Supplement #10

1. $y = x^3 - 6x^2 - 15x + 50$

$y' = 3x^2 - 12x - 15$

$0 = 3(x^2 - 4x - 5)$

$0 = (x + 1)(x - 5)$

$0 = (x + 1)$ & $0 = (x - 5)$

$-1 - 6 + 15 + 50$ $\quad$ $125 - 150 - 75 + 50$

MAX: $(-1, 58)$ & MIN:$(5, -50)$

-6-12 => - Frown $\quad$ 30-12 = > + Smile

$y'' = 6x - 12$

$0 = 6(x - 2) \rightarrow 0 = x - 2$

$8 - 24 - 30 + 50$

IP: $(2, 4)$

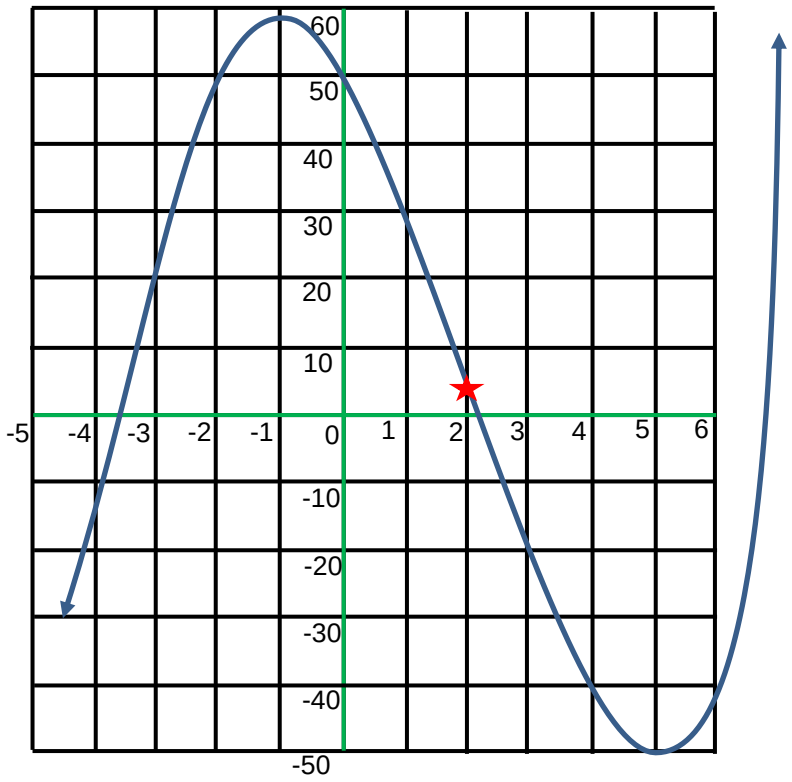
Y.I: $(0,50)$

E.B.: As $x \rightarrow +\infty$ $\quad$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $\quad$ $f(x) \rightarrow -\infty$

