Homework #3

1. $y = (x^3+x^2+1)^5$; $\boxed{y' = 5(x^3+x^2+1)^4(3x^2+2x)}$

2. $y = \sqrt{x^2+1} = (x^2+1)^{1/2}$; $y' = \frac{1}{2}(x^2+1)^{-1/2}(2x) = \boxed{x(x^2+1)^{-1/2} = \frac{x}{\sqrt{x^2+1}}}$

3. $f(t) = \frac{2}{t-3t^3} = 2(t-3t^3)^{-1}$; $f'(t) = \boxed{-2(t-3t^3)^{-2}(1-9t^2) = \frac{-2(1-9t^2)}{(t-3t^3)^2}}$

4. $\frac{d}{dP}\left(\sqrt{1-3P}\right) = \frac{d}{dP}(1-3P)^{1/2} = \frac{1}{2}(1-3P)^{-1/2}(-3)\,dP = \boxed{-\frac{3}{2}(1-3P)^{-1/2}dP = -\frac{3}{2\sqrt{1-3P}}\,dP}$

5. $\frac{d}{dx}(5x+1)^4 = 4(5x+1)^3(5)\,dx = \boxed{20(5x+1)^3\,dx}$

6. $y = (x+1)(x^3+5x+2)$; $\boxed{y' = 1(x^3+5x+2) + (x+1)(3x^2+5)}$

$u = x+1 \quad v = x^3+5x+2$

$u' = 1 \quad v' = 3x^2+5$

7. $y = x^2(7x-1)^2$ $\quad y' = 2x(7x-1)^2 + x^2[14(7x-1)] = \boxed{2x(7x-1)^2 + 14x^2(7x-1)}$

$u = x^2 \quad v = (7x-1)^2$

$u' = 2x \quad v' = 2(7x-1)(7)$

$= 14(7x-1)$

8. $y = (2x+1)^{5/2}(4x-1)^{3/2}$

$u = (2x+1)^{5/2} \qquad v = (4x-1)^{3/2}$

$u' = \frac{5}{2}(2x+1)^{3/2}(2) \qquad v' = \frac{3}{2}(4x-1)^{1/2}(4)$

$= 5(2x+1)^{3/2} \qquad = 6(4x-1)^{1/2}$

$\boxed{y' = 5(2x+1)^{3/2}(4x-1)^{3/2} + 6(2x+1)^{5/2}(4x-1)^{1/2}}$

9. $f(x) = (x^2+3)(x^2-3)^{10}$; $\boxed{y' = 2x(x^2-3)^{10} + 20x(x^2+3)(x^2-3)^9}$

$u = x^2+3 \qquad v = (x^2-3)^{10}$

$u' = 2x \qquad v' = 10(x^2-3)^9(2x)$

$\qquad\qquad = 20x(x^2-3)^9$

10. $\frac{d}{dx}\left[x^7(3x^4+12x-1)^2\right] = \boxed{\left[7x^6(3x^4+12x-1)^2 + \underbrace{24x^7(x^3+1)}_{2x^7(12x^3+12)}(3x^4+12x-1)\right]dx}$

$u = x^7 \qquad v = (3x^4+12x-1)^2$

$u' = 7x^6 \qquad v' = 2(3x^4+12x-1)(12x^3+12)$

$\qquad\qquad = 24(x^3+1)(3x^4+12x-1)$

11. $y = \dfrac{x^2+2x-1}{x^2+2x-2}$; $\boxed{y' = \dfrac{(2x+2)(x^2+2x-2) - (2x+2)(x^2+2x-1)}{(x^2+2x-2)^2}}$

$u = x^2+2x-1 \qquad v = x^2+2x-2$

$u' = 2x+2 \qquad v' = 2x+2$

12. $y = \dfrac{3x^2+5x+1}{3-x^2}$; $\boxed{y' = \dfrac{(6x+5)(3-x^2) + 2x(3x^2+5x+1)}{(3-x^2)^2}}$

$u = 3x^2+5x+1 \qquad v = 3-x^2$

$u' = 6x+5 \qquad v' = -2x$

13. $y = \dfrac{x+3}{(2x+1)^2}$; $\boxed{y' = \dfrac{(2x+1)^2 - 4(x+3)(2x+1)}{(2x+1)^4} = \overbrace{\dfrac{-(2x+11)}{(2x+1)^3}}^{\text{simplified}}}$

