Final Exam Review

1. $f(x) = (2x^2+x)^3$ Power rule: $\boxed{f'(x) = 3(2x^2+x)^2(4x+1)}$

2. $h(x) = \left(x+\frac{1}{x}\right)^2 = (x+x^{-1})^2$; $h'(x) = 2(x+x^{-1})(1-x^{-2})$

Power rule

$$\boxed{h'(x) = 2\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)}$$

3. $f(x) = \frac{x}{e^x}$ Quotient rule: $u=x$ $v=e^x$ $f'(x) = \frac{e^x - xe^x}{(e^x)^2}$

$u'=1$ $v'=e^x$

$$f'(x) = \frac{e^x(1-x)}{e^{2x}} = \boxed{\frac{1-x}{e^x}}$$

4. $f(x) = \frac{e^x}{e^{3x}+1}$ Quotient rule: $u=e^x$ $v=e^{3x}+1$ $f'(x) = \frac{e^x(e^{3x}+1) - 3e^x(e^{3x})}{(e^{3x}+1)^2}$

$u'=e^x$ $v'=3e^{3x}$

$$f'(x) = \frac{e^{4x}+e^x-3e^{4x}}{(e^{3x}+1)^2} = \boxed{\frac{e^x - 2e^{4x}}{(e^{3x}+1)^2}}$$

5. $y = \ln(e^{4x}+3)$ $\boxed{y' = \frac{4e^{4x}}{e^{4x}+3}}$

6. $f(x) = \ln\left[\dfrac{(2x^3+1)(x^2+2)^3}{\sqrt[3]{x^5+3x^2-6}}\right]$

$$f(x) = \ln(2x^3+1) + 3\ln(x^2+2) - \frac{1}{3}(x^5+3x^2-6)$$

$$f'(x) = \frac{6x^2}{2x^3+1} + \frac{3(2x)}{x^2+2} - \frac{5x^4+6x}{3(x^5+3x^2-6)}$$

$$f'(x) = \frac{6x^2}{2x^3+1} + \frac{6x}{x^2+2} - \frac{5x^4+6x}{3(x^5+3x^2-6)}$$

7. $f(x) = \left[\dfrac{(2x^3+1)(x^2+2)^3}{\sqrt[3]{x^5+3x^2-6}}\right]$

$$\ln[f(x)] = \ln\left[\frac{(2x^3+1)(x^2+2)^3}{\sqrt[3]{x^5+3x^2-6}}\right]$$

$$\frac{f'(x)}{f(x)} = \frac{6x^2}{2x^3+1} + \frac{6x}{x^2+2} - \frac{5x^4+6x}{3(x^5+3x^2-6)}$$

multiply both sides by $f(x)$:

$$f'(x) = \left[\frac{6x^2}{2x^3+1} + \frac{6x}{x^2+2} - \frac{5x^4+6x}{3(x^5+3x^2-6)}\right]\overbrace{\frac{(2x^3+1)(x^2+2)^3}{\sqrt[3]{x^5+3x^2-6}}}^{f(x)}$$

8. $f(x) = (2x^3+1)(x^2+2)^3$ $\quad u = 2x^3+1 \quad v = (x^2+2)^3$

$u' = 6x^2 \quad v' = 3(x^2+2)^2(2x) = 6x(x^2+2)^2$

$f'(x) = 6x^2(x^2+2)^3 + (2x^3+1)6x(x^2+2)^2 \Leftarrow$ Product Rule

Rearranging:

$$f'(x) = 6x^2(x^2+2)^3 + 6x(2x^3+1)(x^2+2)^2$$

9. $\int\left(e^{-\pi x}+7-\frac{1}{x}-\frac{3}{7x^4}\right)dx$

$= \int e^{-\pi x}\,dx + \int 7\,dx - \int \frac{1}{x}\,dx - \frac{3}{7}\int x^{-4}\,dx$