$\uparrow (-\infty,-1) \cup (5, +\infty)$ $\quad$ $\downarrow (-1,5)$

Frown: $(-\infty,2)$ $\quad$ Smile: $(2,+\infty)$



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2. $y = x^3 - 6x^2 + 40$

$y' = 3x^2 - 12x$

$0 = 3x(x - 4)$

$0 = 3x$ & $0 = (x - 4)$

$0 - 0 + 40$ $\quad$ $64 - 96 + 40$

MAX: $(0, 40)$ & MIN:$(4,8)$

0-12 => - Frown $\quad$ 24-12 = > + Smile

$y'' = 6x - 12$

$0 = 6(x - 2) \rightarrow 0 = x - 2$

$8 - 24 + 40$

IP: $(2, 24)$

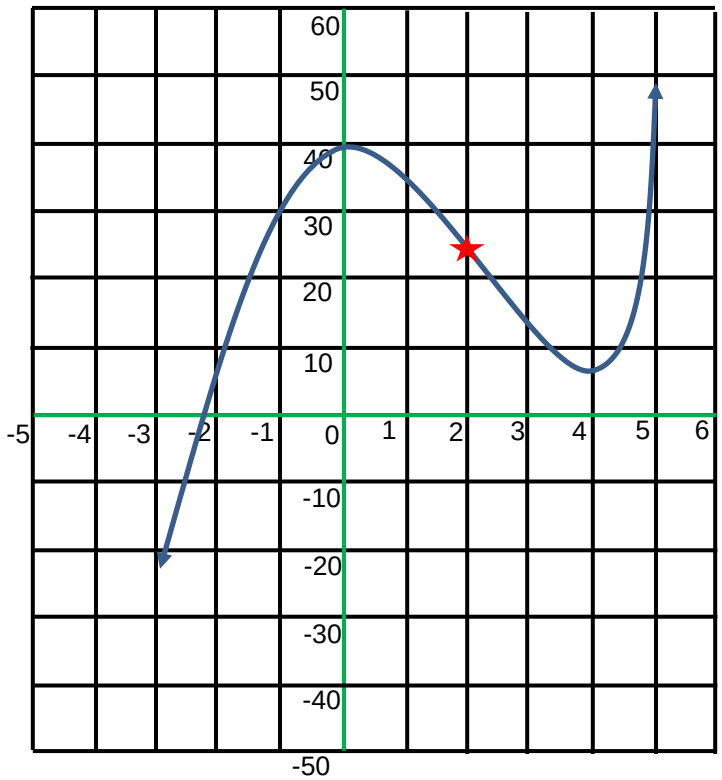
Y.I: $(0,40)$

E.B.: As $x \rightarrow +\infty$ $\quad$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $\quad$ $f(x) \rightarrow -\infty$

$\downarrow (0,4)$ $\quad$ $\uparrow (-\infty,0) \cup (4, +\infty)$

Frown: $(-\infty,2)$ $\quad$ Smile: $(2,+\infty)$



Supplement #10 (continued)

3. $y = 100 + 36x - 6x^2 - x^3$

$y' = 36 - 12x - 3x^2$

$0 = 3(12 - 4x - x^2)$

$0 = (6 + x)(2 - x)$

$0 = (6 + x)$ & $0 = (2 - x)$

100 +216 –216 –216 100 +72 – 24 – 8

MIN: (–6, –116) & MAX: (2, 140)

-12+36 => + Smile -12 -12 = > - Frown

$y'' = -12 - 6x$

$0 = -6(2 + x) \rightarrow 0 = 2 + x$

100 –72 – 24 + 8

IP: (–2 , 12)

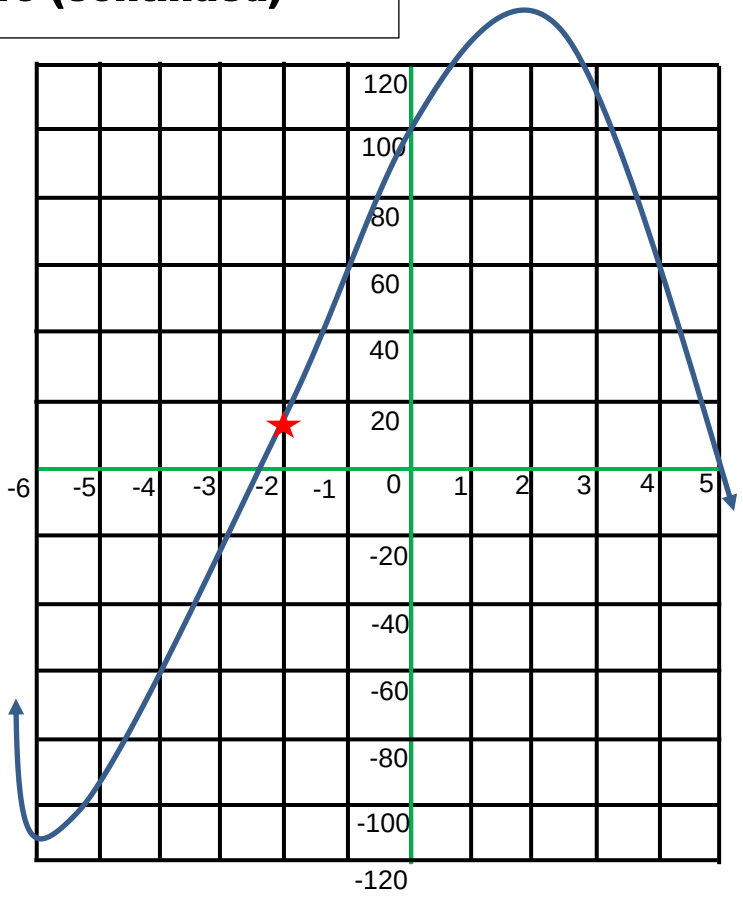
Y.I: (0,100)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$

↓ $(-\infty,-6) \cup (2, +\infty)$ ↑ $(-6,2)$

Smile : $(-\infty,-2)$ Frown: $(-2,+\infty)$



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4. $y = x^3 - 3x^2 - 9x + 7$

$y' = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = (x - 3)(x + 1)$

$0 = (x - 3)$ & $0 = (x + 1)$

27 – 27 – 27 +7 -1 -3 +9 +7

MIN: (3, –20) & MAX:(–1,12)

18 - 6 => + Smile -6 - 6 = > - Frown

$y'' = 6x - 6$

$0 = 6(x - 1) \rightarrow 0 = x - 1$

1 – 3 -9 +7

IP: (1 , –4)

