

Supplement #5

Simplify the following expressions.

1. $\ln 5 + \ln x$
2. $\ln x^5 - \ln x^3$
3. $\frac{1}{2} \ln 9$
4. $3 \ln \frac{1}{2} + \ln 16$
5. $\ln 4 + \ln 6 - \ln 12$
6. $\ln 2 - \ln x + \ln 3$
7. $e^{2 \ln x}$
8. $\frac{3}{2} \ln 4 - 5 \ln 2$
9. $5 \ln x - \frac{1}{2} \ln y + 3 \ln z$
10. $e^{\ln x^2 + 3 \ln y}$
11. $\ln x - \ln x^2 + \ln x^4$
12. $\frac{1}{2} \ln xy + \frac{3}{2} \ln \frac{x}{y}$
13. Which is larger, $2 \ln 5$ or $3 \ln 3$?
14. Which is larger, $\frac{1}{2} \ln 16$ or $\frac{1}{3} \ln 27$?

19. Which of the following is the same as $4 \ln 2x$?

- (a) $\ln 8x$ (b) $8 \ln x$
 (c) $\ln 8 + \ln x$ (d) $\ln 16x^4$

20. Which of the following is the same as $\ln(9x) - \ln(3x)$?

- (a) $\ln 6x$ (b) $\ln(9x)/\ln(3x)$
 (c) $6 \cdot \ln(x)$ (d) $\ln 3$

21. Which of the following is the same as $\frac{\ln 8x^2}{\ln 2x}$?

- (a) $\ln 4x$ (b) $4x$
 (c) $\ln 8x^2 - \ln 2x$ (d) none of these

22. Which of the following is the same as $\ln 9x^2$?

- (a) $2 \cdot \ln 9x$ (b) $3x \cdot \ln 3x$
 (c) $2 \cdot \ln 3x$ (d) none of these

Solve the given equation for x .

23. $\ln x - \ln x^2 + \ln 3 = 0$ 24. $\ln \sqrt{x} - 2 \ln 3 = 0$
 25. $\ln x^4 - 2 \ln x = 1$ 26. $\ln x^2 - \ln 2x + 1 = 0$
 27. $\ln \sqrt{x} = \sqrt{\ln x}$ 28. $3 \ln x - \ln 3x = 0$
 29. $(\ln x)^2 - 1 = 0$ 30. $2(\ln x)^2 + \ln x - 1 = 0$
 31. $\ln(x+1) - \ln(x-2) = 1$
 32. $\ln[(x-3)(x+2)] - \ln(x+2)^2 - \ln 7 = 0$

Differentiate.

33. $y = \ln[(x+5)(2x-1)(4-x)]$
 34. $y = \ln[(x+1)(2x+1)(3x+1)]$
 35. $y = \ln[(1+x)^2(2+x)^3(3+x)^4]$
 36. $y = \ln[e^{2x}(x^3+1)(x^4+5x)]$
 37. $y = \ln \left[\frac{e^{5x}(x+4)(3x-2)}{x+1} \right]$
 38. $y = \ln \left[\frac{\sqrt{x}(x+1)^2(x+2)^3}{4x+1} \right]$
 39. $y = \ln \left[\frac{(5x+1)(4x+1)(\ln x)}{\sqrt{2x+1}} \right]$
 40. $y = \ln \left[\frac{x^5 e^{4x} \sqrt{3x+1}}{1-x^2} \right]$
 41. $y = \ln(3x+1) \ln(5x+1)$ 42. $y = (\ln 4x)(\ln 2x)$

Use logarithmic differentiation to differentiate the following functions.

43. $f(x) = (x+1)^4(4x-1)^2$ 44. $f(x) = e^x(3x-4)^8$
 45. $f(x) = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}$
 46. $f(x) = \frac{(x-2)^3(x-3)^4}{(x+4)^5}$ 47. $f(x) = 2^x$
 48. $f(x) = 10^x$ 49. $f(x) = x^x$ 50. $f(x) = x^{1/x}$

Answers: Supplement #5

2. $\ln x^2$ 4. $\ln 2$ 6. $\ln \frac{6}{x}$ 8. $\ln \frac{1}{4}$ 10. $x^2 y^3$

12. $\ln(\sqrt{xy} \cdot \frac{\sqrt{x}}{y\sqrt{y}}) = \ln \frac{x^2}{y}$ 14. $\ln 4 > \ln 3$

20. $\ln(9x) - \ln(3x) = \ln \frac{9x}{3x} = \boxed{\ln 3} \text{ (D)}$

22. $\ln 9x^2 = \ln (3x)^2 = \boxed{2 \ln 3x} \text{ (C)}$

24. $\ln \sqrt{x} = \ln 9$; $\boxed{x=81}$ 26. $\ln \frac{x}{2} = -1$; $\frac{1}{e} = \frac{x}{2}$; $\boxed{x = \frac{2}{e}}$

28. $\ln \frac{x^3}{3x} = 0$; $1 = \frac{x^2}{3}$; $\boxed{x = \pm \sqrt{3}}$ $\ln(-\sqrt{3})$ undef.