$u = x+3 \qquad v = (2x+1)^2$

$u' = 1 \qquad v' = 2(2x+1)(2) = 4(2x+1)$

constants diff to zero

14. $f(x) = \dfrac{7}{9} + \dfrac{x^2+x+1}{x^5+1}$; $y' = \emptyset + \boxed{\dfrac{(2x+1)(x^5+1) - 5x^4(x^2+x+1)}{(x^5+1)^2}}$

$u = x^2+x+1 \qquad v = x^5+1$

$u' = 2x+1 \qquad v' = 5x^4$

15. $\dfrac{d}{dx}\left(\dfrac{\sqrt{x}}{\sqrt{x}+4}\right)$; $y' = \dfrac{\frac{1}{2}x^{-1/2}(x^{1/2}+4) - \frac{1}{2}x^{1/2}(x^{-1/2})}{(\sqrt{x}+4)^2} = \dfrac{\frac{1}{2} + 2x^{-1/2} - \frac{1}{2}}{(\sqrt{x}+4)^2}$

$u = x^{1/2} \qquad v = x^{1/2}+4$

$u' = \frac{1}{2}x^{-1/2} \qquad v' = \frac{1}{2}x^{-1/2}$

$= \dfrac{2\,dx}{\sqrt{x}(\sqrt{x}+4)^2} \Leftarrow$ simplified

16. $x^2 - 2y^2 = 16$; $2x - 4yy' = 0$; $-4yy' = -2x$; $y' = \frac{-2x}{-4y} = \boxed{\frac{x}{2y}}$

17. $x^2 - \overset{u}{2x}\overset{v}{y} = 6$; $2x - 2y - 2xy' = 0$; $-2xy' = 2y - 2x$; $y' = \frac{2y-2x}{-2x}$

$u = -2x \quad v = y$

$u' = -2 \quad v' = y'$

$\boxed{y' = \frac{x-y}{x} = 1 - \frac{y}{x}}$

18. $x^2y^2 - xy = 8$; $\overset{u'v}{2xy^2} + \overset{uv'}{2x^2yy'} - \overset{u'v}{y} - \overset{uv'}{xy'} = 0$

$2x^2yy' - xy' = -2xy^2 + y$; $y'(2x^2y - x) = -2xy^2 + y$

$y' = \frac{-2xy^2 + y}{2x^2y - x} = \frac{-y(2xy-1)}{x(2xy-1)} = \boxed{-\frac{y}{x}}$ ← Fully Simplified

19. $x^{1/2} + y^{1/2} = 1$; $\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$; $\frac{1}{2}y^{-1/2}y' = -\frac{1}{2}x^{-1/2}$

$y' = -\frac{x^{-1/2}}{y^{-1/2}} = \boxed{-\frac{\sqrt{y}}{\sqrt{x}}}$

20. $p + \frac{1}{6}x^3 = 48$; $\frac{dp}{dt} + \frac{1}{6}(3x^2)\frac{dx}{dt} = 0$; $\frac{dp}{dt} + \frac{1}{2}x^2\frac{dx}{dt} = 0$

$\frac{dp}{dt} = -\frac{1}{2}x^2\frac{dx}{dt}$; $\frac{dp}{dt} = -\frac{1}{2}(4)^2(5) = \boxed{-\$40/\text{week}}$

21. $C'(x) = .04x + 150$

(A) $C(x) = \int C'(x)\,dx = \int (.04x + 150)\,dx = .02x^2 + 150x + C$

C is given to be \$500, so $\boxed{C(x) = .02x^2 + 150x + 500}$

(B) $C(x) = \int_{10}^{12} C'(x)\,dx = \int_{10}^{12} (.04x + 150)\,dx = \left[.02x^2 + 150x\right]_{10}^{12}$

$= .02(12)^2 + 150(12) - [.02(10)^2 + 150(10)]$

$= 1802.88 - 1502 = \boxed{\$300.88}$

22 $f(x) = 2x+3 ; [0, 2] \Rightarrow \frac{1}{2-0}\int_0^2 (2x+3)\,dx = \frac{1}{2}\left[x^2+3x\right]_0^2$

$= \frac{1}{2}\left[(4+6)-0\right] = \frac{1}{2}(10) = \boxed{5}$

23. $f(x) = 2x^2-3 ; [1, 3] \Rightarrow \frac{1}{3-1}\int_1^3 (2x^2-3)\,dx = \frac{1}{2}\left[\frac{2}{3}x^3-3x\right]_1^3$

$= \frac{1}{2}\left[(18-9)-\left(\frac{2}{3}-3\right)\right] = \frac{1}{2}\left[9+2\tfrac{1}{3}\right] = \frac{1}{2}\left(\frac{34}{3}\right) = \boxed{\frac{17}{3}}$