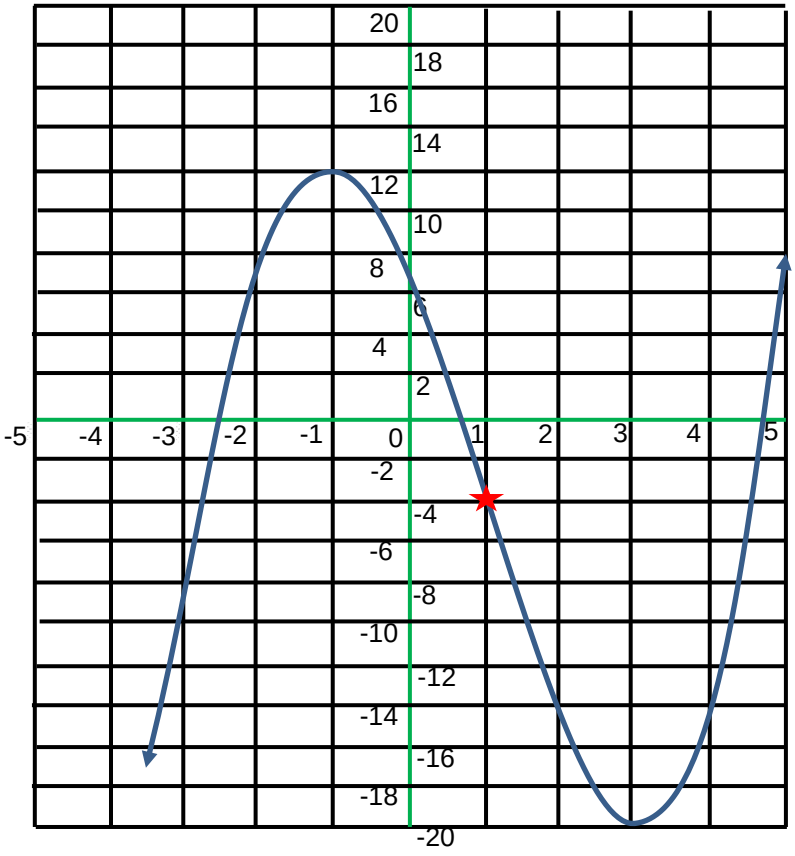
Y.I: (0,7)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$

↓$(-1,3)$ ↑$(-\infty,-1) \cup (3, +\infty)$

Frown: $(-\infty,1)$ Smile: $(1,+\infty)$



Supplement #10 (continued)

5. $y = x^3 - 3x + 1$

$y' = 3x^2 - 3$

$0 = 3(x^2 - 1)$

$0 = (x + 1)(x - 1)$

$0 = (x + 1)$ & $0 = (x - 1)$

$-1 + 3 + 1$ $\quad$ $1 - 3 + 1$

MAX: $(-1, 3)$ & MIN:$(1, -1)$

-6=> - Frown $\quad$ 6 = > + Smile

$y'' = 6x$

$0 = 6x \rightarrow \ 0 = x$

$0 - 0 + 1$

IP: $(0 , 1)$

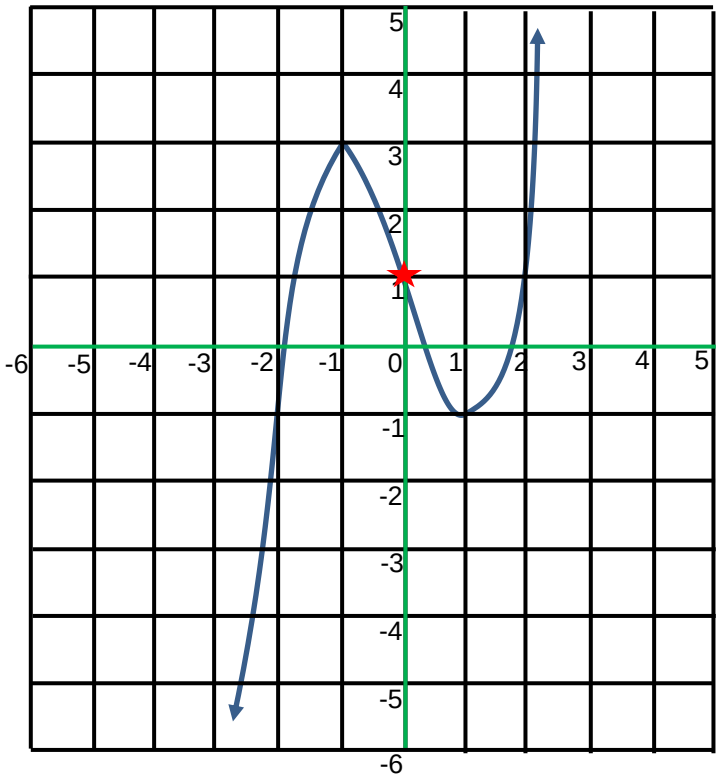
Y.I: $(0,1)$

E.B.: As $x \rightarrow +\infty$ $\quad f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $\quad f(x) \rightarrow -\infty$