30. $2a^2 + a - 1 = 0$; $(2a-1)(a+1) = 0$; $a = \frac{1}{2}$, $a = -1$
 $\ln x = \frac{1}{2}$; $\boxed{x = \sqrt{e}}$ $\ln x = -1$; $\boxed{x = \frac{1}{e}}$

32. $\ln \frac{(x-3)(x+2)}{7(x+2)^2} = 0$; $1 = \frac{x-3}{7(x+2)}$; $7x+14 = x-3$; $6x = -17$; $\boxed{x = -\frac{17}{6}}$

34. $y = \ln[(x+1)(2x+1)(3x+1)] = \ln(x+1) + \ln(2x+1) + \ln(3x+1)$

$$y' = \frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1}$$

36. $y = \ln[e^{2x}(x^3+1)(x^4+5x)] = 2x + \ln(x^3+1) + \ln(x^4+5x)$

$$y' = 2 + \frac{3x^2}{x^3+1} + \frac{4x^3+5}{x^4+5x}$$

Answers: Supplement #5 (continued)

$$38. y = \ln x^{1/2} + \ln(x+1)^2 + \ln(x+2)^3 - \ln(4x+1)$$

$$y' = \frac{1}{2x} + \frac{2(x+1)}{(x+1)^2} + \frac{3(x+2)^2}{(x+2)^3} - \frac{4}{4x+1}$$

$$y' = \frac{1}{2x} + \frac{2}{(x+1)} + \frac{3}{(x+2)} - \frac{4}{4x+1}$$

$$40. y = \ln x^5 + 4x + \ln(3x+1)^{1/2} - \ln(1-x^2)$$

$$y' = \frac{5x^4}{x^5} + 4 + \frac{3}{2(3x+1)} + \frac{2x}{1-x^2} = \frac{5}{x} + 4 + \frac{3}{2(3x+1)} + \frac{2x}{(1-x^2)}$$

$$42. y = \ln(4x) \ln(2x); y' = \frac{4 \ln 2x}{4x} + \frac{2 \ln(4x)}{2x} = \frac{\ln 2x}{x} + \frac{\ln 4x}{x}$$

$$44. f(x) = e^x (3x-4)^8; \ln f(x) = \ln e^x + \ln(3x-4)^8$$

$$\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)} = \left[1 + \frac{24(3x-4)^7}{(3x-4)^8} \right] / f(x)$$

$$= f(x) \frac{d}{dx} \ln f(x) = e^x (3x-4)^8 \left(1 + \frac{24}{3x-4} \right)$$

$$46. f(x) = \frac{(x-2)^3(x-3)^4}{(x+4)^5}; \ln f(x) = \ln(x-2)^3 + \ln(x-3)^4 - \ln(x+4)^5$$

$$\frac{d}{dx} \ln f(x) = \left[\frac{3}{x-2} + \frac{4}{x-3} - \frac{5}{x+4} \right] \frac{(x-2)^3(x-3)^4}{(x+4)^5}$$

$$48. f(x) = 10^x; \ln f(x) = \ln 10^x; \frac{d}{dx} \ln f(x) = \frac{x 10^{x-1}}{10^x} (10^x) = \boxed{x 10^{x-1}} \\ = x \ln(10); \frac{d}{dx} x \ln(10) = \ln 10 (10^x) = \boxed{10^x \ln 10}$$

$$50. f(x) = x^{1/x}; \ln f(x) = x^{-1} \ln x; \frac{d}{dx} \ln f(x) = \left[\frac{\ln x}{x^2} + \frac{1}{x^2} \right] x^{1/x} \\ = x^{1/x} \left[\frac{1 + \ln x}{x^2} \right]$$

Answers: Supplement #5 (continued)

1. $\ln 5x$ 3. $\ln 3$ 5. $\ln 2$ 7. x^2 9. $\ln \left[\frac{x^5 z^3}{\sqrt{y}} \right]$ 11. $\ln(x^3)$

