}}{1\text{ gallon}}) \times (\frac{100\text{ cents}}{1\text{ dollar}}) = 12.5\text{ cents} \tag{2.15}$$

When several conversion factors are needed to transform the given units into the desired units, a graphical representation of the process, the conversion "map" if you will, becomes a useful guide. Fig. 2.4 shows a conversion map for the conversion from pints to cents in the last example. Note how the conversion factors connect one unit to another (along the dashed route).

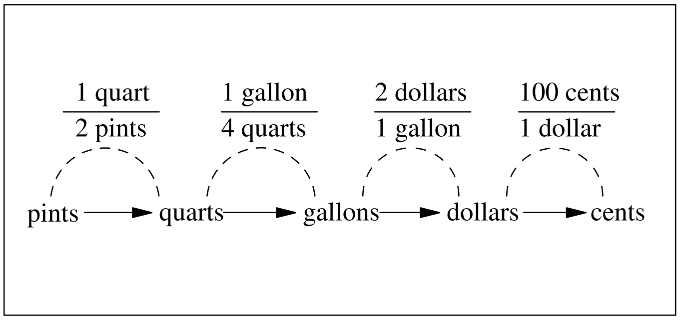

Fig. 2.4 Pints to Cents Conversion Map

With all this under our belt, we can return to the situation which generated the discussion of unit conversions, namely conversion between British and SI units.

Example 2.8 A dime has a diameter of about 1.8 centimeter, a thickness of about 1.4 millimeters, a mass of about 2.3 grams and contains about 1/30 of a mol of atoms. What are the measurements of a dime in British units?

$$\text{Diameter:}\ \ 1.8\ \text{cm} \times \left(\frac{1 \times 10^{-2}\text{m}}{1\ \text{cm}}\right) \times \left(\frac{1\ \text{yd}}{.9144\ \text{m}}\right) \times \left(\frac{3\ \text{ft}}{1\ \text{yd}}\right) \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = .71\ \text{in}$$

$$\text{Thickness:}\ \ 1\ \text{mm} \times \left(\frac{1 \times 10^{-3}\text{m}}{1\ \text{mm}}\right) \times \left(\frac{1\ \text{yd}}{.9144\ \text{m}}\right) \times \left(\frac{3\ \text{ft}}{1\ \text{yd}}\right) \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = .055\ \text{in}$$

$$\text{Mass:}\ \ 2.3\ \text{g} \times \left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) \times \left(\frac{1\ \text{lb}}{.454\ \text{kg}}\right) \times \left(\frac{16\ \text{oz}}{1\ \text{lb}}\right) = .081\ \text{oz}$$

The mol is a measure of the number of atoms and has the same value in British and SI units.

$$\text{Amount: } 1/30 \text{ mol} \times \left(\frac{6.0221367\text{x}10^{23} \text{ atoms}}{1 \text{ mol}}\right) = 2.0\text{x}10^{22} \text{ atoms}$$

2.7. Multidimensional Maps

Example 2.7 involved several conversions, not merely between units of the same type of quantity, but between different classes of quantities as well. Thus pints, quarts and gallons are all different measures of volume, and dollars and cents are different measures of monetary value. But the conversion between dollars and gallons changes the *type*, or class of measurement, from volume to money. The conversion map shown in Fig. 2.4 could be modified to reflect the qualitative differences in the units, say with monetary units on an upper level and volume units on a lower level, the two levels connected by the conversion between dollars (money) and gallons (volume). A two-dimensional conversion map would result in this case.

For another example of a conversion map involving different types of measurements, a set of mass conversions is presented in Fig. 2.5 as a two-dimensional map which displays conversions between smaller and larger mass units vertically and metric and British units horizontally. Solid lines indicate conversion in either direction is possible, but the conversion factor is given for only one of the directions, as the inverse form is easy to construct.