$\uparrow (-\infty,-1) \cup (1, +\infty)$ $\quad \downarrow (-1,1)$

Frown: $(-\infty,0)$ $\quad$ Smile: $(0,+\infty)$



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6. $y = -2x^3 + 3x^2 + 12x + 2$

$y' = -6x^2 + 6x + 12$

$0 = -6(x^2 - x - 2)$

$0 = (x - 2)(x + 1)$

$0 = (x - 2)$ & $0 = (x + 1)$

$-16 + 12 + 24 + 2$ $\quad$ $2 + 3 - 12 + 2$

MAX: $(2, 22)$ & MIN:$(-1,-5)$

-24 + 6 => - Frown $\quad$ 12 + 6 = > + Smile

$y'' = -12x + 6$

$0 = -6(2x - 1) \rightarrow \ 0 = 2x - 1$

$-1/4 + 3/4 + 6 + 2$

IP: $(\frac{1}{2} , 8\frac{1}{2})$

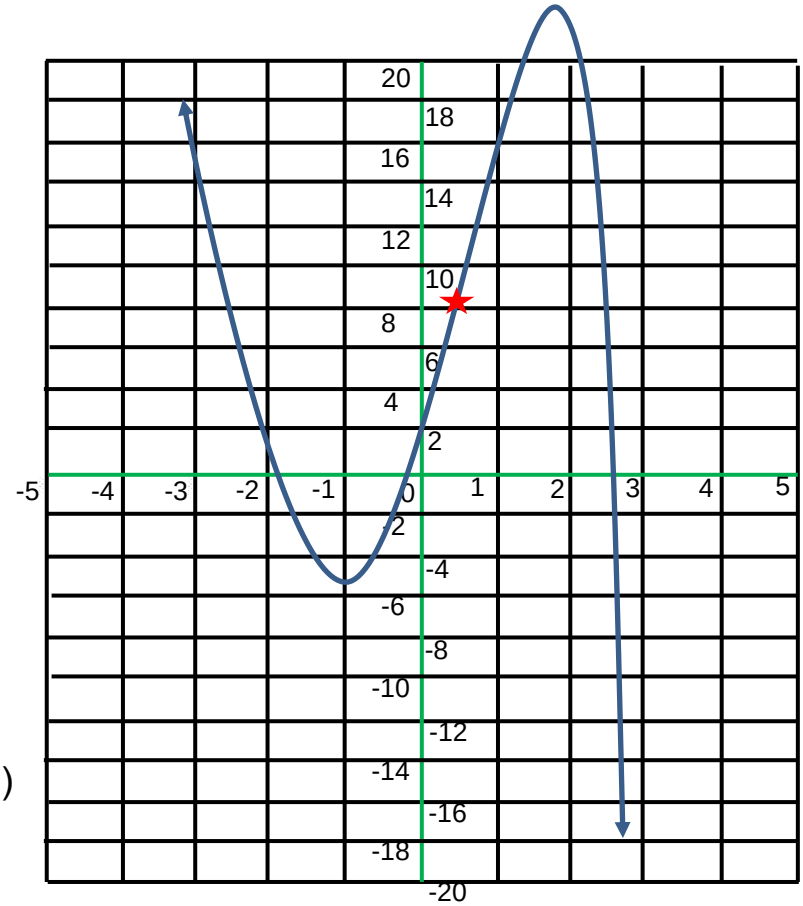
Y.I: $(0,2)$

E.B.: As $x \rightarrow +\infty$ $\quad f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$ $\quad f(x) \rightarrow +\infty$

$\uparrow (-1,2)$ $\quad \downarrow (-\infty,-1) \cup (2, +\infty)$

Smile: $(-\infty,\frac{1}{2})$ $\quad$ Frown: $(\frac{1}{2},+\infty)$



Supplement #10 (continued)

7. $y = 2x^3 + 3x^2 + 1$

$y' = 6x^2 + 6x$

$0 = 6x(x + 1)$

$0 = 6x$ & $0 = (x + 1)$

0 + 0 + 1 –2 + 3 + 1

MIN: (0, 1) & MAX:(–1, 2)

0+6 => + Smile -12 + 6 = > - Frown

$y'' = 12x + 6$

$0 = 6(2x + 1) \rightarrow 0 = 2x + 1$

–1/4 +3/4 + 1

IP: (–½ , 1½)

