Exam 2 Review

SQUARE ROOT

1. $x^{\ln x} = e \Rightarrow \ln[x^{\ln x}] = \ln e \Rightarrow \ln x(\ln x) = 1 \Rightarrow (\ln x)^2 = 1$

$\ln x = \pm 1 \Rightarrow e^{\ln x} = e^{\pm 1} \Rightarrow \boxed{x = e, 1/e}$

2. $3e^{2x} = 15 \Rightarrow e^{2x} = 5 \Rightarrow \ln e^{2x} = \ln 5 \Rightarrow 2x = \ln 5 \Rightarrow \boxed{x = \frac{\ln 5}{2}}$

3. $\ln x^2 - \ln(2x) + 1 = 0 \Rightarrow \ln \frac{x^2}{2x} = -1 \Rightarrow \frac{1}{2}x = \frac{1}{e} \Rightarrow \boxed{x = \frac{2}{e}}$

4. $(6 + 4x)e^x - e^x(5x + 7) = 0 \rightarrow (6 + 4x)e^x = e^x(5x + 7)$ divide both sides by e^x.
$6 + 4x = 5x + 7 \rightarrow -x = 1 \rightarrow x = -1$

5. $\ln\sqrt{x} - 2\ln 3 = 0 \Rightarrow \frac{1}{2}\ln x = \ln 9 \Rightarrow \ln x = \ln 81 \Rightarrow \boxed{x = 81}$

6. $2\ln(5) = \ln 25$.
$3\ln(3) = \ln 27$ ← LARGER

7. $\frac{1}{2}\ln(16) = \ln\sqrt{16} = \ln 4$ ← LARGER
$\frac{1}{3}\ln(27) = \ln\sqrt[3]{27} = \ln 3$

8. $y = ln\left(\frac{x}{7-4x^3}\right) = \ln(x) - \ln(7-4x^3) \rightarrow y' = \frac{1}{x} - \frac{-12x^2}{7-4x^3} = \frac{1}{x} + \frac{12x^2}{7-4x^3}$

9. $f(x) = \ln e^x + \frac{1}{2}\ln(x+1) + 2\ln(x^2+2x+3) - \ln(4x^2)$

$f'(x) = 1 + \frac{1}{2(x+1)} + \frac{2(2x+2)}{x^2+2x+3} - \frac{2}{x}$

10. $f(x) = 10^x$; $\ln y = \ln 10^x = x\ln 10$; $y' = \boxed{10^x \ln 10}$

11. $f(x) = (x^2+5)^6(x^3+7)^8(x^4+9)^{10}$

$y' = (x^2+5)^6(x^3+7)^8(x^4+9)^{10}\left[\frac{12x}{x^2+5} + \frac{24x^2}{x^3+7} + \frac{40x^3}{x^4+9}\right]$

12. $y = xe^{x^2}$ $\quad y' = e^{x^2} + 2x^2e^{x^2} = \boxed{e^{x^2}(1+2x^2)}$

$u = x \quad v = e^{x^2}$

$u' = 1 \quad v' = 2xe^{x^2}$

13. $y = \dfrac{\sqrt{x}+1}{e^{2x}}$ $\qquad y' = \dfrac{\dfrac{e^{2x}}{2\sqrt{x}} - 2e^{2x}(\sqrt{x}+1)}{(e^{2x})^2}$

$u = x^{1/2}+1 \qquad v = e^{2x}$

$u' = \frac{1}{2}x^{-1/2} \qquad v' = 2e^{2x}$

$y' = \dfrac{e^{2x}}{2\sqrt{x}\,e^{4x}} - \dfrac{2e^{2x}(\sqrt{x}+1)}{e^{4x}} = \boxed{\dfrac{1}{2\sqrt{x}\,e^{2x}} - \dfrac{2(\sqrt{x}+1)}{e^{2x}}}$