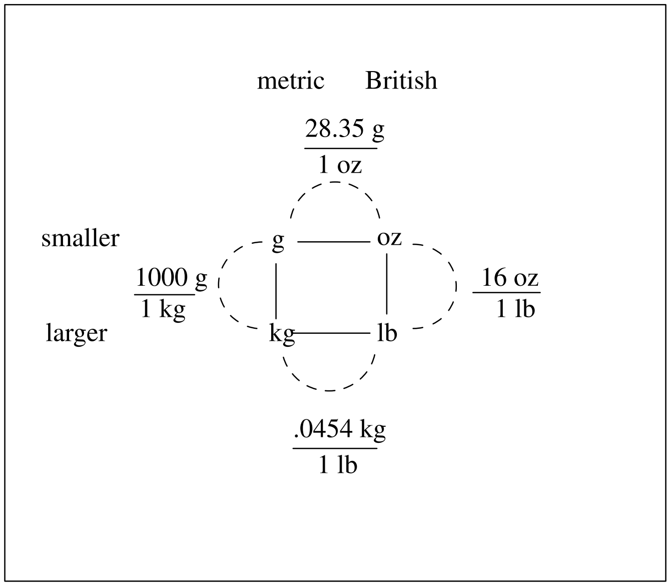

Fig. 2.5 A Mass Conversion Map

To apply the Units Conversion Heuristic to a general map such as this, locate the vertices of the given and requested quantities and construct a path which connects them together. Note that there may be more than one connecting path when the full set of possible conversion factors are available. For example, using Fig. 2.5, grams can be converted to pounds either through kilograms or through ounces. Both routes will yield the same final answer because there is consistency in the conversion factors along each path (and therefore redundancy in the amount of information displayed).

Extensions to more than two different types of units may be displayed using more levels (easier for two-dimensional displays) or more dimensions (limited to three) for humans.

2.8. Conversions and Formulas

Measurable quantities that are defined as ratios may be thought of as conversion factors. **Velocity**, defined as distance divided by time, is such a quantity.[11]

$$\boxed{v \equiv \frac{d}{t} \qquad (2.18)}$$

Note that a rearrangement shows velocity to be a proportionality factor between distance and time:

$$d = vt \tag{2.19}$$

This may be represented graphically in map form.

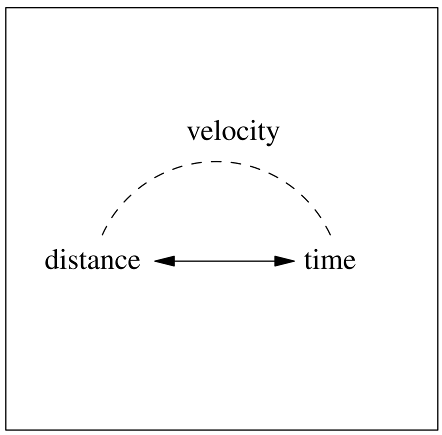

Fig. 2.6 A Velocity Conversion Map

[11] Distance and velocity involve direction as well as magnitude (*vectors*) sometimes symbolized with embolding (**d, v**) or arrows ($\vec{d}, \vec{v}$). Speed is the magnitude of velocity. We use the symbol v for speed to avoid ambiguity with the SI symbol for seconds, s.

To determine the time it takes to go a certain distance given a velocity, one can rearrange the definition according to the rules of algebra and solve for the time:

$$t = \frac{d}{v} \tag{2.20}$$

Alternatively, one can multiply the distance by the inverse velocity as a conversion factor.

Example 2.9 How long would it take to travel 250 miles at an average velocity of 65 miles per hour?

The Units Conversion Heuristic first says to identify the given quantity (250 miles), and the requested quantity (time in hours). The second step says to identify the conversion between the given units (miles) and the requested units (hours), which is the velocity, 65 mph. The last step is to multiply the given quantity by the conversion factor (inverse velocity):

$$250 \text{ mi} \times \left(\frac{1 \text{ hr}}{65 \text{ mi}}\right) = \frac{250}{65} \text{ hr} = 3.8 \text{ hr} \tag{2.21}$$

Note that the Units Conversion Heuristic gives the same result as solving the velocity equation for time, Eq. (2.20).

