

# Solving Equations and Inequalities with Absolute Values

“a”, “b”, “c” and “d” represents a positive numbers, they can be any number.

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## Absolute Values and Equalities

$|x| = a$   $\longrightarrow$  You will have two solutions:  $-a$  and  $+a$ .

$|x| = -b$   $\longrightarrow$  You will get no solution:  $\emptyset$ , because absolute values are always positive.

$|x| = 0$   $\longrightarrow$  You get one solution:  $0$

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*If you have something more complicated, like:*

$|x + a - b| = c$   $\longrightarrow$  You need to break this into two equations, one with  $+c$  on the right and one with  $-c$  on the right.

$x + a - b = c$                       and                       $x + a - b = -c$     then solve for  $x$  in both

$x = c - a + b$                       and                       $x = -c - a + b$     this will give you two values for  $x$

You need to substitute those back into your original equation:  $|x + a - b| = c$  and make sure both of them work. If they both work, then you have two solutions, if only one of them works, then you have one solution, if neither of them work, then you have no solution. Note: Almost always, both solutions will work out.

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*Some other more complicated examples:*

$|x + a - b| = -c$   $\longrightarrow$  You have no solution.

$|x + a - b| = 0$   $\longrightarrow$  You will have one solution.

$|dx + a - b| = c$                        $\longrightarrow$                       You will have two solutions.

$|dx + a - b| = -c$                        $\longrightarrow$                       You will have no solutions.

$|dx + a - b| = 0$                        $\longrightarrow$                       You will have one solution.

$d|x| + a - b = c$   $\longrightarrow$  You need to solve for the absolute value first to see how many solutions you have:

$d|x| + a - b = c$                       and add  $b$  to both sides of the equation

$d|x| + a = c + b$                       then subtract  $a$  from both sides of the equation

$d|x| = c + b - a$                       then divide both sides by  $d$

$|x| = (c + b - a) \div d$

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## Solving Equations and Inequalities with Absolute Values

Now, if the right hand side is positive, you have two solutions, if it is negative, you have no solutions, if it is zero, you have one solution. Just like the simple versions above.

$|dx - a| + b = c \longrightarrow$  Get the absolute value by itself on the left hand side:

$$|dx - a| = c - b$$

If the right hand side is positive, then you have two solutions. If the right hand side is negative, you have no solutions. If the right hand side is zero, you have one solution.

$d|x + a| = c \longrightarrow$  You need to get the absolute value alone on the left side, so that means dividing by  $d$ .

$$|x + a| = c/d$$

So, just like everything else, if  $c/d$  is positive, then you have two solutions. If  $c/d$  is negative, you have no solutions. If  $c/d$  is zero, you have one solution.

*The moral of the story:* You should get your absolute value alone on one side with a number on the other side. If your number is positive, you have two solutions. If your number is negative, you have no solutions. If your number is zero, you have one solution. Then you solve the equation like in the second box above. You should always go back and check your answers.

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### Inequalities and Absolute Values

$|x| > c \longrightarrow$  You are going to break this into an “or” or a “union” statement.

$x > c$  or  $x < -c$  Notice that changing the sign also changes the direction of the inequality sign.

Your answer will be two intervals:  $(-\infty, -c) \cup (c, \infty)$

$|x| < c \longrightarrow$  You are going to break this into an “and” or an “intersection” statement.

$x < c$  and  $x > -c$  You are looking for the place where these two intervals overlap only.

Your answer will be one interval:  $(-c, c)$

$|x| < -c \longrightarrow$  An absolute value cannot be less than a negative number, so you will have no solution.

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*Special Cases:*

$|x| < 0 \longrightarrow$  This will have no solutions.

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## Solving Equations and Inequalities with Absolute Values

$|x| \leq 0$   $\longrightarrow$  This will have one solution  $[0,0]$ .

For more complicated problems, solve like you would for the equations (getting the absolute value on one side and a number on the other). *Remember to flip the inequality if you multiply or divide by a negative number.* Then use the examples above.