Homework #5

1. $y = \ln(2x)$; $y' = \frac{2}{2x} = \boxed{\frac{1}{x}}$

2. $y = (\ln(2x))^2$; $y' = 2\ln(2x) \cdot \frac{1}{x} = \boxed{\frac{2\ln(2x)}{x}}$

3. $y = \ln\left(\frac{x}{1+x^2}\right) = \ln x - \ln(1+x^2)$; $\boxed{y' = \frac{1}{x} - \frac{2x}{1+x^2}}$

4. $y = \ln|x^3 - 7x^2 - 3|$; $\boxed{y' = \frac{3x^2 - 14x}{x^3 - 7x^2 - 3}}$

5. $y = \ln(\sqrt{x}) = \ln(x^{1/2}) = \frac{1}{2}\ln(x)$; $y' = \frac{1}{2} \cdot \frac{1}{x} = \boxed{\frac{1}{2x}}$

6. $y = e^7$; $\boxed{y' = \emptyset}$ constants differentiate to zero

7. $y = e^{7x}$; $\boxed{y' = 7e^{7x}}$

8. $y = x^3 e^x$; $\boxed{y' = 3x^2 e^x + x^3 e^x = x^2 e^x (3 + x)}$
   $u = x^3 \quad v = e^x$
   $u' = 3x^2 \quad v' = e^x$

9. $y = e^{-5x^2}$; $\boxed{y' = -10x e^{-5x^2}}$

10. $y = \frac{e^x}{\ln x}$ $\qquad y' = \frac{e^x \ln x - \frac{1}{x} e^x}{(\ln x)^2} = \boxed{\frac{e^x}{\ln x} - \frac{e^x}{x(\ln x)^2}}$
    $u = e^x \quad v = \ln x$
    $u' = e^x \quad v' = 1/x$

11. $y = x(\ln x)^2$ $\qquad y' = 1(\ln x)^2 + x \cdot \frac{2\ln x}{x} = \boxed{(\ln x)^2 + 2\ln x = \ln x[\ln(x) + 2]}$
    $u = x \quad v = (\ln x)^2$
    $u' = 1 \quad v' = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$

12. $y = \frac{x}{(\ln x)^2}$ $\qquad y' = \frac{1 \cdot (\ln x)^2 - x \cdot \frac{2\ln x}{x}}{((\ln x)^2)^2} = \frac{(\ln x)^2 - 2\ln x}{(\ln x)^4}$
    $u = x \quad v = (\ln x)^2$
    $u' = 1 \quad v' = 2\ln x \cdot \frac{1}{x} = \frac{2\ln x}{x}$
    $\boxed{\frac{\ln x - 2}{(\ln x)^3} = \frac{1}{(\ln x)^2} - \frac{2}{(\ln x)^3}}$

13. $y = \ln\left[e^{2x}(x^3+1)(x^4+5x)\right] = \ln e^{2x} + \ln(x^3+1) + \ln(x^4+5x)$

$y = 2x + \ln(x^3+1) + \ln(x^4+5x)$

$\boxed{y' = 2 + \frac{3x^2}{x^3+1} + \frac{4x^3+5}{x^4+5x}}$

14. $y = \ln\left[\frac{e^{5x}(x+4)(3x-2)}{x+1}\right] = \ln e^{5x} + \ln(x+4) + \ln(3x-2) - \ln(x+1)$

$\Rightarrow 5x + \ln(x+4) + \ln(3x-2) - \ln(x+1)$

$\boxed{y' = 5 + \frac{1}{x+4} + \frac{3}{3x-2} - \frac{1}{x+1}}$

15. $y = \ln\left[\frac{\sqrt{x}(x+1)^2(x+2)^3}{4x+1}\right] = \ln x^{1/2} + \ln(x+1)^2 + \ln(x+2)^3 - \ln(4x+1)$

$= \frac{1}{2}\ln x + 2\ln(x+1) + 3\ln(x+2) - \ln(4x+1)$

$\boxed{y' = \frac{1}{2x} + \frac{2}{x+1} + \frac{3}{x+2} - \frac{4}{4x+1}}$

16. $y = \ln\left[\frac{(5x+1)(4x+1)(\ln x)}{\sqrt{2x+1}}\right] = \ln(5x+1) + \ln(4x+1) + \ln(\ln x) - \ln(2x+1)^{1/2}$

$= \ln(5x+1) + \ln(4x+1) + \ln(\ln x) - \frac{1}{2}\ln(2x+1)$

$y' = \frac{5}{5x+1} + \frac{4}{4x+1} + \frac{\frac{1}{x}}{\ln x} - \frac{1}{2}\cdot\frac{2}{2x+1} = \boxed{\frac{5}{5x+1} + \frac{4}{4x+1} + \frac{1}{x\ln x} - \frac{1}{2x+1}}$