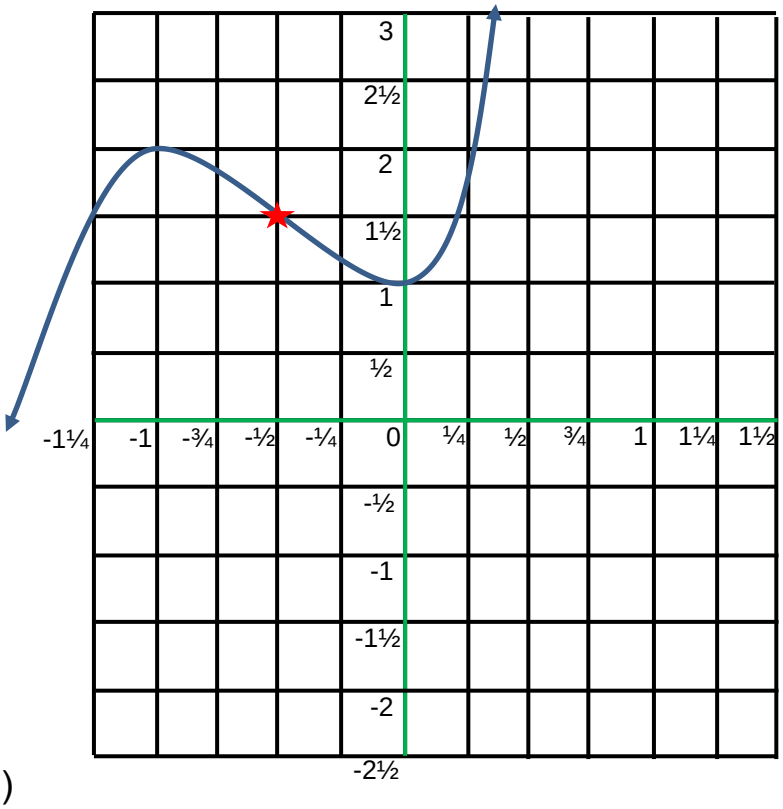
Y.I: (0,1)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$

↑ $(-\infty,-1) \cup (0, +\infty)$ ↓ $(-1,0)$

Frown: $(-\infty,-½)$ Smile: $(-½,+\infty)$



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8. $y = 6 + 3x - x^2 - \frac{1}{3}x^3$

$y' = 3 - 2x - x^2$

$0 = (3 + x)(1 - x)$

$0 = (3 + x)$ & $0 = (1 - x)$

6 – 9 -9 +9 6 + 3 – 1 – 1/3

MIN: (–3, –3) & MAX:$(1, 7\frac{2}{3})$

-2 + 6 => + Smile -2 - 2 = > - Frown

$y'' = -2 - 2x$

$0 = -2(1 + x) \rightarrow 0 = 1 + x$

6 – 3 - 1 +1/3

IP: $(-1, 2\frac{1}{3})$

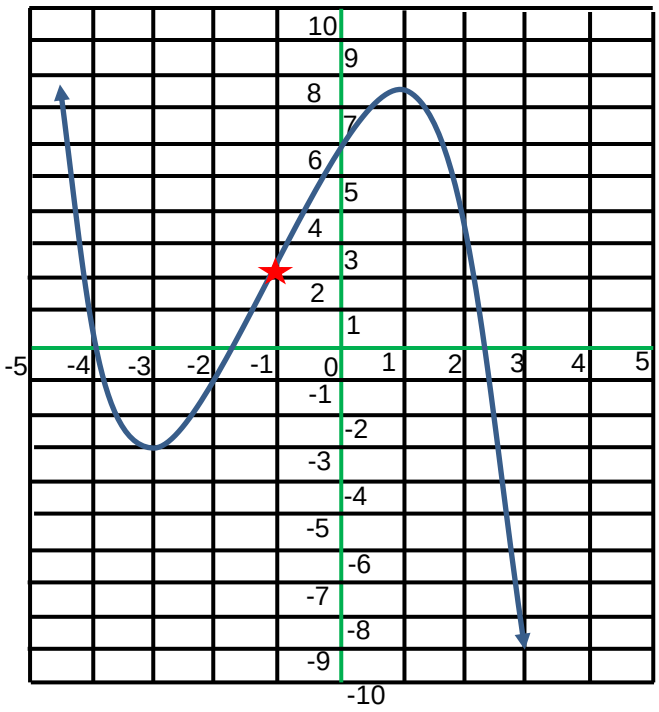
Y.I: (0,6)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$

↑ (–3,1) ↓ $(-\infty,-3) \cup (1, +\infty)$

Smile: $(-\infty,-1)$ Frown: $(-1,+\infty)$



Supplement #10 (continued)

9. $y = x^3 + 3x^2 - 24x$

$y' = 3x^2 + 6x - 24$

$0 = 3(x^2 + 2x - 8)$

$0 = (x + 4)(x - 2)$

$0 = (x + 4)$ & $0 = (x - 2)$

-64 + 48 + 96 8 + 12 -48

MAX: (–4,80) & MIN:(2,–28)

