

Homework #2

Integration By Substitution

$$1. \int 4x(2x^2+1)^7 dx \Rightarrow \int u^7 du = \frac{1}{8}u^8 + C = \boxed{\frac{1}{8}(2x^2+1)^8 + C}$$

let $u = 2x^2 + 1$
 $du = 4x dx$

$$2. \int (3x^2 - 2x + 1)(x^3 - x^2 + x)^4 dx \Rightarrow \int u^4 du = \frac{1}{5}u^5 + C$$

let $u = x^3 - x^2 + x$
 $du = (3x^2 - 2x + 1) dx$

$$= \boxed{\frac{1}{5}(x^3 - x^2 + x)^5 + C}$$

$$3. \int \frac{3x^2 + 2}{(x^3 + 2x)^2} dx \Rightarrow \int \frac{1}{u^2} du = \int u^{-2} du = -u^{-1} + C$$

let $u = x^3 + 2x$
 $du = (3x^2 + 2) dx$

$$= \boxed{-\frac{1}{x^3 + 2x} + C}$$

$$4. \int 3x^2(x^3 + 2)^{3/2} dx \Rightarrow \int u^{3/2} du = \frac{2}{5}u^{5/2} + C$$

let $u = x^3 + 2$
 $du = 3x^2 dx$

$$= \boxed{\frac{2}{5}(x^3 + 2)^{5/2} + C}$$

$$5. \int \frac{x^2}{\sqrt{x^3 - 1}} dx \Rightarrow \frac{1}{3} \int \frac{1}{\sqrt{u}} du = \frac{1}{3} \int u^{-1/2} du = \frac{2}{3}u^{1/2} + C$$

let $u = x^3 - 1$
 $du = 3x^2 dx$
 $\frac{1}{3} du = x^2 dx$

$$= \boxed{\frac{2}{3}\sqrt{x^3 - 1} + C}$$

$$6. \int (x^3 - 2x)^2 (3x^2 - 2) dx = \int u^2 du = \frac{1}{3} u^3 + C = \boxed{\frac{1}{3} (x^3 - 2x)^3 + C}$$

let $u = x^3 - 2x$
 $du = (3x^2 - 2) dx$

$$7. \int \frac{4x}{(2x^2 + 3)^3} dx = \int \frac{1}{u^3} du = \int u^{-3} du = -\frac{1}{2} u^{-2} + C$$

let $u = 2x^2 + 3$
 $du = 4x dx$

$$= \boxed{-\frac{1}{2(2x^2 + 3)^2} + C}$$

$$8. \int 3t^2 \sqrt{t^3 + 2} dt = \int u^{1/2} du = \frac{2}{3} u^{3/2} + C = \boxed{\frac{2}{3} (t^3 + 2)^{3/2} + C}$$

let $u = t^3 + 2$
 $du = 3t^2 dt$

$$9. \int x(x^2 - 1)^9 dx = \frac{1}{2} \int u^9 du = \frac{1}{20} u^{10} + C = \boxed{\frac{1}{20} (x^2 - 1)^{10} + C}$$

let $u = x^2 - 1$
 $du = 2x dx; \frac{1}{2} du = x dx$

$$10. \int_0^1 20x(2x^2 + 1)^4 dx \Rightarrow 5 \int u^4 du = u^5 \Rightarrow (2x^2 + 1)^5 \Big|_0^1 = 3^5 - 1^5$$

let $u = 2x^2 + 1$
 $du = 4x dx; 5du = 20x dx$

$$= 243 - 1 = \boxed{242}$$

$$11. \int_{-1}^0 12(x^2 - 2x)(x^3 - 3x^2 + 1)^3 dx \Rightarrow 4 \int u^3 du = u^4 \Rightarrow (x^3 - 3x^2 + 1)^4 \Big|_{-1}^0$$

let $u = x^3 - 3x^2 + 1$
 $du = (3x^2 - 6x) dx = 3(x^2 - 2x) dx$
 $4du = 12(x^2 - 2x) dx$

$$= 1^4 - (-1 - 3 + 1)^4 = 1 - (-3)^4 = 1 - 81 = \boxed{-80}$$