17. $y = \ln\left[\frac{x^5 e^{4x}\sqrt{3x+1}}{1-x^2}\right] = \ln x^5 + \ln e^{4x} + \ln(3x+1)^{1/2} - \ln(1-x^2)$

$= \ln x^5 + 4x + \frac{1}{2}\ln(3x+1) - \ln(1-x^2)$

$y' = \frac{5x^4}{x^5} + 4 + \frac{1}{2}\cdot\frac{3}{3x+1} - \frac{-2x}{1-x^2} = \boxed{\frac{5}{x} + 4 + \frac{3}{2(3x+1)} + \frac{2x}{1-x^2}}$

18. $f(x) = 2^x$; $y = 2^x$; $\ln y = \ln 2^x = x\ln(2)$

$\frac{1}{y}\cdot y' = \ln 2$; $y' = y\ln 2 = \boxed{2^x \ln(2)}$

19. $f(x) = e^x(3x-4)^8$; $y = e^x(3x-4)^8$; $\ln y = \ln[e^x(3x-4)^8]$

$\ln y = \ln e^x + \ln(3x-4)^8 = x + 8\ln(3x-4)$

$\frac{1}{y}\cdot y' = 1 + 8\cdot\frac{3}{3x-4}$; $y' = y\left(1 + \frac{24}{3x-4}\right) = e^x(3x-4)^8\left(1 + \frac{24}{3x-4}\right)$

20. $f(x) = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}$; $y = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}$; $\ln y = \ln\left[\frac{(x+1)(2x+1)(3x+1)}{(4x+1)^{1/2}}\right]$

$\ln y = \ln(x+1) + \ln(2x+1) + \ln(3x+1) - \frac{1}{2}(4x+1)$

$\frac{1}{y} \cdot y' = \frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} - \frac{1}{2} \cdot \frac{4}{4x+1}$; $y' = y\left[\frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} - \frac{2}{4x+1}\right]$

$$\boxed{y' = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}\left[\frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} - \frac{2}{4x+1}\right]}$$

21. $\int \frac{x^5 + 2x^2 - 1}{x}\,dx = \int \left(x^4 + 2x - \frac{1}{x}\right)dx = \boxed{\frac{1}{5}x^5 + x^2 - \ln|x| + C}$

22. $\int \left[\frac{2}{x} + 3e^x\right]dx = \boxed{2\ln|x| + 3e^x + C}$

23. $\int \left(e^{3x} - \frac{4}{x} + 3 - \frac{8}{x^3}\right)dx = \int \left(e^{3x} - \frac{4}{x} + 3 - 8x^{-3}\right)dx$

$= \frac{1}{3}e^{3x} - 4\ln|x| + 3x - \frac{8}{-2}x^{-2} + C = \boxed{\frac{1}{3}e^{3x} - 4\ln|x| + 3x + \frac{4}{x^2} + C}$

24. $\int_{\ln 2}^{\ln 5} \frac{e^{-4x}e^{2x}}{e^{-5x}}\,dx = \int_{\ln 2}^{\ln 5} e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_{\ln 2}^{\ln 5} = \frac{1}{3}\left[e^{3\ln 5} - e^{3\ln 2}\right]$

$= \frac{1}{3}\left[e^{\ln 5^3} - e^{\ln 2^3}\right] = \frac{1}{3}\left[5^3 - 2^3\right] = \frac{1}{3}[125 - 8] = \frac{117}{3} = \boxed{39}$

25. $f(x) = \frac{1}{x}$; $\left[\frac{1}{3}, 3\right] \Rightarrow \frac{1}{3 - 1/3}\int_{1/3}^{3} \frac{1}{x}\,dx = \frac{3}{8}\ln x\Big|_{1/3}^{3}$

$= \frac{3}{8}\left[\ln 3 - \ln\left(\frac{1}{3}\right)\right] = \frac{3}{8}\ln\left(\frac{3}{\frac{1}{3}}\right) = \boxed{\frac{3}{8}\ln 9}$

26.

<u>Colony 1</u>

$r = \frac{\ln 2}{21} = .033$

<u>Colony 2</u>

$r = \frac{\ln 2}{33} = .021$

$A_1 = 1000e^{.033t} \Leftarrow$ set The 2 eqns equal & solve for t $\Rightarrow A_2 = 710{,}000e^{.021t}$

$$\frac{100\cancel{0}e^{.033t}}{10\cancel{00}} = \frac{710{,}000e^{.021t}}{1000}$$

$$\frac{e^{.033t}}{e^{.021t}} = \frac{710\,\cancel{e^{.021t}}}{\cancel{e^{.021t}}} \Rightarrow e^{.033t-.021t} = 710$$

$$\ln e^{.012t} = \ln 710 \Rightarrow .012t = \ln(710);\ t = \frac{\ln(710)}{.012} = 547$$

$\boxed{\text{547 minutes OR 9.1 hours}}$