-24+6 => - Frown 12 + 6 = > + Smile

$y'' = 6x + 6$

$0 = 6(x + 1) \rightarrow 0 = x + 1$

–1 +3 + 24

IP: (–1 , 26)

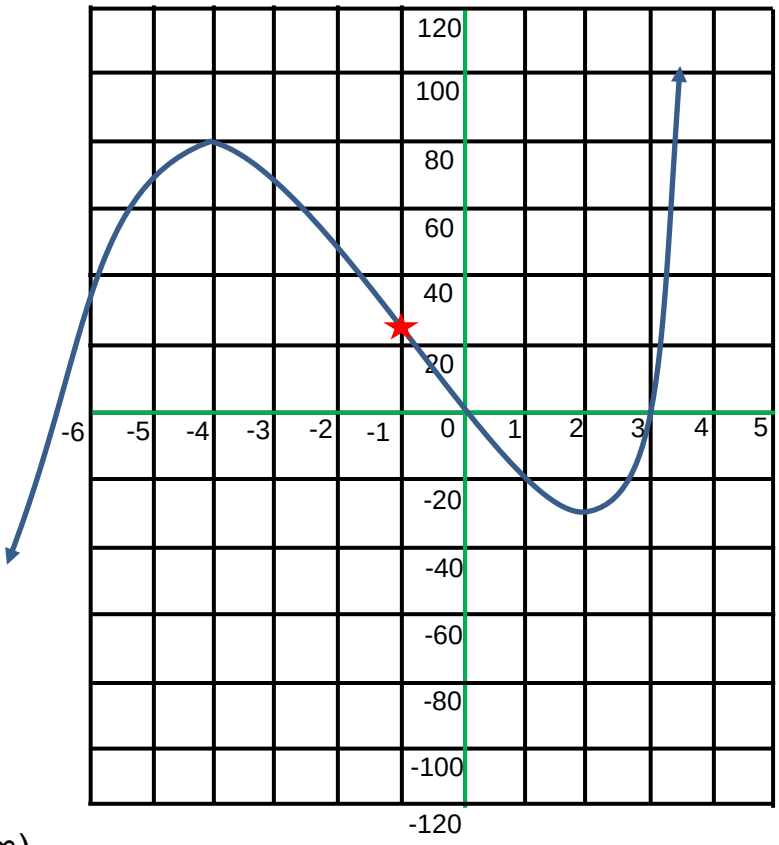
Y.I: (0,0)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$

↑ $(-\infty,-4) \cup (2, +\infty)$ ↓ $(-4,2)$

Frown: $(-\infty,-1)$ Smile: $(-1,+\infty)$



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10. $y = -2x^3 + 9x^2 + 60x - 200$

$y' = -6x^2 + 18x + 60$

$0 = -6(x^2 - 3x - 10)$

$0 = (x - 5)(x + 2)$

$0 = (x - 5)$ & $0 = (x + 2)$

-250+225+300-200 16 + 36 – 120 – 200

MAX: (5, 75) & MIN:(–2,–268)

-60 + 18 => - Frown 24 +18 = > + Smile

$y'' = -12x + 18$

$0 = -6(2x - 3) \rightarrow 0 = 2x - 3$

-27/4 +81/4 + 90 -200

IP: (1½,–96½)

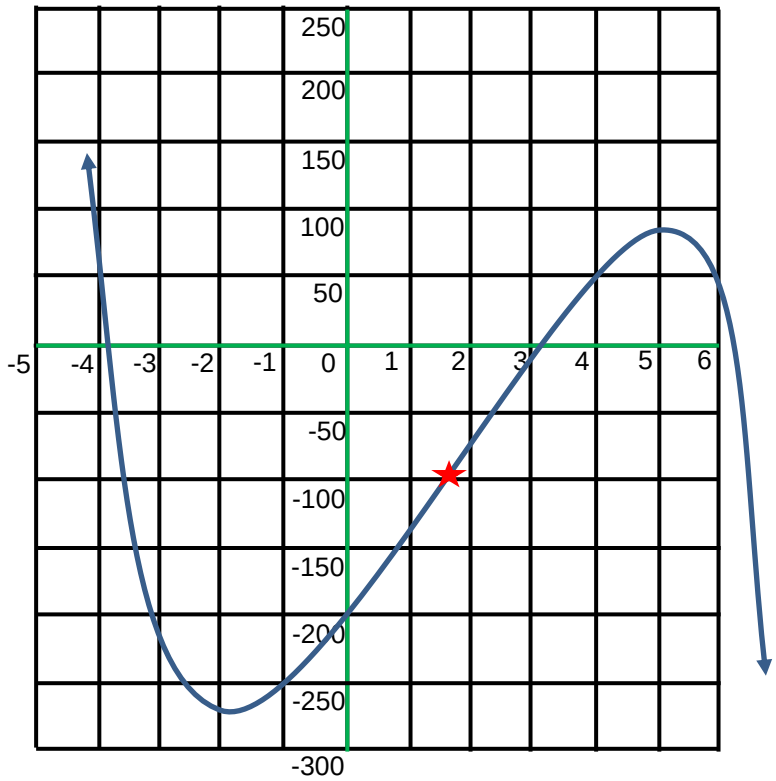
Y.I: (0,–200)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow -\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$