14. $\int e^{-x/2}\,dx = \int e^{-\frac{1}{2}x}\,dx = \boxed{-2e^{-x/2}+C}$

15. $\int\left(\frac{5}{x}-\frac{x}{5}\right)dx = 5\int\frac{1}{x}\,dx - \frac{1}{5}\int x\,dx = \boxed{5\ln|x| - \frac{1}{10}x^2 + C}$

16. $\left(\int \dfrac{x^3}{x} - \dfrac{2x^2}{x} + \dfrac{3x}{x} - \dfrac{7}{x}\right)dx = \int x^2\,dx - 2\int x\,dx + \int 3\,dx - 7\int \dfrac{1}{x}\,dx$

$= \dfrac{x^3}{3} - 2\dfrac{x^2}{3} + 3x - 7ln|x| + C = \dfrac{1}{3}x^3 - x^2 + 3x - 7ln|x| + C$

17. $\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u$

let $u = x^2$

$\frac{1}{2}du = x\,dx$

Apply the limits, the average value adjustment, and finish the problem.

$\dfrac{1}{3-0}\left(\dfrac{1}{2}\,e^{x^2}\right)\Big|_0^3 = \dfrac{1}{3}\left[\dfrac{1}{2}\left(e^{3^2} - e^{0^2}\right)\right]$

$= \dfrac{1}{6}(e^9 - 1)$

18. $\int \dfrac{(1+e^{-x})^3}{e^x}\,dx = -\int u^3\,du = -\frac{1}{4}u^4 + C$

let $u = 1+e^{-x}$ $\qquad = \boxed{-\frac{1}{4}(1+e^{-x})^4 + C}$

$-du = e^{-x}dx = \frac{1}{e^x}\,dx$

19. $\displaystyle\int_0^{\ln 3} \frac{e^x+e^{-x}}{e^{2x}}\,dx = \int_0^{\ln 3}(e^{-x}+e^{-3x})\,dx = -e^{-x} - \frac{1}{3}e^{-3x}\Big|_0^{\ln 3}$

$= -\dfrac{1}{e^x} - \dfrac{1}{3e^{3x}}\Big|_0^{\ln 3} = -\dfrac{1}{e^{\ln 3}} - \dfrac{1}{3e^{\ln(3^3)}} - \left[-\dfrac{1}{e^0} - \dfrac{1}{3e^0}\right]$

$= -\dfrac{1}{3} - \dfrac{1}{3(27)} - \left[-1 - \dfrac{1}{3}\right] = \dfrac{-27-1}{81} + \dfrac{81}{81} + \dfrac{27}{81}$

$= \boxed{\dfrac{80}{81}}$

20. $\int_2^6 \left(\frac{3}{32}x^2 - x + 200\right)dx = \frac{1}{32}x^3 - \frac{1}{2}x^2 + 200x \Big|_2^6$

$= 6\frac{3}{4} - 18 + 1200 - \left(\frac{1}{4} - 2 + 400\right) = \boxed{\$790.50}$

21. (a) $41,787 = Pe^{.065(22)}$; $P = \frac{41787}{e^{.065(22)}} = \boxed{\$10,000}$

(b) $A = 10000e^{.065t}$ $\quad A' = 650e^{.065t} = 650e^{.065(9.05)}$

$18000 = 10000e^{.065t}$ $\quad = \boxed{\$1170.00/\text{yr.}}$

$1.8 = e^{.065t}$

$\frac{\ln 1.8}{.065} = t = 9.05\text{yrs}$

Trevor Method: 18,000 * .065 = $1170.00

22. $r = \frac{\ln 3}{30} = .03662$ $\quad A = 1000e^{.03662t}$

$1,000,000 = 1000e^{.03662t}$; $1000 = e^{.03662t}$

$\frac{\ln 1000}{.03662} = \boxed{187 \text{ minutes} \approx 3 \text{ hrs.}}$

23. $A_p = 70e^{.94t} = A_B = 700000e^{.14t}$

$\frac{e^{.94t}}{e^{.14t}} = 10000$; $e^{.8t} = 10000$; $t = \frac{\ln 10000}{.8}$

$\boxed{t = 11\frac{1}{2} \text{ hrs.}}$