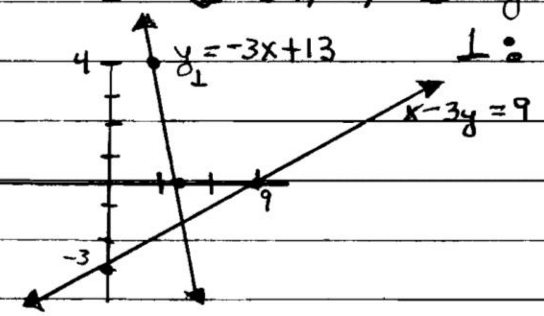
Exam 1 - Review

1. $x - 3y = 9$; $y = \frac{1}{3}x - 3$ XI $(9,0)$ YI $(0,-3)$

① $m_{\perp} = -3$ ② $(4,1)$ ③ $y - 1 = -3(x-4)$; $y_{\perp} = -3x + 13$

$\perp$: XI $(\frac{13}{3}, 0)$ YI: $(0, 13)$



2. $\lim\limits_{x \to 5} \frac{2x-10}{x^2-25} = \lim\limits_{x \to 5} \frac{2(x-5)}{(x+5)(x-5)} = \lim\limits_{x \to 5} \frac{2}{x+5} = \frac{2}{10} = \boxed{1/5}$

3. $\lim\limits_{x \to 6} \frac{x^2-6x}{x^2-5x-6} = \lim\limits_{x \to 6} \frac{x(x-6)}{(x-6)(x+1)} = \lim\limits_{x \to 6} \frac{x}{x+1} = \boxed{6/7}$

4. $y = 3x^3 - 5x^2 + x + 3$ @ $x = 1$

① $y' = 9x^2 - 10x + 1$; $y'(1) = 9 - 10 + 1 = 0$

② $y(1) = 3 - 5 + 1 + 3 = 2$ $(1, 2)$

③ $y - 2 = 0(x - 1)$; $\boxed{y = 2}$

5. $f(t) = 5t - \sqrt{t} = 5t - t^{1/2}$; $f'(t) = 5 - \frac{1}{2\sqrt{t}}$; $f'(4) = 5 - \frac{1}{2\sqrt{4}} = 4\frac{3}{4}$

$\boxed{4\,3/4 \text{ gal/hr.}}$

6. $s(t) = 160t - 16t^2$

(a) $s'(t) = 160 - 32t$; $s'(0) = 160 - 32(0) = \boxed{160 \text{ fps}}$

(b) $s'(2) = 160 - 32(2) = \boxed{96 \text{ fps}}$

(c) $0 = 160t - 16t^2 = 16t(10 - t)$; $t = 0$ sec $t = \boxed{10 \text{ sec}}$

(d) s'(10) = 160 – 32(10) = 160 – 320 = –160 smashes at –160fps (neg. since it's going down)

7. $s(t) = -16t^2 + 32t + 128$; $s'(t) = -32t + 32$

$0 = -16(t^2 - 2t - 8)$ $s'(4) = -32(4) + 32$

$0 = -16(t - 4)(t + 2)$ $= \boxed{-96 \text{ fps}}$

$t = 4$ sec $t = -2$ sec

8. a) $x^2 - y^2 = 1$; $2x - 2yy' = 0$; $-2yy' = -2x$; $\boxed{y' = \frac{x}{y}}$

(b) $x^3 + y^3 - 6 = 0$; $3x^2 + 3y^2y' - 0 = 0$; $3y^2y' = -3x^2$

$\boxed{y' = \frac{-x^2}{y^2}}$

9. a) $y = (4x-1)(3x+1)^4$ $\quad u = 4x-1 \quad v = (3x+1)^4$

$u' = 4 \quad v' = 4(3x+1)^3 \cdot 3 = 12(3x+1)^3$

$y' = 4(3x+1)^4 + 12(4x-1)(3x+1)^3$

b) $y = \frac{x^2 - 6x}{x-2}$ $\quad u = x^2 - 6x \quad v = x - 2$

$u' = 2x - 6 \quad v' = 1$

$$y' = \frac{(2x-6)(x-2) - (x^2 - 6x)}{(x-2)^2} = \frac{x^2 - 4x + 12}{(x-2)^2}$$

