Differentiation

Example: $y = 3x^5$ $\quad 3x^{5-1} \Longrightarrow 5 * 3x^4 \rightarrow$ Answer: $y' = 15x^4$

0. Constants, all by themselves, differentiate to zero. Example: $y = 7e^2$ {notice: NO VARIABLE} $y' = 0$

1. Constant multiple rule: $\frac{d}{dx}[k * f(x)] = k * \frac{d}{dx}[f(x)]$, k a constant Example: $\frac{dy}{dx}[3x^2] = 3\frac{dy}{dx}[x^2] = 3 * 2x = 6x$

2. Sum rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$ Example: $\frac{dy}{dx}[3x^2 + 4x^5] = \frac{dy}{dx}[3x^2] + \frac{dy}{dx}[4x^5] = 6x + 20x^4$

3. General power rule: $\frac{d}{dx}[g(x)]^r = r * [g(x)]^{r-1} * \frac{d}{dx}[g(x)]$ "OI!" (outer•inner)

4. Product rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \Longrightarrow$ **u'v + uv'**

The Product Rule you must rhyme, E I E I O,
It's u-prime v plus u v-prime, E I E I O.

OLD MACDONALD KNOWS CALCULUS

5. Quotient rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \Longrightarrow \frac{u'v - uv'}{v^2}$

Lo dee Hi **minus** Hi dee Lo,
Square the bottom and away we go!

6. Chain rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \Longrightarrow \frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$

7. $\frac{d}{dx}(e^{kx}) = ke^{kx}$ Example: $y = e^{5x} \Longrightarrow y' = 5e^{5x}$

8. $\frac{d}{dx}e^{g(x)} = g'(x)e^{g(x)} \Longrightarrow$ (differentiate only the exponent, then just copy the 'e' with the old exponent, exactly as it is)
Example: $y = e^{x^2-3x+4} \Longrightarrow y = (2x-3)e^{x^2-3x+4}$

9. $\frac{d}{dx}ln|x| = \frac{1}{x}, x \neq 0$

10. $\frac{d}{dx}[\ln g(x)] = \frac{g'(x)}{g(x)}$

Differentiate the argument of the log OVER the argument of the log copied exactly.
Example: $y = ln(3x^2 - 2x + 1)$
$y' = \frac{6x - 2}{3x^2 - 2x + 1}$

Note: 1st split complex log eqn. into separate, individual logs using log properties, **THEN** differentiate each log piece separately according to rule #10.

Example: $y = ln\left[\frac{x^5 e^{4x}\sqrt{3x+1}}{1+x^2}\right]$ Recall: $\ln(e^{4x}) = 4x$

$= 5\ln(x) + 4x + \frac{1}{2}\ln(3x+1) - \ln(1+x^2)$

ANSWER: $y' = \frac{5}{x} + 4 + \frac{3}{2(3x+1)} - \frac{2x}{1+x^2}$

(a) **Logarithmic Differentiation:** $f'(x) = f(x)\left[\frac{d}{dx}\ln f(x)\right]$

11.

Prime Notation	$\frac{d}{dx}$ Notation
$f'(x)$	$\frac{d}{dx}f(x)$
y'	$\frac{dy}{dx}$
$f''(x)$	$\frac{d^2}{dx^2}f(x)$
y''	$\frac{d^2y}{dx^2}$

Example:

$y = x^2(x+1)^4(x^2+2)^3$

$\ln(y) = ln\left[x^2(x+1)^4(x^2+2)^3\right]$ ←Take "ln of both sides

$\ln(y) = 2\ln(x) + 4\ln(x+1) + 3\ln(x^2+2)$ ←Use properties of logs to simplify eqn.

$\frac{y'}{y} = = \frac{2}{x} + \frac{4}{x+1} + \frac{3(2x)}{x^2+2}$ ←Differentiate (implicitly) both sides

$y' = y\left[\frac{2}{x} + \frac{4}{x+1} + \frac{6x}{x^2+2}\right]$ ←Isolate y' by multiplying both sides by "y"

$y' = x^2(x+1)^4(x^2+2)^3\left[\frac{2}{x} + \frac{4}{x+1} + \frac{6x}{x^2+2}\right]$ ←Recall: $y = x^2(x+1)^4(x^2+2)^3$

12. 'Marginal' means differentiate! -- To get **from** marginal back to original (cost, profit, revenue), integrate!
   (a) If C(x) is the cost, then C'(x) is the marginal cost
   (b) If R(x) is the revenue, then R'(x) is the marginal revenue
   (c) If P(x) is the profit, then P'(x) is the marginal profit.

13a). **Differentiate** position to get velocity (**RATE**)

13b). Differentiate velocity to get acceleration

13c). Integrate acceleration to get velocity

13d). Integrate velocity to get position

To find a rate, DIFFERENTIATE!

Differentiate ↓ / Integrate ↑

s(t)	Position	s(t)
s'(t)	Velocity	s'(t)
s"(t)	Acceleration	s"(t)

Implicit Differentiation

1. Differentiate each term of the equation **with respect to x**.
   a. Whenever you differentiate the 'y' variable, tack on a y' to its derivative.
2. Get all the y' terms on one side, all the non-y' terms on the other side of the equal sign.
3. Factor out the common y'.
4. Divide both sides by the y' factor to get y' all by itself on one side of the equal sign.
5. Example: $2x^2 - 4x + 3y^2 - y = 16$ → $4x - 4 + 6yy' - 1y' = 0$ → $6yy' - 1y' = -4x + 4$

   $y'(6y - 1) = -4x + 4$ → $y' = \dfrac{-4x + 4}{(6y - 1)}$ ← **ANSWER**

Chain Rule Problems

CALCULUS DEVIL

1. Differentiate each term of the equation **with respect to t(time)**.
   a. That is, differentiate every variable, then tack on that variable's letter with a prime.
2. Plug in all given values and solve for the remaining variable (or its prime)
3. Example: $3x^2y - 6x + 20y^3 = 0$ **(Note: First term will require product rule)**
   a. $6xx'y + 3x^2y' - 6x' + 60y^2y' = 0$ **OR** $6x\frac{dx}{dt} \cdot y + 3x^2\frac{dy}{dt} - 6\frac{dx}{dt} + 60y^2\frac{dy}{dt} = 0$

Line Information

1. To find the equation of a line:
   a) Find a point on the line: (x_1,y_1)
   b) Find the slope, m
   c) Write the equation of the line: $y - y_1 = m(x - x_1)$
2. To find the slope:
   a) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
   b) **First derivative (y') = slope**
3. $y = mx + b$ ⟹ slope - intercept form of a line
4. $y - y_1 = m(x - x_1)$ ⟹ point-slope form of a line
5. $m_{parallel} = m$ Parallel lines have the same slope. **DON'T CHANGE THE SLOPE!**
6. $m_{perpendicular} = -\dfrac{1}{m}$ Flip old slope over **AND CHANGE THE SIGN**
7. X-intercept: (x,0) **plug in 'zero' for y, and solve for x**
8. Y-intercept: (0,y) **plug in 'zero' for x, and solve for y**

Not finding the slope would be a crime. simply find, y-prime!

