Supplement #8

1. $p = \$2$

$R = p \cdot x = 2x$ $\qquad R = 2x$

$C(x) = x^2 - 2x - 12$ $\qquad -C(x) = -x^2 + 2x + 12$

$P(x) = -x^2 + 4x + 12$

$P'(x) = -2x + 4 = 0$

$\boxed{x = 2}$ $\quad P(x) = -(2)^2 + 4(2) + 12 = -4 + 8 + 12 = \boxed{\$16}$ ✓

2. $p = 2 - .001x$

$R = p \cdot x = -.001x^2 + 2x$

$R' = -.002x + 2 = 0 \Rightarrow \boxed{x = 1000}$ $\quad p = 2 - .001(1000) = \boxed{\$1}$

3. $p = 256 - 50x$

$R = p \cdot x = -50x^2 + 256x$

$-C(x) = -56x - 182$

$P(x) = -50x^2 + 200x - 182$

$P'(x) = -100x + 200 = 0$

$\boxed{x = 2}$ $\quad p = 256 - 50(2) = \boxed{\$156}$ $\quad P = -50(2)^2 + 200(2) - 182$

$= -200 + 400 - 182 = \boxed{\$18}$ ✓

4. (items, price) $(10000, 2)$ ① $m = \frac{2 - 2.4}{10000 - 8000} = -\frac{.4}{2000} = -\frac{1}{5000} = -.0002$

② $(8000, 2.4)$ ③ $y - 2.4 = -.0002(x - 8000)$; $y = -.0002x + 4$

$p = -.0002x + 4$

(a) $R = p \cdot x = -.0002x^2 + 4x$

$R' = -.0004x + 4 = 0$

$\boxed{x = 10000}$ $\quad p = -.0002(10000) + 4 = \boxed{\$2}$ per burger

(b) $C(x) = .6x + 1000$ $\qquad R = -.0002x^2 + 4x$

$-C(x) = -.6x - 1000$

$P(x) = -.0002x^2 + 3.4x - 1000$

$P'(x) = -.0004x + 3.4 = 0$

$\boxed{x = 8500}$ $\quad p = -.0002(8500) + 4 = \boxed{\$2.30}$ per burger

$P = -.0002(8500)^2 + 3.4(8500) - 1000$

$= -14450 + 28900 - 1000 = \boxed{\$13,450}$

5. (items, price) (4000, 50) ① $m = \frac{50-52}{4000-3800} = -\frac{2}{200} = -\frac{1}{100} = -.01$

② (3800, 52) ③ $y - 52 = -.01(x - 3800)$; $y = -.01x + 90$

$p = -.01x + 90$

$R = p \cdot x = -.01x^2 + 90x$

$R' = -.02x + 90 = \emptyset$; $\boxed{x = 4500}$ TICKETS

$p = -.01(4500) + 90 = \boxed{\$45}$ per TICKET

6. (a) $p = 60 - 10^{-5}x$

$R = -10^{-5}x^2 + 60x$

$-C(x) = \quad -30x - 7 \cdot 10^6$

$P(x) = -10^{-5}x^2 + 30x - 7 \cdot 10^6$

$P'(x) = -2 \cdot 10^{-5}x + 30 = \emptyset$; $x = 15 \cdot 10^5 = \boxed{1.5 \cdot 10^6}$ kw-hr

$p = 60 - 10^{-5}(1.5 \cdot 10^6) = 60 - 15 = \boxed{\$45}$ per kw-hr

(b) $R = -10^{-5}x^2 + 60x$

$-C(x) = \quad -40x - 7 \cdot 10^6$

$P(x) = -10^{-5}x^2 + 20x - 7 \cdot 10^6$

$P'(x) = -2 \cdot 10^{-5}x + 20 = \emptyset$; $x = 10 \cdot 10^5 = \boxed{1 \cdot 10^6}$ kw-hr

$p = 60 - 10^{-5}(1 \cdot 10^6) = 60 - 10 = \boxed{\$50}$ per kw-hr

$\$50/\text{kwhr} - \$45/\text{kwhr} = \boxed{\$5/\text{kw} \cdot \text{hr}}$ price increase to customer