$= -\frac{1}{\pi}e^{-\pi x} + 7x - \ln|x| - \frac{3}{7}\frac{x^{-3}}{-3} + C$

$= -\frac{1}{\pi}e^{-\pi x} + 7x - \ln|x| + \frac{1}{7x^3} + C$

10

save this for the very end

$\frac{1}{1-(-1)}\int_{-1}^{1}\frac{x^3-2}{(x^4-8x+3)^3}\,dx$ let $u = x^4-8x+3$

$du = (4x^3-8)\,dx$

SAME AS

$du = 4(x^3-2)\,dx$

$\frac{1}{4}\int\frac{1}{u^3}\,du$ $\frac{1}{4}du = (x^3-2)\,dx$

$\frac{1}{4}\int u^{-3}\,du = \frac{1}{4}\cdot\frac{u^{-2}}{-2} = -\frac{1}{8}u^{-2} = -\frac{1}{8}\cdot\frac{1}{u^2}$

substituting back in for u:

$-\frac{1}{8}\cdot\frac{1}{(x^4-8x+3)^2}\Big|_{-1}^{1}$

$1:\ -\frac{1}{8}\left(\frac{1}{((1)^4-8(1)+3)^2}\right) = -\frac{1}{8}\left(\frac{1}{(1-8+3)^2}\right) = -\frac{1}{8}\left(\frac{1}{(-4)^2}\right) = -\frac{1}{8}\cdot\frac{1}{16}$

$-1:\ -\frac{1}{8}\left(\frac{1}{((-1)^4-8(-1)+3)^2}\right) = -\frac{1}{8}\left(\frac{1}{(1+8+3)^2}\right) = -\frac{1}{8}\left(\frac{1}{(12)^2}\right) = -\frac{1}{8}\cdot\frac{1}{144}$

subtract lower limit from upper limit: (−1/8 is a common factor)

$-\frac{1}{8}\left(\frac{1}{16}-\frac{1}{144}\right) = -\frac{1}{8}\left(\frac{9}{144}-\frac{1}{144}\right) = -\frac{1}{\cancel{8}}\left(\frac{\cancel{8}}{144}\right) = -\frac{1}{144}$

Now, average value of the function:

$\frac{1}{2}\left(-\frac{1}{144}\right) = \boxed{-\frac{1}{288}}$

11. $\int_4^9 5x\sqrt{x}\,dx = 5\int_4^9 x^{3/2}\,dx$ Recall: $x^1\cdot x^{1/2} = x^{1+1/2}$

$= 5\cdot\frac{2}{5}x^{5/2}\Big|_4^9 = 2x^{2\,1/2}\Big|_4^9 = 2x^2\sqrt{x}\Big|_4^9$

$= 2(81)(3) - 2(16)(2) = 486 - 64 = \boxed{422}$

12. $y = e^{2x-3}$ @ $x = \frac{3}{2}$ ① Slope: $y' = 2e^{2x-3}$; $y'(\frac{3}{2}) = 2e^{3-3} = \boxed{2}$ SLOPE

② $y(3/2) = e^{2(3/2)-3} = e^0 = 1$ Point: $\boxed{(3/2, 1)}$

③ $y - 1 = 2(x - \frac{3}{2}) \Rightarrow \boxed{y = 2x - 2}$

13. $y = x^3 - \frac{3}{2}x^2 - 6x$; $y' = 3x^2 - 3x - 6$; $y'' = 6x - 3$

$y(2) = 2^3 - \frac{3}{2}(2)^2 - 6(2) = -10$ $\quad y' = 3(x^2 - x - 2)$ $\quad y'' = 3(2x - 1)$

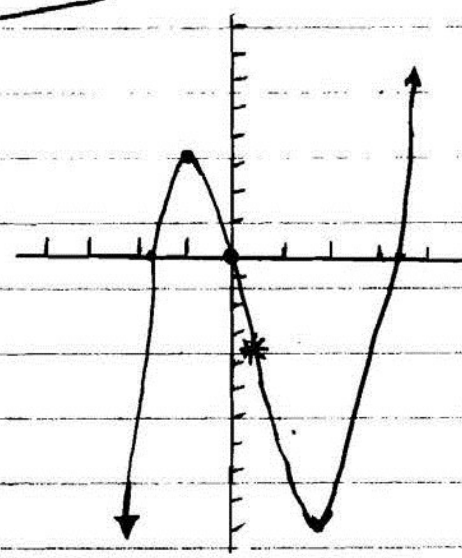
$y(-1) = (-1)^3 - \frac{3}{2}(-1)^2 - 6(-1) = 3\frac{1}{2}$ $\quad y' = 3(x-2)(x+1)$ $\quad (\frac{1}{2}, -3\frac{1}{4})$ I.P.