To Find the Equation of a Tangent Line

1. Find the slope:
   a. Take the first derivative of f(x)
   b. **Plug x-value into first derivative and solve**.
   c. <u>That is your slope</u>.
2. Find the point:
   a. Plug x-value into the ORIGINAL EQUATION, f(x), and solve for f(x) (**<u>this is the y-value of the point</u>**).
   b. Your point is the value (x,f(x))
3. Write the equation of the line: $y - y_1 = m(x - x_1)$
   a. Solve for y in terms of x, to put it into slope-intercept form, y = mx + b.

Integration

Example: $y' = \int 15x^4\,dx$ $\underline{15}\,x^{4+1} \Longrightarrow \frac{15}{5}x^5 \rightarrow$ Answer: $y = 3x^5 + C$

0. $\int k\,dx = kx + C$, k is a constant → Constants pick up the variable when integrated. $\int 7\,dx = 7x + C$

1. $\int kx^r\,dx = k\int x^r\,dx = k * \frac{1}{r+1}x^{r+1} + C,\ r \neq 1$
2. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
3. $\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} + C,\ n \neq 1$
4. $\int e^{kx}dx = \frac{1}{k}e^{kx} + C,\ k \neq 0$ ⟹ (divide by whatever number is in front of the 'x' then just copy the 'e' with the old exponent, exactly as it is) {Example: $\int e^{-5x}\,dx = \frac{1}{-5}e^{-5x} + C$
5. $\int \frac{1}{x}dx = \ln|x| + C,\ x \neq 0$
6. $\int \frac{1}{a+bx}dx = \frac{1}{b}\ln|a+bx| + C$
7. $\int_a^b f(x)dx = F(b) - F(a)$ ⟹ Fundamental Theorem of Calculus (i.e. the area under the curve **between** two points, a & b)

To find the value in BETWEEN, simply use the CURVY THING! $\int$

When the power of the denominator is positive one, The natural log is your best chum!

CALCULUS LINCOLN

Initial Value Problems

$\int f'(x) = f(x) + C$; then use given f(a) = b, that is, the point (a,b) to solve for 'C'

Example: $f'(x) = 3x^2 - 4x + 8$; $f(1) = 9$

$\int (3x^2 - 4x + 8)dx = x^3 - 2x^2 + 8x + C$, that is, $y = x^3 - 2x^2 + 8x + C$

We're given f(1) = 9, or the point, (1,9), so plugging in for x & y gives: $9 = (1)^3 - 2(1)^2 + 8(1) + C$

Solve for C and rewrite the equation: $9 = 1 - 2 + 8 + C$ → $9 = 7 + C$ → $C = 2$ **ANSWER: $f(x) = x^3 - 2x^2 + 8x + 2$**

Average Value of a Function

$\frac{1}{b-a}\int_a^b f(x)dx = f(x);\ [a,b]$ ⟹ The average value integral

Example: $f(x) = \sqrt{x}\,;[0,4] \rightarrow \frac{1}{4-0}\int_0^4 \sqrt{x}\,dx = \frac{1}{4}\int_0^4 x^{\frac{1}{2}}\,dx = \frac{1}{4}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 = \frac{1}{4}\left[\frac{2}{3}x\sqrt{x}\right]_0^4$

$= \frac{1}{4}\left[\frac{2}{3}4\sqrt{4} - \frac{2}{3}0\sqrt{0}\right] = \frac{1}{4}\left[\frac{2}{3}4(2) - \frac{2}{3}0(0)\right] = \frac{1}{4}\left[\frac{2}{3}(8) - \frac{2}{3}(0)\right] = \frac{1}{4}\left[\frac{16}{3}\right] = \frac{4}{3}$

$= \frac{1}{4}\left[\frac{2}{3}4\sqrt{4} - \frac{2}{3}0\sqrt{0}\right] = \frac{1}{4}\left[\frac{2}{3}4(2) - \frac{2}{3}0(0)\right] = \frac{1}{4}\left[\frac{2}{3}(8) - \frac{2}{3}(0)\right] = \frac{1}{4}\left[\frac{16}{3}\right] = \frac{4}{3}$ ← **ANSWER**

A Buddhist or Confucian would integrate by SUBSTITUTION!

Integration by Substitution

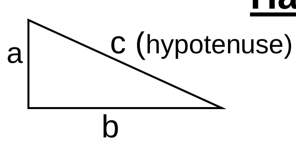
1. Define u = g(x), where g(x) is chosen so that g'(x)dx will get rid of a nasty term in the integral.
2. Replace g(x) with u and dx with the adjusted g'(x)dx with du.
3. Integrate the resulting function of u.
4. Rewrite the answer in terms of x by replacing u by g(x).

Example: $\int \frac{4x\,dx}{(x^2+3)^5}$; let $u = x^2+3$; $du = 2x\,dx$, so $2du = 4x\,dx$

Rewrite integral: $\int \frac{2du}{u^5} = 2\int u^{-5}du = 2\frac{u^{-4}}{-4} = \frac{u^{-4}}{-2} = -\frac{1}{2u^4}$

Substituting back in for u: $-\frac{1}{2(x^2+3)^4}$ ← **ANSWER**

Handy Formulae



1. Pythagorean Theorem: $a^2 + b^2 = c^2$
2. Quadratic Formula: Given $0 = ax^2 + bx + c$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Oompa, Loompa, Doompa Dee Dormula, To Solve This Equation, Use the Quadratic formula!