7. $p = 150 - .02x$

$R = p \cdot x = -.02x^2 + 150x$

$-C(x) = \quad -100x - 300$

$P(x) = -.02x^2 + 140x - 300$

$P'(x) = -.04x + 140 = \emptyset$; $\boxed{x = 3500 \text{ units}}$

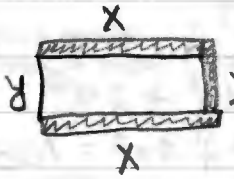
$p = 150 - .02(3500) = \boxed{\$80/\text{unit}}$

$P = -.02(3500)^2 + 140(3500) - 300$

$-245000 + 490000 - 300 = \boxed{\$244,700}$

✓

Supplement #8 (continued)

8. 

$A = 75 = x \cdot y \Rightarrow \frac{75}{x} = y$

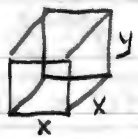
$\text{Cost} = 5y + 10y + 10x + 10x = 15y + 20x$

$\text{Cost} = 15\left(\frac{75}{x}\right) + 20x = \frac{1125}{x} + 20x = 1125x^{-1} + 20x$

$\text{Cost}' = -1125x^{-2} + 20 = \emptyset$

$\frac{-1125}{x^2} = -20 \; ; \; -1125 = -20x^2 \; ; \; 56.25 = x^2 \; ; \; \boxed{7.5 = x}$

$y = 75/x = 75/7.5 = \boxed{10}$ $\boxed{10 \text{ ft} \times 7.5 \text{ ft.}}$

9. 

$V = 12 = x^2 y \; ; \; y = \frac{12}{x^2}$

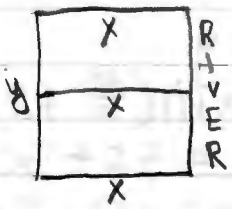
$\text{Cost} = \underset{\text{Top}}{\$2x^2} + \underset{\text{Bottom}}{\$1x^2} + \underset{\text{4 sides}}{\$1(4(xy))} = 3x^2 + 4xy$

$\text{Cost} = 3x^2 + 4x\left(\frac{12}{x^2}\right) = 3x^2 + \frac{48}{x} = 3x^2 + 48x^{-1}$

$\text{Cost}' = 6x - 48x^{-2} = 6x - \frac{48}{x^2} = \emptyset$

$6x = \frac{48}{x^2} \; ; \; 6x^3 = 48 \; ; \; x^3 = 8 \; ; \; \boxed{x = 2} \; ; \; y = \frac{12}{x^2} = \frac{12}{4} = \boxed{3}$

$\boxed{\text{Box is } 2 \times 2 \times 3}$

10. 

$A = x \cdot y$

$\text{Cost} = 1500 = \$6y + 3(\$5)x$

$1500 = 6y + 15x$

$1500 - 6y = 15x$

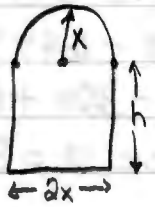
$100 - \frac{2}{5}y = x$

$A = y\left(100 - \frac{2}{5}y\right) = 100y - \frac{2}{5}y^2$

$A' = 100 - \frac{4}{5}y = \emptyset \; ; \; \frac{4}{5}y = 100 \; ; \; \boxed{y = 125} \; ; \; 100 - \frac{2}{5}(125) = x$

$\boxed{50 = x}$

$\boxed{\text{pasture is } 125 \text{ ft} \times 50 \text{ ft.}}$

11. 

Circumfrance $= 2\pi x$ Area $= \pi x^2$ ⟸ For a circle

$\text{Perimeter} = 14 = 2x + 2h + \frac{1}{2}(2\pi x) = 2x + 2h + \pi x$

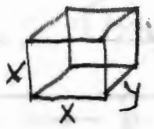
$h = \frac{1}{2}(14 - 2x - \pi x) = \boxed{7 - x - \frac{1}{2}\pi x}$

$\text{Area} = 2xh + \frac{1}{2}\pi x^2 = 2x\left(7 - x - \frac{1}{2}\pi x\right) + \frac{1}{2}\pi x^2$

$\text{Area} = 14x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2 = 14x - 2x^2 - \frac{1}{2}\pi x^2$

$\text{Area}' = 14 - 4x - \pi x = \emptyset \; ; \; 14 = 4x + \pi x = x(4 + \pi)$

$$\boxed{x = \frac{14}{4 + \pi}}$$

12. 

