Supplement 12
Implicit Differentiation and the Chain Rule

1. An offshore oil well is leaking oil onto the ocean surface, forming a circular oil slick about .005 meters thick. If the radius of the slick is r meters, then the volume of oil spilled is $\mathbf{V = .005\pi r^2}$ cubic meters. Suppose that the oil is leaking at **a constant rate of** 20 cubic meters per hour. Find the rate at which the radius of the oil slick is increasing at a time when the radius is 50 meters.

2. Animal physiologists have determined experimentally that the weight, W (in kilograms), and the surface area, S (in square meters) of a typical horse are related by the empirical equation, $S = 0.1W^{\frac{2}{3}}$. How fast is the surface area of a horse increasing at a time when the horse weighs 350 kg and is gaining weight **at the rate of** 200 kg per year?

3. Suppose that a tiny, experimental, kitchen appliance company's monthly sales and advertising expenses are approximately related by the equation, **$xy - 6x + 20y = 0$**, where x is dollars spent on advertising and y is dishwashers sold. Currently, the company is spending $10 dollars on advertising and is selling 2 dishwashers. If the company plans to increase monthly advertising expenditures **at the rate of** $1.50 per month, how fast will sales rise?

4. Suppose that the price, p (in dollars), and the weekly sales, x (in thousands of units), of a certain commodity satisfy the demand equation, **$2p^3 + x^2 = 4500$**. Determine the rate at which sales are changing at a time when x = 50, p = 10, and the price is **falling** **at the rate of** $.50 per week.

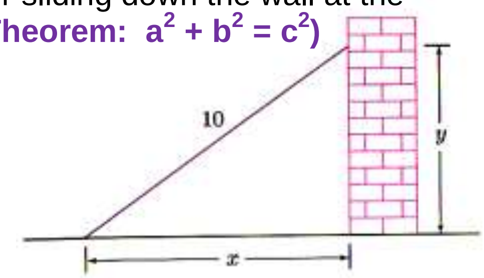
5. Suppose that the price, p (in dollars), and the demand, x (in thousands of units), of a commodity satisfy the demand equation, **$6p + x + xp = 94$**. How fast is the demand changing at a time when x = 4, p = 9, and the price is rising **at the rate of** $2/week?

6. Suppose that in Boston, the wholesale price, p, of oranges (in dollars per crate), and the daily supply, x crates, are related by the equation, **$px + 7x + 8p = 328$**. If there are 4 crates available today at a price of $25, and if the supply is changing **at the rate of** $-\mathbf{0.3}$ crates per day, at what rate is the price changing?

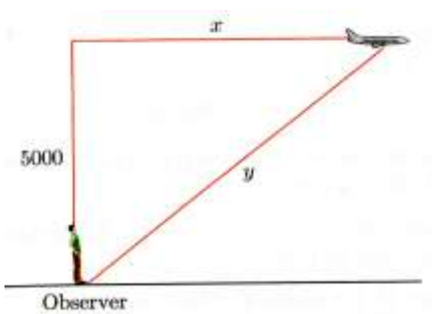
7. Under certain conditions (called adiabatic expansion) the pressure, P, and the volume, V, of a gas such as oxygen satisfy the equation, $P^5V^7 = k$, where k is a constant. Suppose that at some moment, the volume of the gas is 4 liters, the pressure is 200 units, and the pressure is increasing **at the rate of** 5 units per second. Find the (time) rate at which the volume is changing.

8. The volume, V, of a spherical cancer tumor is given by, $V = \frac{\pi x^3}{6}$, where x is the diameter of the tumor. A physician estimates that the diameter is growing **at the rate of** 0.4 mm per day, at a time when the diameter is already 10 mm. How fast is the volume of the tumor changing at that time?

9. Referring to the accompanying diagram, suppose that the foot of a 10' ladder is being pulled along the ground that **the rate of** 3 ft./sec. How fast is the top end of the ladder sliding down the wall at the time when the foot of the ladder is 8 feet from the wall? **(Pythagorean Theorem: $a^2 + b^2 = c^2$)**



10. An airplane flying at 390 ft./sec. $\left(\frac{dx}{dt}\right)$ at an altitude of 5000 feet flew directly over an observer. Referring to the accompanying diagram, how fast is the distance from the observer to the airplane $\left(\frac{dy}{dt}\right)$ changing at the time when the airplane is 13,000 feet (y = 13,000) from the observer?



Answers

1. $\frac{40}{\pi}$ m/hr.
2. 1.89 m^2/yr.
3. $\frac{1}{5}$ dishwasher/month (sales will rise by 20%)
4. 3 thousand units/week
5. – 2 thousand units/week
6. $.80/day
7. –0.07 liters/sec or DECREASING at $\frac{1}{14}$ liters/sec
8. 20 π mm/day
9. –4 ft./sec. (negative means sliding DOWN)
10. 360 ft./sec.