Guidelines for Integration by Substitution:

Choose only ONE of these four situations:

1. Let u equal the highest power polynomial expression.

OR

2. If an $e^{\text{polynomial}}$ is present, let u equal **JUST** the polynomial part of the exponent, **NOT** the whole "e-term."

OR

3. If there is an $(e^{\text{polynomial}}\text{ expression})^{\text{Power}}$, let u equal the "$e^{\text{polynomial}}$ expression," i.e. **ONLY** what is in the parentheses.

OR

4. If a natural log is present, let u equal the ln(argument).

Uber Important!!! The substituted expressions MUST match the expressions of the integral EXACTLY. If they don't, they **MUST be adjusted** (by multiplying or dividing by a constant).

Examples (corresponding to guideline numbers above):

1. $\int (x^3 - 2x)^2(3x^2 - 2)dx$

$\int u^2\, du$

Let: $u = x^3 - 2x$ This is the highest power polynomial expression.

$du = (3x^2 - 2)dx$ Notice how this matches EXACTLY the expression in **RED** in the original problem.

Now, substitute into the original problem, u (the green) and du (the purple) to get a much easier problem to integrate.

2. $\int xe^{-x^2}dx$

$\int -\frac{1}{2}e^u du$

$-\frac{1}{2}\int e^u du$

Let: $u = -x^2$ This is just the polynomial part of the exponent.

$du = -2xdx$ Notice this does **NOT** match EXACTLY the expression in **RED** in the original problem; it will have to be ADJUSTED until it does match EXACTLY.

$-\frac{1}{2}du = xdx$ By dividing both sides by 2, an EXACT match is achieved.

Now, substitute into the original problem, u (the green) and du (the purple) to get a much easier problem to integrate.

Notice: Pull the constant, $-\frac{1}{2}$ outside the integral to make things easier.

3. $\int \frac{e^{3x}+x^2}{(e^{3x}+x^3)^3}dx$

$\int \frac{\frac{1}{3}du}{u^3} = \frac{1}{3}\int u^{-3}du$

Let: $u = e^{3x} + x^3$ This is just what is in the parentheses, NOTHING ELSE!.

$du = 3e^{3x} + 3x^2$ Notice this does **NOT** match EXACTLY the expression in **RED** in the original problem; it will have to be ADJUSTED until it does match EXACTLY.

$du = 3(e^{3x} + x^2)$

$\frac{1}{3}du = (e^{3x} + x^2)dx$ By dividing both sides by 3, an EXACT match is achieved.

Now, substitute into the original problem, u (the green) and du (the purple) to get a much easier problem to integrate.

Notice: Pull the constant, $\frac{1}{3}$ outside the integral to make things easier.

4. $\int \frac{\ln 5x}{x}dx$

$\int u\,du$

Let: $u = \ln(5x)$ This is the ln(argument).

$du = \frac{1}{x}dx$ Notice how this matches EXACTLY the expression in **RED** in the original problem.

Now, substitute into the original problem, u (the green) and du (the purple) to get a much easier problem to integrate.

Properties of Exponents

1. $a^0 = 1$
2. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
3. $a^{-m} = \frac{1}{a^m}$
4. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
5. $a^m \bullet a^n = a^{m+n}$
6. $\left(a^m\right)^n = a^{m\bullet n}$
7. $\frac{a^m}{a^n} = a^{m-n}$
8. $\sqrt[n]{a^m} = \sqrt[n]{a}^m = a^{m/n}$
9. (negative)$^{\text{Odd}}$ = negative

   (negative)$^{\text{Even}}$ = positive

Examples:

$\frac{3}{x} = 3x^{-1}$

$\frac{3}{(x + x^2)} = 3(x + x^2)^{-1}$

$\frac{3}{\sqrt{x}} = \frac{3}{x^{\frac{1}{2}}} = 3x^{-\frac{1}{2}}$

Examples:

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$x^2\sqrt{x} = x^2 x^{\frac{1}{2}} = x^{2\frac{1}{2}} = x^{\frac{5}{2}}$

$\frac{1}{\sqrt[4]{x^3}} = \frac{1}{x^{\frac{3}{4}}} = x^{-\frac{3}{4}}$

Properties of Logarithms

For all real numbers a, b, N, M>0, a≠1:

1. ln(1) = 0
2. ln(e) = 1
3. $\ln(e^x) = x$
4. $e^{\ln(x)} = x$, x>0
5. ln(M·N) = ln(M) + ln(N)
6. $\ln(N^m) = m\ln(N)$
7. $\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$
8. $e^x = a$ ⟹ ln(a) = x
9. $\log_b x = y \rightarrow b^y = x,\ \ b > 0,\ b \neq 1$

   Example: $\log_2 16 = ? \rightarrow ? = 4$ since $2^4 = 16$

Example:

5. $\ln(x^2+1)(x^3 - 3) = \ln(x^2+1) + \ln(x^3 - 3)$
6. $\ln(x^2+1)^3(x^3 - 3)^2$

   $= 3\ln(x^2+1) + 2\ln(x^3 - 3)$
7. $\ln\frac{(x^2+1)^2}{\left(\sqrt[3]{x^3-3}\right)} = 2\ln(x^2+1) - \frac{1}{3}\ln(x^3-3)$

Compound Interest & Exponential Growth/Decay

Simple Interest:

1. I = Prt ⟹ I = $ interest earned; P = principal (starting amount); r = interest rate; t = time (in yrs.)
2. A = P(1+ rt) ⟹ A = $ final amount; P = principal (starting amount); r = interest rate; t = time (in yrs.)

Compound 'n' times per year:

3. $A = P\left(1+\frac{r}{n}\right)^{nt}$ ⟹ **A = $ final amount; P = principal (starting amount); r = interest rate; n = # times compounded per year; t = time (in yrs.)**

Compound Continuously, Population Growth, Radioactive Decay:

4. $A(t) = Pe^{rt}$ ⟹ A = final amount; P = starting amount or present value; r = growth/decay constant or interest rate, t = time

   (a) Rate: A'(t) = r·A(t)

5. Doubling time: $r = \frac{\ln(2)}{doubling\ time}$ Half-life: $r = \frac{\ln\left(\frac{1}{2}\right)}{half-life}$

   Tripling time: $r = \frac{\ln(3)}{tripling\ time}$ Quadrupling time: $r = \frac{\ln(4)}{quadrupling\ time}$, etc.

