Supplement #2

23. Refer to Fig. 20.

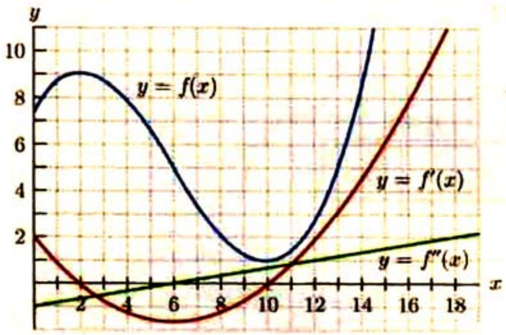

Figure 20

(a) Looking at the graph of $f'(x)$, determine whether $f(x)$ is increasing or decreasing at $x = 9$. Look at the graph of $f(x)$ to confirm your answer.

(b) Looking at the values of $f'(x)$ for $1 \leq x < 2$ and $2 < x \leq 3$, explain why the graph of $f(x)$ must have a relative maximum at $x = 2$. What are the coordinates of the relative maximum point?

(c) Looking at the values of $f'(x)$ for x close to 10, explain why the graph of $f(x)$ has a relative minimum at $x = 10$.

(d) Looking at the graph of $f''(x)$, determine whether $f(x)$ is concave up or concave down at $x = 2$. Look at the graph of $f(x)$ to confirm your answer.

(e) Looking at the graph of $f''(x)$, determine where $f(x)$ has an inflection point. Look at the graph of $f(x)$ to confirm your answer. What are the coordinates of the inflection point?

(f) Find the x-coordinate of the point on the graph of $f(x)$ at which $f(x)$ is increasing at the rate of 6 units per unit change in x.

Exercises 25–36 refer to Fig. 22, which contains the graph of $f'(x)$, the derivative of the function $f(x)$.

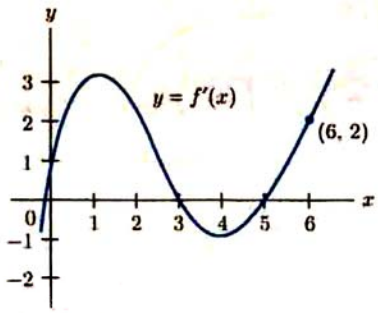

Figure 22

25. Explain why $f(x)$ must be increasing at $x = 6$.

26. Explain why $f(x)$ must be decreasing at $x = 4$.

27. Explain why $f(x)$ has a relative maximum at $x = 3$.

28. Explain why $f(x)$ has a relative minimum at $x = 5$.

29. Explain why $f(x)$ must be concave up at $x = 0$.

30. Explain why $f(x)$ must be concave down at $x = 2$.

31. Explain why $f(x)$ has an inflection point at $x = 1$.

32. Explain why $f(x)$ has an inflection point at $x = 4$.

33. If $f(6) = 3$, what is the equation of the tangent line to the graph of $y = f(x)$ at $x = 6$?

34. If $f(6) = 8$, what is an approximate value of $f(6.5)$?

Answers:

23. (a) ↓ (b) $1 \leq x < 2$ + $2 < x \leq 3$ − $\therefore$ Max @ $x = 2$ $(2, 9)$ (c) $f'(x)$ switches from − to +

(d) $f''(x)$ is − $\therefore$ ∩ (e) $x = 6$ $(6, 5)$ (f) 15

25. $f'(6)$ + 27. $f'(3)$ goes + to − 29. $f''(0)$ is + $\therefore$ ∪ 31. $f''(1) = 0$

33. $y - 3 = 2(x - 6)$ 35. $y - 3 = x$ $f(.25) \approx 3.25$

26. $f'(4) < 0$ 28. $f'(5) = 0$ & $f''(5) > 0$ 30. $f''(2) < 0$ 32. $f''(4) = 0$

34. $f(6.5) = 9$ 35. $f(.25) = 3.25$ 36. $f'(0) = 1$; $y - 3 = 1(x - 0)$ $\boxed{y = x + 3}$