Another common quantity that is related to conversion factors is **percentage**, which is defined as the fraction of the part out of the whole times 100. The factor of 100 is a convenience for magnifying decimal fractions between zero and one to numbers between 0 and one hundred.

$$\boxed{\% \equiv 100 \left(\frac{P}{W}\right) \qquad (2.22)}$$

The most sensible way to deal with percentages is to convert them to fractions as parts out of 100 total parts, and apply the fractions as a conversion factors.

Example 2.10 If ocean water is 1.5% salt by mass, how many pounds of ocean water would be needed to provide one pound of salt?

A conversion map sketches the solution route using conversion factors:

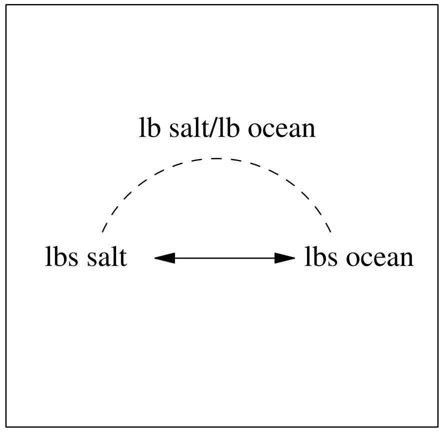

Fig. 2.7 An Ocean Water Conversion Map

Converting the percentage to a fraction, we arrive at 1.5 pounds of salt per 100 pounds of ocean water. Applying this information to the Units Conversion Heuristic gives the desired result.

$$1 \text{ lb salt} \times \left(\frac{100 \text{ lb ocean water}}{1.5 \text{ lb salt}}\right) = 67 \text{ lb ocean water} \quad (2.23)$$

Note again that the conversion factor is set up to cancel the (given) units of lb salt and introduce the (requested) units of lb ocean water. The corresponding rearranged equation would be $W = \frac{100 \text{ P}}{\%}$.

Density is another example of a measurable ratio for which problems can be solved equivalently using algebraic formulas or using conversion factors.

$$D \equiv \frac{M}{V} \qquad (2.24)$$

Again, density is the proportionality factor between volume and mass, M = DV.

Example 2.11 What is the mass in pounds of 1 liter of mercury?

The density of mercury is 13.5 g/cc (cubic centimeter), which can be considered a factor which converts between mass and volume.

$$1 \text{ liter mercury} \times \left(\frac{1000 \text{ cm}^3}{1 \text{ liter}}\right) \times \left(\frac{13.6 \text{ g mercury}}{1 \text{ cm}^3 \text{ mercury}}\right) \times \left(\frac{1 \text{ lb}}{453.59 \text{ g}}\right) = 30.0 \text{ lb mercury} \qquad (2.25)$$

Note that the conversion factor method takes care of all the calculational steps at once. The formula M = DV can only be applied to the conversion between cubic centimeters and grams, neither of which are the units given or sought for in this example. Thus unit conversions need to be made in any case.

The **specific heat capacity** (SH) is a factor which converts a change in temperature (Δt) for a given mass in grams (m) of a substance into a change in heat energy (ΔH). The **molar heat capacity** (C) is similar to the specific heat capacity, except the amount of substance is measured in mols instead of grams. The value of the specific heat capacity depends on the substance and its temperature, but remains fairly constant for small temperature changes.

$$\Delta H = mSH\Delta t \qquad (2.26a)$$
$$\Delta H = nC\Delta t \qquad (2.26b)$$

Example 2.12 How much heat is gained when 1 kg of water is heated from 20 °C to 100 °C ?

The specific heat of liquid water is 4.184 J/g-°C, so the increase in heat energy is

$$1 \text{ kg} \left(\frac{10^3 \text{ g}}{1 \text{ kg}}\right)\left(\frac{4.184 \text{ J}}{\text{g °C}}\right)(100 \text{ °C} - 20 \text{ °C})\left(\frac{1 \text{ kJ}}{10^3 \text{ J}}\right) = 335 \text{ kJ}$$

This gives a feeling for the magnitude of energy in kJ to heat a pan of water to boiling.