Note: $half-life = \frac{\ln\left(\frac{1}{2}\right)}{r}$

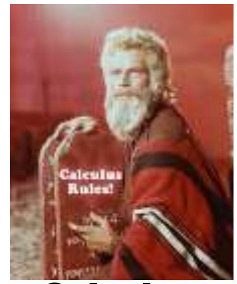
$doubling\ time = \frac{\ln(2)}{r}$

$tripling\ time = \frac{\ln(3)}{r}$, etc.

To avoid a career digging dirt,
just remember, APERT!

How to solve Max/Min Problems:

1. Revenue: $R(x)$ = items ∗ price. Price abbreviated with the letter, p. Items represented by the letter, x. The Capital '∗' above means "times."



Calculus Prophet

2. Profit: $P(x)$ = Revenue – Cost.
3. To find the number of items needed to get a maximum or minimum:
   (a) Differentiate either the revenue or profit equation.
      (1) If trying to maximize revenue, differentiate revenue equation.
      (2) If trying to maximize profit, differentiate profit equation.
   (b) Set the first derivative equal to zero
   (c) Solve for 'x.'
4. To find the actual maximum or minimum value for revenue or profit:
   (a) Take the x-value from 3.(c) above
   (b) Plug that x-value into the original revenue or profit equation.
5. To find the ideal price to charge:
   (a) Take the x-value from 3.(c) above
   (b) Plug that x-value into the original price equation.
6. Example:

   (a) We are told the price of a item, as a function of the number of items, x, is:

   $$p(x) = -3x + 14$$

   (b) Then the revenue equation, as a function of the number of items, x, will be:

   $$R(x) = \text{items} * \text{price} = x[-3x + 14] = -3x^2 + 14x.$$

   (c) If we are told the cost to produce a item, as a function of the number of items, x is:

   $$C(x) = x^2 - 10x + 22$$

   (d) Then the profit equation, as a function of the number of items, x, will be:

   $$P(x) = R(x) - C(x) = [-3x^2 + 14x] - [x^2 - 10x + 22]$$

   $$P(x) = -4x^2 + 24x - 22$$

   (e) To maximize the profit, take the derivative of the equation in (d) above, set it equal to zero, and solve for 'x.' This tells you the perfect number of items to produce to maximize your profit:

   $$P'(x) = -8x + 24 = 0 \Longrightarrow -8x = -24 \Longrightarrow x = 3 \text{ items}$$

      (1) This means when you make 3 items, you will have maximized your profit.

   (f) The actual maximum profit would be:

   $$P(3) = -4(3)^2 + 24(3) - 22 = \$14$$

   (g) You should price your items at:

   $$p(3) = -3(3) + 14 = \$5$$

   (h) If you charge \$5/item, you can expect to sell 3 items, thus maximizing your profits at \$14.

Example: that

The price of selling an item, x, is given by: **$p = -4x + 30$**. The total cost (in dollars) for a company to produce and sell x items per week is **$C(x) = x^2 + 20x - 11$**. How many items must be sold to maximize the profit? What is that maximum profit? How much should be charged per item to reach that maximum profit? Graph the profit function labeling the vertex, points of symmetry and all intercepts.

Revenue is item times price: R(x) = items ∗ price → x(**–4x + 30**) → **$R(x) = -4x^2 + 30x$**

Profit is Revenue minus Cost:

$P(x) = R(x) - C(x) = [-4x^2 + 30x] - [x^2 + 20x - 11]$ → **$P(x) = -5x^2 + 10x + 11$**

X-value of the vertex (which represents the number of items you need to sell in order to maximize your profit) is found by setting the first derivative of the profit equal to zero and solving for x.

$P'(x) = -10x + 10 = 0$ → $x = 1$ item (exciting, isn't it?)

Y-value of the vertex (which represents the actual maximum profit) is found by plugging in the x-value from the step above into the ORIGINAL Profit function.

$P(x) = -5x^2 + 10x + 11$ → $P(1) = -5(1)^2 + 10(1) + 11 = 16$ → **Vertex: (1, 16)** ← This means if you sell 1 item, you will maximize your profit at $16.

Plug the x-value of the vertex into the price equation to find out how much to charge per item in order to maximize your profit.

$p = -4x + 30$ → $p = p = -4(1) + 30$ → $p = \$26$/item

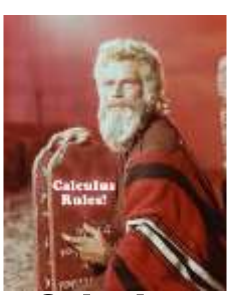
Y-intercept: $P(0) = -5(0)^2 + 10(0) + 11 = 11$ → **Y.I.: (0, 11)**

X-Intercepts: $0 = -5x^2 + 10x + 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{10^2 - 4(-5)(11)}}{2(-5)} = = \frac{-10 \pm \sqrt{100 + 220}}{-10} = = \frac{-10 \pm \sqrt{320}}{-10} = \frac{-10 \pm \sqrt{64 * 5}}{-10}$$

$$\frac{-10 \pm 8\sqrt{5}}{-10} = \frac{-2(5 \pm 4\sqrt{5})}{-10} = \frac{(5 \pm 4\sqrt{5})}{5} = \frac{5}{5} \pm \frac{4\sqrt{5}}{5} = 1 \pm \frac{4\sqrt{5}}{5} \rightarrow X.I.: \left(1 + \frac{4\sqrt{5}}{5}, 0\right) \quad \left(1 - \frac{4\sqrt{5}}{5}, 0\right)$$

