Homework #4—Solution Guide—Log & e's Integration

1. $\int \frac{x^2-1}{x^3-3x+1}\,dx \Rightarrow \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u|+C$

$= \boxed{\frac{1}{3}\ln|x^3-3x+1|+C}$

let $u = x^3-3x+1$

$du = (3x^2-3)\,dx$

$du = 3(x^2-1)\,dx$

$\frac{1}{3}du = (x^2-1)\,dx$

2. $\int x^2 e^{x^3-1}\,dx \Rightarrow \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C$

let $u = x^3-1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$= \boxed{\frac{1}{3}e^{x^3-1}+C}$

3. $\int \frac{e^{2x}}{1+e^{2x}}\,dx \Rightarrow \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|+C$

let $u = 1+e^{2x}$

$du = 2e^{2x}\,dx$

$\frac{1}{2}du = e^{2x}\,dx$

$= \boxed{\frac{1}{2}\ln|1+e^{2x}|+C}$

4. $\int \frac{e^{-1/x}}{x^2}\,dx \Rightarrow \int e^u\,du = e^u+C = \boxed{e^{-1/x}+C}$

let $u = -1/x = -x^{-1}$

$du = x^{-2}\,dx = \frac{1}{x^2}\,dx$

5. $\int \frac{e^x - e^{-x}}{(e^x+e^{-x})^{3/2}}\,dx \Rightarrow \int \frac{1}{u^{3/2}}\,du = \int u^{-3/2}\,du = -2u^{-1/2}+C$

let $u = e^x+e^{-x}$

$du = (e^x - e^{-x})\,dx$

$= \boxed{\frac{-2}{\sqrt{e^x+e^{-x}}}+C}$

6. $\int \frac{(\ln x)^3}{x}\,dx \Rightarrow \int u^3\,du = \frac{1}{4}u^4+C = \boxed{\frac{1}{4}(\ln x)^4+C}$

let $u = \ln x$

$du = \frac{1}{x}\,dx$

7. $\int \frac{1}{x(\ln x)^2}\,dx \Rightarrow \int \frac{1}{u^2}\,du = \int u^{-2}du = -u^{-1}+C = -\frac{1}{u}+C$

let $u = \ln x$

$du = \frac{1}{x}\,dx$ $= \boxed{-\frac{1}{\ln x}+C}$

8. $\int \frac{(\ln x)^{7/2}}{x}\,dx \Rightarrow \int u^{7/2}\,du = \frac{2}{9}u^{9/2}+C$

let $u = \ln x$

$du = \frac{1}{x}\,dx$ $= \boxed{\frac{2}{9}(\ln x)^{9/2}+C}$

9. $\int \frac{x^4}{1-x^5}\,dx = -\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln|u|+C = \boxed{-\frac{1}{5}\ln|1-x^5|+C}$

let: $u = 1-x^5$

$du = -5x^4dx$; $-\frac{1}{5}du = x^4dx$

10. $\int \frac{e^x}{1+e^x}\,dx = \int \frac{1}{u}\,du = \ln|u|+C = \boxed{\ln(1+e^x)+C}$

let: $u = 1+e^x$

$du = e^x dx$

11. $\int \frac{e^{3x}+x^2}{(e^{3x}+x^3)^3}\,dx = \frac{1}{3}\int \frac{1}{u^3}\,du = \frac{1}{3}\int u^{-3}du = \frac{1}{3}\cdot\frac{1}{-2}u^{-2}+C = -\frac{1}{6u^2}+C$

let: $u = e^{3x}+x^3$ $= \boxed{-\frac{1}{6(e^{3x}+x^3)^2}+C}$

$du = (3e^{3x}+3x^2)dx$

$du = 3(e^{3x}+x^2)dx$

$\frac{1}{3}du = (e^{3x}+x^2)dx$

12. $\int \frac{1}{x\ln x}\,dx \Rightarrow \int \frac{1}{u}\,du = \ln|u| \Rightarrow \boxed{\ln|\ln x|+C}$

let: $u = \ln x$

$du = \frac{1}{x}\,dx$

13. $\int_0^1 \frac{x}{x^2+3}\,dx \Rightarrow \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| \Rightarrow \frac{1}{2}\ln|x^2+3|\Big|_0^1$

let $u = x^2+3$ $= \frac{1}{2}[\ln(4)-\ln(3)]$

$du = 2x\,dx$; $\frac{1}{2}du = x\,dx$ $= \boxed{\frac{1}{2}\ln\left(\frac{4}{3}\right)}$