$y(1) = (\frac{1}{2})^3 - \frac{3}{2}(\frac{1}{2})^2 - 6(\frac{1}{2}) = -3\frac{1}{4}$ $\quad (2, -10)$ $\quad (-1, 3\frac{1}{2})$

$(2, -10) \Rightarrow y'' = 6(2) - 3 = +$ ∪ MIN

$(-1, 3\frac{1}{2}) \Rightarrow y'' = 6(-1) - 3 = -$ ∩ MAX

YI $(0, 0)$ As $x \to \infty$ $y \to \infty$, $x \to -\infty$ $y \to -\infty$



↑ $(-\infty, -1) \cup (2, \infty)$

↓ $(-1, 2)$

∪ $(\frac{1}{2}, \infty)$

∩ $(-\infty, \frac{1}{2})$

14. Increasing: $(-8, -2) \cup (0, 6)$

Decreasing: $(-\infty, -8) \cup (-2, 0) \cup (6, \infty)$

Rel max: $x = -2$, $x = 6$

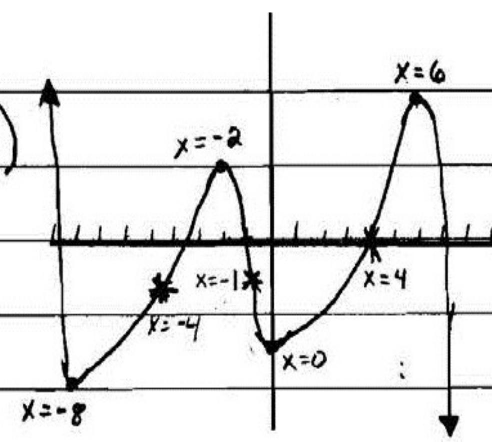
Rel min: $x = -8$, $x = \emptyset$

Smile: $(-\infty, -4) \cup (-1, 4)$

frown: $(-4, -1) \cup (4, \infty)$

Degree: $-x^5$ (2 more than the 3 bumps on $f'(x)$ is 5)

I.P.s: $x = -4$, $x = -1$, $x = 4$



15. $p = 20 - 2x \qquad R(x) = -2x^2 + 20x$

$$\begin{aligned} -C(x) &= \quad -4x - 12 \\ \hline P(x) &= -2x^2 + 16x - 12 \end{aligned}$$

$P'(x) = -4x + 16 = -4(x-4)$; $\boxed{x = 4 \text{ Buckets}}$

$\boxed{P(4) = -2(4)^2 + 16(4) - 12 = -32 + 64 - 12} = \boxed{\$20 \text{ Profit}}$

$p = 20 - 2(4) = 20 - 8 = \boxed{\$12/\text{Bucket}}$

16. $\int_3^6 \left(26 - \frac{2}{3}t\right) dt = \left(26t - \frac{1}{3}t^2\right)_3^6 = 26(6) - \frac{1}{3}(6)^2 - \left[26(3) - \frac{1}{3}(3)^2\right]$

$= 156 - 12 - [78 - 3] = \boxed{69 \text{ Buckets}}$

17. (a) Vertex: FIRST DERIVATE EQUALS ZERO!