(2.8 , 0) **(–0.8 , 0)**

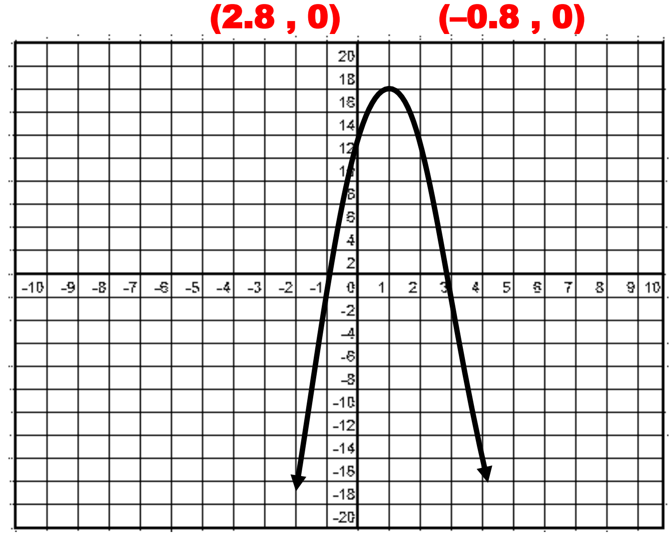


Calculus Prophet

Items to Sell to Maximize Profit: **1 item**

Price to Charge per item: **$26/item**

Maximum Profit: **$16**



How to Graph a Polynomial

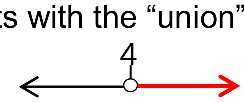
1. Take **FIRST** & **SECOND** derivatives
   (a) Set first derivative = 0 and solve for x.
   (b) Plug in x-value(s) into original function and get y-coordinate(s).
   (c) Plug in **x-value**(s) for each coordinate into second derivative.
      (i) If second derivative is +, then graph "smiles" (min) at that coordinate.
         Note: Ignore the value of the second derivative. We just care if it is '+' or '-'. Nothing else!
      (ii) If second derivative is –, then graph "frowns" (max) at that coordinate.
         Note: Ignore the value of the second derivative. We just care if it is '+' or '-'. Nothing else!
   (d) Set second derivative = 0 and solve for x.
   (e) Plug in x-value(s) into original function and get y-coordinate(s).
      (i) These are the inflection points.
2. Find y-intercept(s) {by setting x = 0 in the original function & solving for y}
3. Find x-intercept(s), if easy (generally only with "even" functions i.e. x^2, x^4, etc.)
   {by setting y = 0 in the original function & solving for x usually by factoring or the quadratic formula}
4. Determine end behavior, asymptotes, etc. (i.e. As x → ∞, y → ?: As x → –∞, y → ?)
   (a) End Behavior - Polynomial Graph:

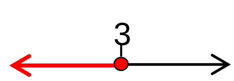
$+x^{even}$	↑↑	As $x \to \infty$	$f(x) \to \infty$
		As $x \to -\infty$	$f(x) \to \infty$
$-x^{even}$	↓↓	As $x \to \infty$	$f(x) \to -\infty$
		As $x \to -\infty$	$f(x) \to -\infty$
$+x^{odd}$	↓↑	As $x \to \infty$	$f(x) \to \infty$
		As $x \to -\infty$	$f(x) \to -\infty$
$-x^{odd}$	↑↓	As $x \to \infty$	$f(x) \to -\infty$
		As $x \to -\infty$	$f(x) \to \infty$

5. Plot all points, end behavior, asymptotes, then graph.

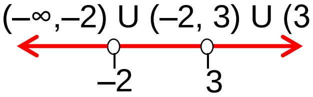
Interval Notation

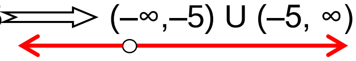
1. Interval Notation: left to right (or down to up). {smallest x, largest x} OR {smallest y, largest y}
   a) > or < or ∞ or –∞ ⟹ round parenthesis: ( )
   b) ≥ or ≤ ⟹ square brackets: [ ]
   c) Join two or more sets with the "union" symbol: U





   d) Examples: 1) x > –4 ⟹ (–4, ∞) 2) x ≤ 3 ⟹ (–∞, 3]
      3) x ≠ –2 & x ≠ 3 ⟹ (–∞,–2) U (–2, 3) U (3, ∞) 4) x ≠ –5 ⟹ (–∞,–5) U (–5, ∞)

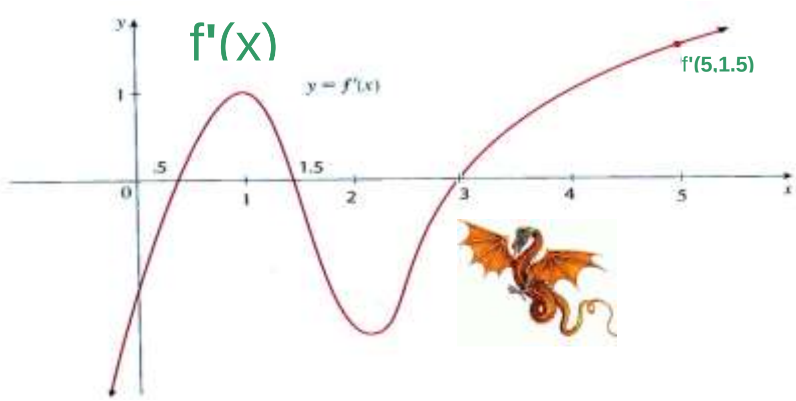


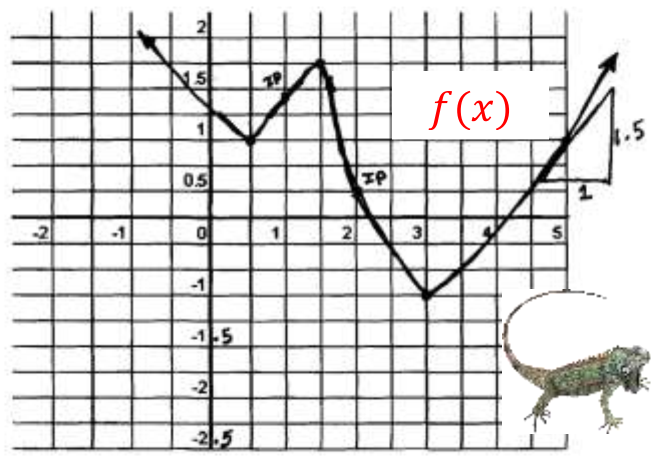


How to Draw a Graph from its Derivative

(How to get an iguana from a dragon shadow)