↑ $(-2,5)$ ↓ $(-\infty,-2) \cup (5, +\infty)$

Smile: $(-\infty,1\frac{1}{2})$ Frown: $(1\frac{1}{2},+\infty)$



Supplement #10 (continued)

11. $y = 7 - 36x + 3x^2 + 2x^3$

$y' = -36 + 6x + 6x^2$

$0 = -6(6 - x - x^2)$

$0 = (3 + x)(2 - x)$

$0 = (3 + x)$ & $0 = (2 - x)$

7 + 108 + 27 – 54 7 – 72 + 12 + 16

MAX: (–3, 88) & MIN:(2, –37)

6 – 36 => - Frown 6 + 24 = > + Smile

$y'' = 6 + 12x$

$0 = 6(1 + 2x) \rightarrow 0 = 1 + 2x$

7 +18 +3/4 – 1/4

IP: (–½ , 25½)

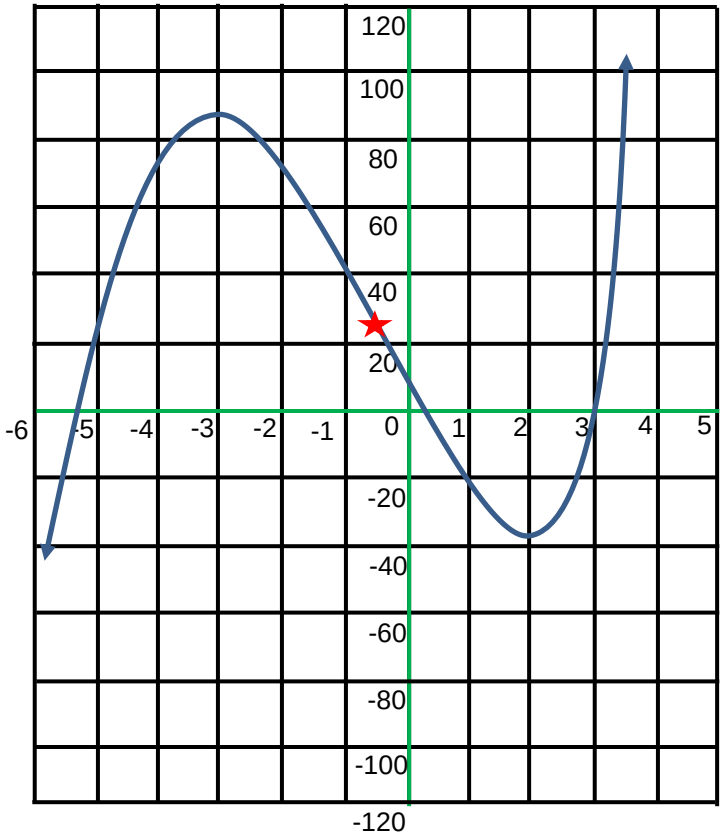
Y.I: (0,7)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$

↑ $(-\infty,-3) \cup (2, +\infty)$ ↓ $(-3,2)$

Frown: $(-\infty,-½)$ Smile: $(-½,+\infty)$



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12. $y = 3x^3 - 9x^2 - 27x + 5$

$y' = 9x^2 - 18x - 27$

$0 = 9(x^2 - 2x - 3)$

$0 = (x - 3)(x + 1)$

$0 = (x - 3)$ & $0 = (x + 1)$

81 – 81 – 81 + 5 -3 – 9 + 27 + 5

MIN: (3, –76) & MAX:(–1,20)

54 – 18 => + Smile -18 - 18 = > - Frown

$y'' = 18x - 18$

$0 = 18(x - 1) \rightarrow 0 = x - 1$

3 – 9 – 27 + 5

IP: (1,–28)