13. $\ln(25) < \ln(27)$

19. $4 \ln 2x = \ln(2x)^4 = \ln(16x^4)$ $\boxed{D}$

21. $\frac{\ln 8x^2}{\ln 2x}$ $\boxed{D}$

23. $\ln x - \ln x^2 + \ln 3 = 0$; $\ln \frac{3x}{x^2} = \ln \frac{3}{x} = 0$; $1 = \frac{3}{x}$; $\boxed{x=3}$

25. $\ln x^4 - 2 \ln x = 1$; $\ln \frac{x^4}{x^2} = 1$; $\ln x^2 = 1$; $e = x^2$; $x = \boxed{\sqrt{e}}$

27. $\ln \sqrt{x} = \sqrt{\ln x}$; $(\ln \sqrt{x})^2 = \ln(x) = (\frac{1}{2} \ln x)^2 = \frac{1}{4} (\ln x)^2$; $1 = \frac{1}{4} \ln x$
 $4 = \ln x$; $\boxed{x=e^4}$

29. $(\ln x)^2 - 1 = 0$; $\ln x = \pm 1$; $\boxed{x = \frac{1}{e} \text{ or } e}$

31. $\ln(x+1) - \ln(x-2) = 1$; $\ln \frac{x+1}{x-2} = 1$; $e(x-2) = x+1$; $xe - 2e = x+1$
 $ex - x = 1 + 2e = x(e-1)$; $\boxed{x = \frac{1+2e}{e-1}}$

33. $y = \ln[(x+5)(2x-1)(4-x)] = \ln(x+5) + \ln(2x-1) + \ln(4-x)$
 $y' = \frac{1}{x+5} + \frac{2}{2x-1} - \frac{1}{4-x}$

35. $y = 2 \ln(1+x) + 3 \ln(2+x) + 4 \ln(3+x)$
 $y' = \frac{2}{1+x} + \frac{3}{2+x} + \frac{4}{3+x}$

37. $y = \ln e^{5x} + \ln(x+4) + \ln(3x-2) - \ln(x+1)$
 $y' = 5 + \frac{1}{x+4} + \frac{3}{3x-2} - \frac{1}{x+1}$

27. $\ln \sqrt{x} = \sqrt{\ln x}$; $\ln(x^{1/2}) = \sqrt{\ln x}$; $(\frac{1}{2} \ln x)^2 = (\sqrt{\ln x})^2$

$\frac{1}{4} (\ln x)^2 = \ln x$; $\frac{1}{4} (\ln x)^2 - \ln x = 0$; $\ln x (\frac{1}{4} \ln x - 1) = 0$

$\ln x = 0$; $\frac{1}{4} \ln x - 1 = 0$

$\boxed{x=1}$

$\frac{1}{4} \ln x = 1$

$\ln x = 4$

$\boxed{x=e^4}$

Answers: Supplement #5 (continued)

$$39. y = \ln(5x+1) + \ln(4x+1) + \ln(\ln x) - \frac{1}{2}\ln(2x+1)$$

$$y' = \frac{5}{5x+1} + \frac{4}{4x+1} + \frac{1}{x \ln x} - \frac{1}{2x+1}$$

$$41. y = \ln(3x+1) \ln(5x+1)$$

$$y' = \frac{3 \ln(5x+1)}{3x+1} + \frac{5 \ln(3x+1)}{5x+1}$$

$$43. \ln[f(x)] = 4 \ln(x+1) + 2 \ln(4x-1)$$

$$\frac{f'(x)}{f(x)} = \frac{4}{x+1} + \frac{8}{4x-1} ; f'(x) = (x+1)^4 (4x-1)^2 \left[\frac{4}{x+1} + \frac{8}{4x-1} \right]$$

$$45. \ln f(x) = \ln(x+1) + \ln(2x+1) + \ln(3x+1) - \frac{1}{2} \ln(4x+1)$$

$$\frac{f'(x)}{f(x)} = \frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} - \frac{2}{4x+1}$$

$$f'(x) = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}} \left[\frac{1}{x+1} + \frac{2}{2x+1} + \frac{3}{3x+1} - \frac{2}{4x+1} \right]$$

$$47. f(x) = 2^x \quad \ln f(x) = x \ln 2 \quad \frac{f'(x)}{f(x)} = \ln 2 ; f'(x) = 2^x \ln(2)$$

$$49. \ln f(x) = x \ln x \quad \frac{f'(x)}{f(x)} = \ln x + 1 ; f'(x) = x^x (\ln(x) + 1)$$