1. The zeros (x-intercepts) of the graph of f'(x) are the maximum or minimum values of f(x).
   (a) If f'(x) goes from negative to positive, f(x) has a minimum.
   (b) If f'(x) goes from positive to negative, f(x)has a maximum.
2. Wherever f'(x) is negative (i.e. BELOW the x-axis), f(x)is DECREASING (has a negative slope). i.e. the ladybug on f(x) is going "downhill"
3. Wherever f'(x) is positive (i.e. ABOVE the x-axis), f(x)is INCREASING (has a positive slope). i.e. the ladybug on f(x) is going "uphill"
4. Wherever f'(x) has a positive SLOPE i.e. the ladybug on f'(x) is going "uphill" (that is, the second derivative f''(x) is positive), f(x)SMILES (concave up).
5. Wherever f'(x) has a negative SLOPE). i.e. the ladybug on f'(x) is going "downhill" (that is, the second derivative f''(x) is negative), f(x) FROWNS (concave down).
6. Wherever f'(x) (the first derivative) has a maximum or a minimum (i.e. the zeros of the second derivative, f''(x)), the graph f(x) has an INFLECTION POINT.
7. The degree of the FUNCTION itself is TWO more than the # of turning points ("bumps") on the first DERIVATIVE graph, f'(x), that is, one more than the DEGREE of f'(x).
8. The Y-values of the FUNCTION are arbitrary and can NOT be determined from the graph of the first derivative, f'(x).
9. Example:





f(x) increasing (.5, 1.5) U (3, ∞)

f(x) decreasing (–∞, .5) U (1.5, 3)

f(x) relative maximum x = 1.5 {no info on 'y' value}

f(x) relative minimum x = .5 & x = 3 {no info on 'y' value}

f(x) concave up (smiles) (–∞, 1) U (2, ∞)

f(x) concave down (frowns) (1, 2)

f(x) inflection points x = 1 & x = 2 {no info on 'y' value}

slope of f(x) at x = 5 1.5

Note: Y-values are arbitrary. X-values are fixed.

Note: There are 2 turning points ("bumps") on f'(x), therefore the **degree** of the FUNCTION is two more than the bumps on f'(x), that is, 4.

P.S. Just adding 1 to the number of bumps on f(x), as you did in Math 121, is not always a reliable method for finding the degree of f(x). You are better served adding 2 to f'(x), which ALWAYS works.

degree of f(x) 4 {two more than the number of "bumps" on f'(x)}

DRAGON – IGUANA INSTRUCTIONS

- **ALWAYS MOVE LEFT TO RIGHT**
- **IGNORE Y-VALUES**

Quickie 3-Step Summary:
1. Plot maxes and mins.
2. Plot I.P.s.
3. Connect the dots.

STEP 1 – Using the zeros (x-intercepts) of f'(x)–Dragon, Identify the max & min x-values for F(x)–Iguana.

1. When f'(x)–Dragon goes **UP** through the x-intercept, draw a **CUP**.
   a. Having an **UP** day? **SMILE**!
2. When f'(x)–Dragon goes **DOWN** through the x-intercept, draw a **FROWN**.
   a. Having a **DOWN** day? **FROWN**!

In other words:

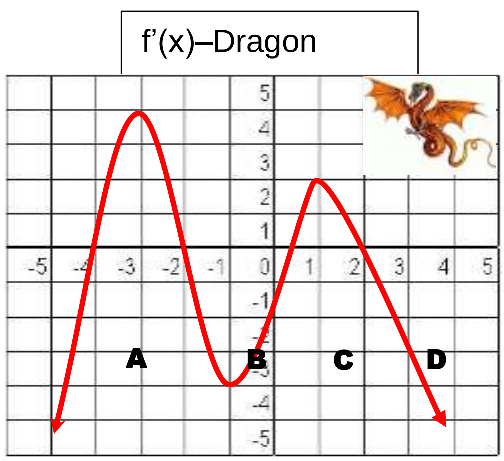
Moving from left to right, Does f'(x)–Dragon go **UP** as it crosses the x-axis?

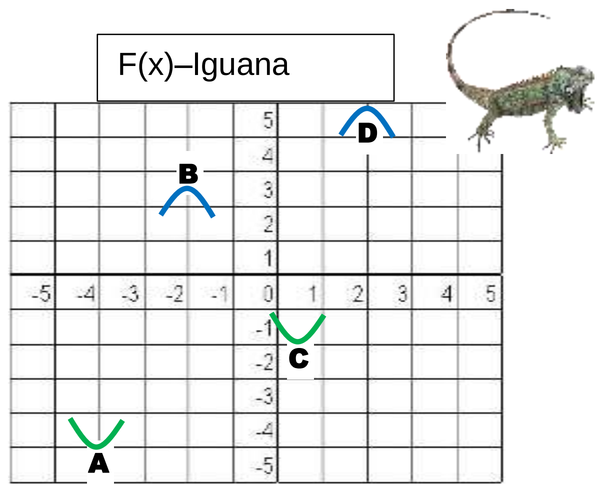
NO → Moving from left to right, Does f'(x)–Dragon go **DOWN** as it crosses the x-axis?

YES (first question) → X-intercept on f'(x)–Dragon is the x-value of the **MINIMUM** on F(x)–Iguana. Draw a **SMILE** at that x-value on your graph of F(x)–Iguana. The y-value is arbitrary → your choice.

YES (second question) → X-intercept on f'(x)–Dragon is the x-value of the **MAXIMUM** on F(x)–Iguana. Draw a **FROWN** at that x-value on your graph of F(x)–Iguana. The y-value is arbitrary → your choice.

EXAMPLE





Refer to the x-intercepts on f'(x)–Dragon →

A. On f'(x)–Dragon, the graph goes **UP** through x = –4, so we **SMILE** at x = –4 on F(x)–Iguana.
B. On f'(x)–Dragon, the graph goes **DOWN** through x = –2, so we **FROWN** at x = –2 on F(x)–Iguana.
C. On f'(x)–Dragon, the graph goes **UP** through x = ½, so we **SMILE** at x = ½ on F(x)–Iguana.
D. On f'(x)–Dragon, the graph goes **DOWN** through x = 2, so we **FROWN** at x = 2 on F(x)–Iguana.

Remember: y-values are arbitrary, so you can put them wherever you want so long as your maxes are above your mins.