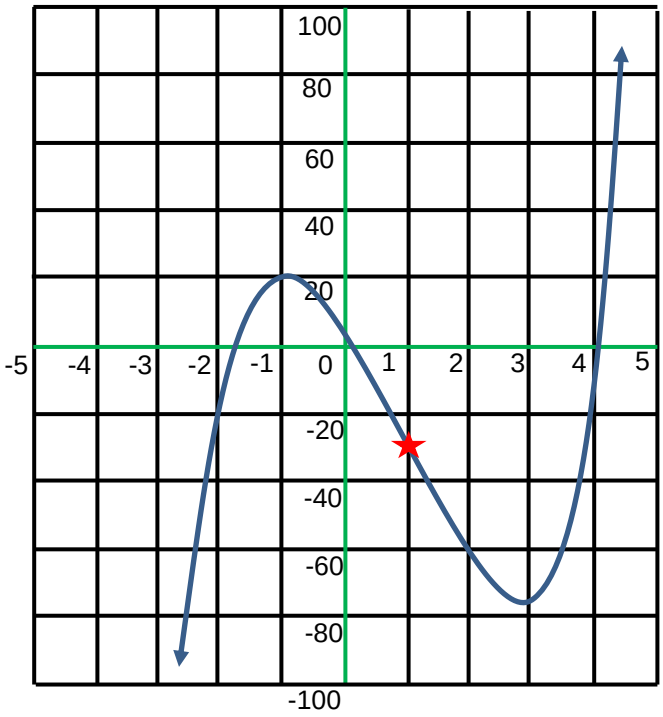
Y.I: (0,5)

E.B.: As $x \rightarrow +\infty$ $f(x) \rightarrow +\infty$

As $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$

↓ $(-1,3)$ ↑ $(-\infty,-1) \cup (3, +\infty)$

Frown: $(-\infty,1)$ Smile: $(1,+\infty)$



Supplement #10 (continued)

13. $y = -x^2 + 2x + 3$

$y' = -2x + 2$

$0 = -2(x - 1)$

$0 = (x - 1)$

-1 + 2 + 3

MAX: (1, 4) ⇐ **VERTEX**

$y'' = -2$

No I.P. Since y" is always –, function always frowns

Y.I: (0,3)

X.I: (3,0) & (–1,0)

$0 = -x^2 + 2x + 3$

$0 = -(x^2 - 2x - 3)$

$0 = (x - 3)(x + 1)$

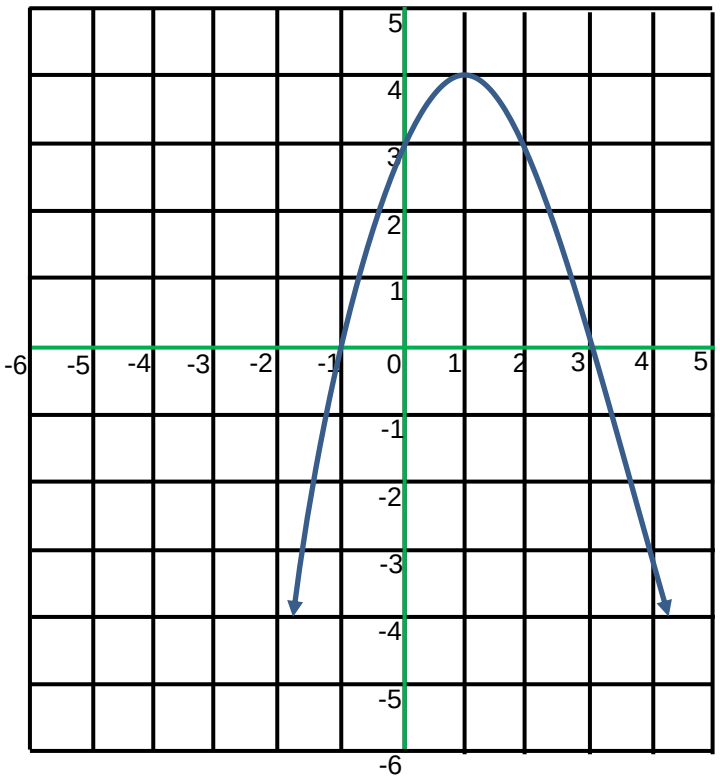
Note: If function doesn't factor, quadratic formula must be used to find X.I.

E.B.: As $x \to +\infty$ $f(x) \to -\infty$

As $x \to -\infty$ $f(x) \to -\infty$

↑ $(-\infty,1)$ ↓ $(1, +\infty)$

Frown: $(-\infty,+\infty)$



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14. $y = x^4 - 4x^3$

$y' = 4x^3 - 12x^2$

$0 = 4x^2 (x - 3)$

$0 = 4x^2$ & $0 = (x - 3)$

0 – 0 81 -108

(0, 0) & MIN:(3,–27)

(0,0) is an I.P. 108-72 = > + Smile

$y'' = 12x^2 - 24x$

$0 = 12x(x - 2)$

$0 = 12x$ & $0 = x - 2$

0 – 0 16 -32

IPs: (0 , 0) & (2,–16)

Y.I: (0,0)

E.B.: As $x \to +\infty$ $f(x) \to +\infty$

As $x \to -\infty$ $f(x) \to +\infty$

↑ $(3, +\infty)$ ↓ $(-\infty,3)$ $(2 - x)$

Smile: $(-\infty,0) \cup (2, +\infty)$ Frown: (0,2)

Note:
I.P. supersedes max or min calculation