10. $\int_{20}^{38} (.1t + 2.4)\,dt = .05t^2 + 2.4t \Big|_{20}^{38}$

$= .05(38)^2 + 2.4(38) - [.05(20)^2 + 2.4(20)] = \boxed{95.4 \text{ trillion!}}$

11. $\int_2^5 (21 - \frac{4}{5}t)\,dt = 21t - \frac{2}{5}t^2 \Big|_2^5 = 21(5) - \frac{2}{5}(5)^2 - [21(2) - \frac{2}{5}(2)^2]$

$= \boxed{54.6 \text{ mowers}}$

12. a) $f(x) = x^2$; $[0,3] \Rightarrow \frac{1}{3-0}\int_0^3 x^2\,dx = \frac{1}{3}\left\{\frac{1}{3}x^3\Big|_0^3\right\}$

$= \frac{1}{9}(3)^3 - \frac{1}{9}(0)^3 = \boxed{3}$

b) $f(x) = 1 - x$; $[-1,1] \Rightarrow \frac{1}{1-(-1)}\int_{-1}^1 (1-x)\,dx = \frac{1}{2}\left\{x - \frac{1}{2}x^2\Big|_{-1}^1\right\}$

$= \frac{1}{2}\left\{1 - \frac{1}{2} - (-1 - \frac{1}{2})\right\} = \frac{1}{2}\{2\} = \boxed{1}$

13. $\int_1^2 (x^2 - 3x + 2)\,dx = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x \Big|_1^2$

$= \frac{1}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2) - [\frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + 2(1)] = \boxed{-\frac{1}{6}}$

Exam 1 – Review (continued)

14. $2\int \frac{x^5+3x^4-1}{x^2}\,dx \;=\; 2\int\left(\frac{x^5}{x^2}+\frac{3x^4}{x^2}-\frac{1}{x^2}\right)dx \;=\; 2\int(x^3+3x^2-x^{-2})\,dx$

$= 2\left(\frac{x^4}{4}+3\frac{x^3}{3}-\frac{x^{-1}}{-1}\right)+C\;\; 2\left(\frac{1}{4}x^4+x^3+x^{-1}\right)+C \;=\; \boldsymbol{\frac{1}{2}x^4+2x^3+\frac{2}{x}+C}$

15. a) $\int 2x(x^2+4)^5\,dx:\;\int u^5\,du = \frac{1}{6}u^6+C = \boxed{\frac{1}{6}(x^2+4)^6+C}$

let $u = x^2+4$

$du = 2x\,dx$

15. b) $\int 24(x^2-2x+1)(x^3-3x^2+3x-7)^3\,dx$

let u = x^3-3x^2+3x-7

then du = $3x^2 - 6x + 3$ ← now factor out a '3'

du = $3(x^2 - 2x + 1)$dx and divide both sides by '3'

$\frac{1}{3}du = (x^2 - 2x + 1)dx$

Now substituting: $24\int u^3 * \frac{1}{3}du \;=\; 24 * \frac{1}{3}\int u^3\,du$

$= 8 * \frac{u^4}{4}+C \;=\; 2u^4+C$

Substitute back x for u:

$\mathbf{2(x^3-3x^2+3x-7)^4+C}$ ← **Answer**

16. $40 = \frac{1}{3}x^3+x^2+3p^2+4p$

$$0 = x^2x' + 2xx' + 6pp' + 4p'$$
$$0 = (12)^2(.3) + 2(12)(.3) + 6(40)p' + 4p'$$
$$0 = 43.2 + 7.2 + 240p' + 4p'$$
$$0 = 50.4 + 244p'$$
$$-50.4 = 244p'$$
$$p' = \frac{-50.4}{244} = -\$0.21/hr.$$