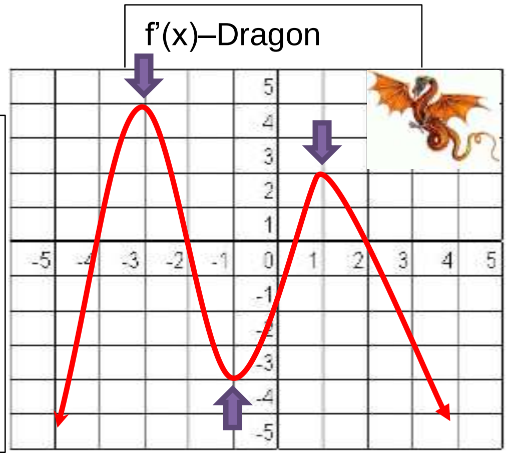
STEP 2 – IDENTIFY THE INFLECTION POINT X-VALUES FOR F(x)–Iguana

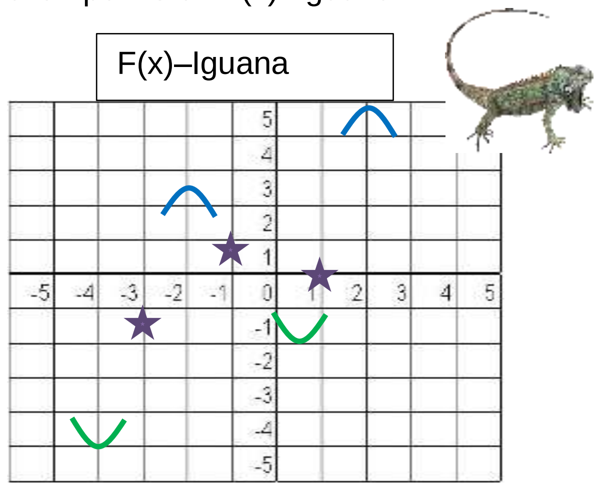
1. The maximums and minimums on f'(x)–Dragon, are the inflection points on F(x)–Iguana.

EXAMPLE

The purple arrows on f'(x)–Dragon indicate the **x-values** of the inflection points (purple stars) on F(x)–Iguana.

Again, as with the max & mins, the y-values for the inflection points are your choice so long as they are between the maxes & mins.

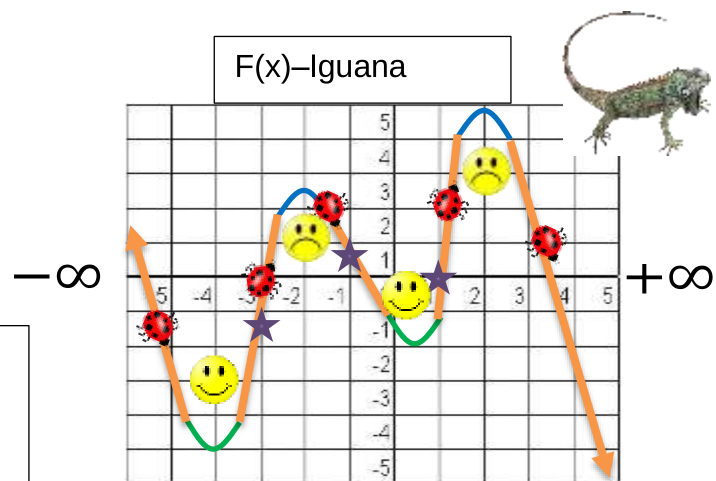




STEP 3 – Connect the dots to complete your F(x)–Iguana

1. Answer all relevant questions regarding F(x)–Iguana.

EXAMPLE



Remember:
Intervals are x-values only!
Moving from left ($-\infty$)
to right ($+\infty$)

1. F(x) decreasing $(-\infty,-4) \cup \left(-2,\frac{1}{2}\right) \cup (2,\infty)$ → Wherever ladybug goes downhill.

   Always increase and decrease between MAX & MIN

2. F(x) increasing $(-4,-2) \cup \left(\frac{1}{2},2\right)$ → Wherever ladybug goes uphill.
3. F(x) relative minimum (x- value only) x = **–4**, x = **½**
4. F(x) relative maximum (x- value only) x = **–2**, x = **2**
5. F(x) concave down (frowns) $(-3,-1) \cup (1,\infty)$ **Always smile & frown between INFLECTION POINTS**
6. F(x) concave up (smiles) $(-\infty,-3) \cup (-1,1)$
7. F(x) inflection points x = **–3**, x = **–1**, x = **1**
8. degree of F(x) $-x^5$ "degree" is **TWO** more than # turning pts. ("bumps") on **f'(x)–Dragon**, not F(x)–Iguana
9. End Behavior: $As \;\; x \to \infty \quad F(x) \to -\infty$

   $x \to -\infty \quad F(x) \to +\infty$

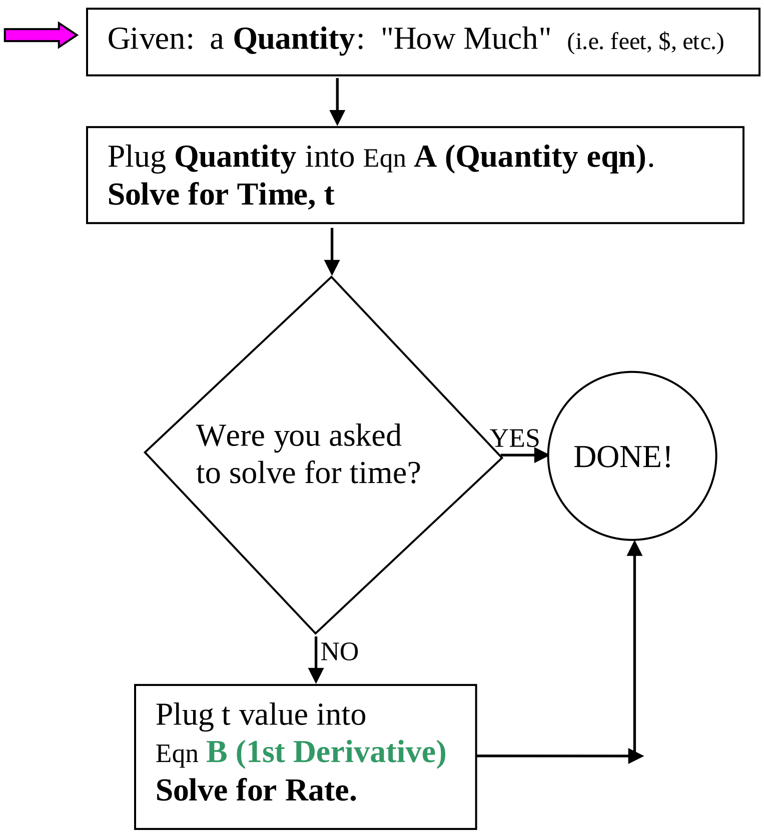
To Find the Rate, DIFFERENTIATE!