2.9. Conversion Tables

Finally, a word needs to be said about where conversion factors can be found (step 2 of the Units Conversion Heuristic). Some conversion factors are memorized through constant use (e.g. monetary and kitchen conversions), while others are less familiar. Some are derived from definitions (the original definition of the yard as the distance from King Henry the VIII's nose to the end of his hand was not exactly three times the length of his foot), others (like density) vary from system to system. The meter was originally "defined" as one ten-millionth the distance from the North Pole to the Equator (of Earth), a very impractical definition, not only in terms of the logistics, but conceptually awkward as well, since the geography of the earth is neither directionally uniform nor constant in time. The parctical definition originally was the distance between two inscribed lines on a platinum bar kept under controlled conditions in Paris. Today the meter is defined in terms of a certain number of wavelengths of light of a certain electronic transition in a certain element, more precise and conceptually no less arbitrary than any other definition.

Since units of measurement are ultimately arbitrary they may not be easy to remember. Therefore rather complete lists of conversions used in various disciplines are published in handbooks and other reference works. A short list of constants and conversion factors is given on the inside back cover.

Summary

Measurements report magnitudes and units. The units of a measurement may be transformed by multiplication factors called conversion factors. Conversion factors are ratios equal to unity derived from equalities, having different units in numerator and denominator. Since they have numbers and units in both the numerator and denominator, the units in the denominator are made to match the units of a given quantity to be converted, and the units of the numerator are the units of the new quantity produced.

The conversion factor approach is a great way to organize problems involving change of units. The only difficulties are recognizing *when* to use conversion factors, and finding a legitimate *path* between the input and the desired output. Clues to identifying problems involving conversion factors are that they have quantitative information, numbers with units, and that the units of the answer differ from the units of the input. The conversion factors themselves come from a variety of sources.

UNIT CONVERSIONS EXERCISES

1. Draw a conversion map for Examples 2.2 and 2.3.
2. Suppose a neighbor wants to ‘‘borrow’’ three eggs, and wants to pay you for them. How much should you charge, if the eggs cost you 80 cents per dozen?
3. Describe carefully the process used to convert a given number of years, days, hours and minutes, to seconds. Describe carefully the process used to convert a given number of seconds to minutes plus hours plus days plus years.
4. How many miles is one light-year?
5. Draw a conversion map for converting between cm, m, in and yd.
6. How could one extend the Mass Map of Fig. 2.5 to include tons?
7. What process should be followed in converting between United States currency and current British currency?
8. What process should be followed in converting between United States currency and old British currency?
9. Show a *density* conversion factor map.
10. What is the average speed of a runner in mi/hr who runs one mile in four minutes?

UNIT CONVERSIONS EXERCISE HINTS

1. Refer to Example 2.1 and Fig. 2.1.
2. What is the itinerary?
3. The order of operations isn’t important for the first conversion, but is for the second conversion. One day is 24 hours, one hour is 60 minutes, one minute is 60 seconds.
4. One light-year is defined to be the distance traveled by light in one sidereal year (at the rate of 2.99792458×10^8 m/s). (Sidereal means with respect to the stars.) One

sidereal year is 365 sidereal days, 6 hours, 9 minutes, 9.55 seconds (i.e. 365.2563695 sidereal days). One sidereal day is 24 hours, one hour is 60 minutes, one minute is 60 seconds.

5. There are 100 cm per m, 36 in per yd, .9144 m per yd and 2.54 cm per in.
6. There are 2000 lb in one ton, and 2000 kg in one (metric) tonne.
7. Current British currency includes the pound, the 10p (10 to the pound), the 25p (4 to the pound), the pence (100 to the pound). The rate of exchange may be taken to be $1 = 1.75 pound Sterling.
8. Before 1965 British currency included the pound, the shilling (20 to the pound), the fbrin (10 to the pound), the half crown (8 to the pound), the pence (12 to the shilling), and the half-penny (two to the pence).
9. Compare the definition formulas for velocity and density and have a look at the velocity conversion map.
10. Express the input information in the form of the ratio of the answer.