

Math 180 - Homework #5

Write the ANSWERS ONLY on this page. Do your calculations/work elsewhere, then NEATLY transfer your answers to this page.

Differentiate the following (2pts. each):

ANSWERS

1. $y = \ln(2x)$

$$y' = \frac{1}{x}$$

2. $y = (\ln(2x))^2$

$$y' = \frac{2 \ln|2x|}{x}$$

3. $y = \ln\left(\frac{x}{1+x^2}\right)$

$$y' = \frac{1}{x} - \frac{2x}{1+x^2}$$

4. $y = \ln|x^3 - 7x^2 - 3|$

$$y' = \frac{3x^2 - 14x}{x^3 - 7x^2 - 3}$$

5. $y = \ln(\sqrt{x})$

$$y' = \frac{1}{2x}$$

6. $y = e^7$

$$y' = 0$$

7. $y = e^{7x}$

$$y' = 7e^{7x}$$

8. $y = x^3 e^x$

$$y' = 3x^2 e^x + x^3 e^x$$

9. $y = e^{-5x^2}$

$$y' = -10xe^{-5x^2}$$

10. $y = \frac{e^x}{\ln x}$ (Remember: No complex fractions!)

$$y' = \frac{e^x}{\ln(x)} - \frac{e^x}{x(\ln x)^2}$$

11. $y = x(\ln x)^2$

$$y' = (\ln|x|)^2 + 2\ln|x| = \ln|x|[\ln|x| + 2]$$

12. $y = \frac{x}{(\ln x)^2}$

$$y' = \frac{\ln(x) - 2}{(\ln x)^3} = \frac{1}{(\ln x)^2} - \frac{2}{(\ln x)^3}$$

13. $y = \ln[e^{2x}(x^3+1)(x^4+5x)]$

$$y' = 2 + \frac{3x^2}{x^3+1} + \frac{4x^3+5}{x^4+5x}$$

14. $y = \ln\left[\frac{e^{5x}(x+4)(3x-2)}{x+1}\right]$

$$y' = 5 + \frac{1}{x+4} + \frac{3}{3x-2} - \frac{1}{x+1}$$

15. $y = \ln\left[\frac{\sqrt{x}(x+1)^2(x+2)^3}{4x+1}\right]$

$$y' = \frac{1}{2x} + \frac{2}{x+1} + \frac{3}{x+2} - \frac{4}{4x+1}$$

16. $y = \ln\left[\frac{(5x+1)(4x+1)(\ln x)}{\sqrt{2x+1}}\right]$ (Remember: No complex fractions!)

$$y' = \frac{5}{5x+1} + \frac{4}{4x+1} + \frac{1}{x \ln x} - \frac{1}{2x+1}$$

17. $y = \ln\left[\frac{x^5 e^{4x} \sqrt{3x+1}}{1-x^2}\right]$

$$y' = \frac{5}{x} + 4 + \frac{3}{2(3x+1)} + \frac{2x}{1-x^2}$$

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Use **logarithmic differentiation** to differentiate the following (2pts. each):

ANSWERS

18. $f(x) = 2^x$

$$y' = 2^x \ln(2)$$

19. $f(x) = e^x (3x-4)^8$

$$y' = e^x (3x-4)^8 \left(1 + \frac{24}{3x-4} \right)$$

20. $f(x) = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}$

$$y' = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}} \left[\frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} - \frac{2}{4x+1} \right]$$

Integrate the following (2pts. each):

ANSWERS

21. $\int \frac{x^5 + 2x^2 - 1}{x} dx$ (Hint: Simplify before integrating)

$$\frac{1}{5} x^5 + x^2 - \ln|x| + C$$

22. $\int \left[\frac{2}{x} + 3e^x \right] dx$

$$2\ln|x| + 3e^x + C$$

23. $\int \left(e^{3x} - \frac{4}{x} + 3 - \frac{8}{x^3} \right) dx$

$$\frac{1}{3} e^{3x} - 4\ln|x| + 3x + \frac{4}{x^2} + C$$

24. $\int_{\ln 2}^{\ln 5} \frac{e^{-4x} e^{2x}}{e^{-5x}} dx$ (Hint: Simplify before integrating) **NO DECIMALS!**

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Find the average value of the function f over the indicated interval $[a,b]$ (2pts.):

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25. $f(x) = \frac{1}{x}; \left[\frac{1}{3}, 3 \right]$ (Write answer as a SINGLE logarithm) **NO DECIMALS!**

$$\frac{3}{8} \ln(9)$$

Word Problem (2pts.):

26. Two different bacteria colonies are growing near a pool of stagnant water. Suppose that the first colony initially has 1000 bacteria and doubles every 21 minutes. The second colony has 710,000 bacteria and doubles every 33 minutes. How much time will elapse until the two colonies are the same size (break-even analysis)? **Round to nearest minute**

547 minutes = 9.1 hours