$s(t) = -96t^2 + 192t + 768$

$s'(t) = -192t + 192$ à $0 = -192t + 192$ —> $1 = t$ —> **Takes 1 second**

(b) $s(1) = -96(1)^2 + 192(1) + 768 = -96 + 192 + 768 =$ **864 feet**

(c) On the ground means height, $s(t) = 0$

$0 = -96t^2 + 192t + 768$ —> $0 = -96(t^2 - 2t - 8) = (t - 4)(t + 2)$ —> **4 sec to splat**

(d) $s'(4) = -192(4) + 192 = -768 + 192 = -$**576 fps** (negative since he's going down)

18. $B^2 = \frac{2}{5} V^2 \sqrt{V} = \frac{2}{5} V^{5/2}$ Recall: $V^2 \overset{+}{\cdot} V^{1/2} = V^{2\,1/2} = V^{5/2}$

$2BB' = \frac{2}{5} \cdot \frac{5}{2} V^{3/2} V' = V^{1\,1/2} V' = V\sqrt{V}\, V'$

$V = 4 \quad V' = 3 \quad B = 0.3$ ⇐ Given Find B'

$2(.3)B' = 4\sqrt{4}(3)$; $B' = \frac{24}{.6} = 40 \text{ ml/min}$

$40 \text{ ml/min} \cdot 3 \text{ min} = 120 \text{ ml}$ so $\boxed{\text{Jill has 3 minutes to live}}$

19. $A = Pe^{rt}$ $\qquad r = \frac{ln\left(\frac{1}{2}\right)}{half-life} = \frac{ln\left(\frac{1}{2}\right)}{5776} = -0.00012$

If you start with 1g, then you end up with 0.72g (72%). If you start with 100g, then you end up with 72g (72%). It doesn't matter how much you start with (it's your choice for "P"), so long as you end up with 72% of that amount (A is 72% of whatever number you chose for "P").

$$0.72 = 1\, e^{-0.00012t} \rightarrow ln(0.72) = -0.00012t \rightarrow \frac{ln(0.72)}{-0.00012} = t$$

$\boxed{t = 2737 \text{ years}}$

Lousy murdering festering scum should fry for eternity in the flames of perdidtion!!

20. a) $3000 = 1230e^{.045t}$; $\frac{3000}{1230} = e^{.045t}$; $\ln \frac{3000}{1230} = .045t$

$.8916 = .045t$

$\boxed{t = 19.8 \text{ yrs.}}$

b) $7600 = 1230e^{.045t}$ $\qquad A = 1230e^{.045t}$

$\frac{7600}{1230} = e^{.045t}$ $\qquad A'(t) = 55.35\, e^{.045t}$

$\ln \frac{7600}{1230} = .045t$ $\qquad A'(40.5) = 55.35e^{.045(40.5)}$

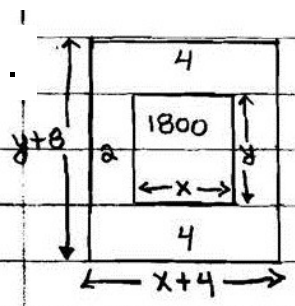
$1.82 = .045t$ $\qquad A'(40.5) = \boxed{\$342.47/\text{yr.}}$

$40.5 \text{ yrs.} = t$

Note: If you don't round any of the numbers in this solution, you will get \$342 even. The extra 47 cents comes from rounding the number from the ln calculation.

20. b) Alternate Method — The "Trevor" Method:
Final Amount is \$7600. Multiply that by the interest, r, 4.5%, converted to a decimal of course: —> 7600 * .045 = \$342.00

Note: If the numbers in the first 21b) (above) weren't rounded, the answer would be the same as the "Trevor" Method answer, which is, actually, more accurate than using e's and ln's!

21. 

$xy = 1800$ $\qquad A_{min} = (y+8)(x+4) = \left(\frac{1800}{x} + 8\right)(x+4)$

$y = \frac{1800}{x}$

First Outer Inner Last

$A_{min} = 1800 + \frac{7200}{x} + 8x + 32$

$A_{min} = 1832 + 7200x^{-1} + 8x$

$A'_{min} = -7200x^{-2} + 8 ➡ 0 = \frac{-7200}{x^2} + 8;\ \frac{7200}{x^2} = 8$

$7200 = 8x^2;\ 900 = x^2;\ x = \boxed{30 \text{ ft.}}$

$y = \frac{1800}{x} = \frac{1800}{30} = \boxed{60 \text{ ft.}}$