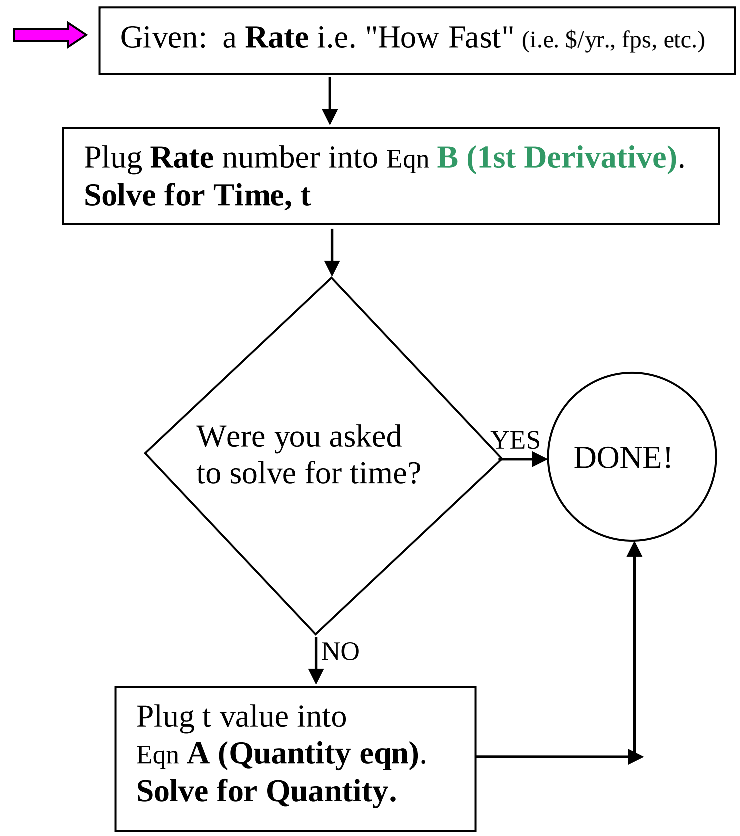
How To Do Height/Velocit y/Temp/Interest Problems

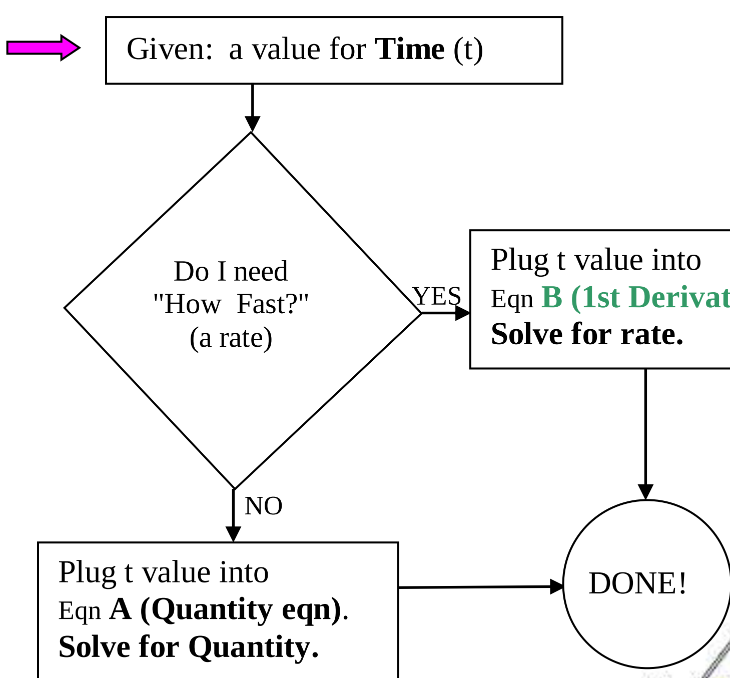
Tips:
1. "On the ground" means Quantity = 0
2. Apex or vertex means Rate = 0
3. Before an event starts, t = 0

Eqn **A**: **QUANTITY**: How many, How much, How far, How high, How hot, etc.
Eqn **B**: **1st Derivative of Eqn. A**: **RATE**, velocity, speed: How Fast, at what rate, etc.

Note: If no explicit value is given for Quantity or Rate, then Time is probably zero (t=0)







Example:
You invest $500 at 3%. At what rate is the balance growing when the amount in the account is $2500?

$A(t) = 500e^{.03t}$

$2500 = 500e^{.03t}$

$\frac{2500}{500} = e^{.03t}$

$ln\left[\frac{2500}{500}\right] = .03t$

53.7 *yrs.*

$A'(t) = 15e^{.03t}$

$A'(53.7) = 15e^{.03(53.7)}$

$A'(53.7) = \$75/yr.$

Faster ("Trevor") Method:
(Amount in the account at time, t) * (interest rate)
$2500 * .03 = $75/yr.
Trevor method **ONLY** works with $A=Pe^{rt}$ problems!

Example:
The height of a meteor in earth's atmosphere is given by: $s(t) = -12t^2 + 24t + 1440$. How fast is it going when it strikes the earth?

$s(t) = -12t^2 + 24t + 1440$
when hits earth height is zero
$0 = -12(t^2 - 2t - 120)$
$0 = -12(t - 12)(t + 10)$
12 seconds = t
–10 seconds = t *Silly Answer (neg. time)*

$s'(t) = -24t + 24$
$s'(12) = -24(12) + 24$
$s'(12) = -264$ fps
negative because meteor is heading **DOWN**