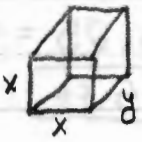
$V = 36 = x^2 y$; $\frac{36}{x^2} = y$

Surface Area (SA) = xy (bottom) + $2xy$ (2 sides) + $2x^2$ (square ends / other 2 sides) (top is open)

$SA = 3xy + 2x^2 = 3x\left(\frac{36}{x^2}\right) + 2x^2 = \frac{108}{x} + 2x^2 = 108x^{-1} + 2x^2$

$SA' = -108x^{-2} + 4x = -\frac{108}{x^2} + 4x = \emptyset$; $4x = \frac{108}{x^2}$; $4x^3 = 108$

$x^3 = 27$; $x = 3$; $y = 36/x^2 = 4$ **Box is 3 x 3 x 4**

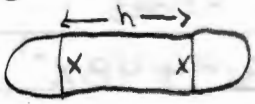
13. 

$120 = x + x + y = 2x + y$; $120 - 2x = y$

$V = x^2 y = x^2(120 - 2x) = 120x^2 - 2x^3$

$V' = 240x - 6x^2 = \emptyset = 6x(40 - x)$; $x \overset{\text{silly}}{=} \emptyset$ OR $x = 40$

$y = 120 - 2(40) = 40$ **Package is 40 x 40 x 40**

14. 

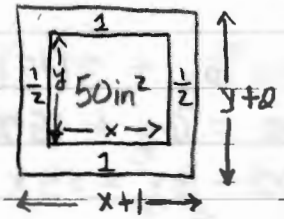
Perimeter = $440 = \pi x + 2h$ (2 hemicircles + 2 sides of rectangle)

Circle circumference: πx

$220 - \frac{1}{2}\pi x = h$

Rectangle Area = $x \cdot h = x(220 - \frac{1}{2}\pi x)$

$\text{Area} = 220x - \frac{1}{2}\pi x^2$; $\text{Area}' = 220 - \pi x = \emptyset$; $\boxed{x = \frac{220}{\pi}}$

15. 

Print = $50 = xy \Rightarrow \frac{50}{x} = y$

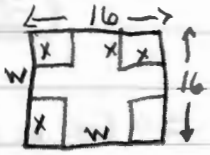
Page = $(x+1)(y+2) = (x+1)\left(\frac{50}{x} + 2\right)$

$\text{Page} = 50 + \frac{50}{x} + 2x + 2 = 52 + 50x^{-1} + 2x$

$\text{Page}' = -50x^{-2} + 2 = -\frac{50}{x^2} + 2 = 0$; $2 = \frac{50}{x^2}$

$2x^2 = 50$; $x^2 = 25$; $x = 5$; $y = \frac{50}{x} = \frac{50}{5} = 10$

Print is 5 x 10 so PAGE is $x+1 = \boxed{6}$ by $y+2 = \boxed{12}$

16. 

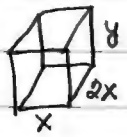
$w = 16 - 2x$ $V = w^2 x = (16 - 2x)^2 x$

$V = (256 - 64x + 4x^2)x = 256x - 64x^2 + 4x^3$

$V' = 256 - 128x + 12x^2 = 0 = 4(3x^2 - 32x + 64)$

$\emptyset = (3x - 8)(x - 8)$; $\boxed{x = 8/3}$ OR $x = 8$ ← silly since cardboard is 16 x 16

Supplement #8 (continued)

17. 

Surface Area = $27 = \underbrace{2(2x^2)}_{\text{Top Bottom}} + \underbrace{2(2xy)}_{\text{2 sides}} + \underbrace{2(xy)}_{\text{other 2 sides}}$

$27 = 4x^2 + 4xy + 2xy = 4x^2 + 6xy$

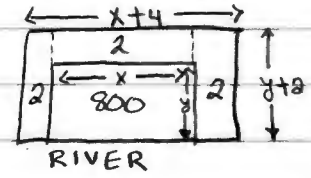
$27 - 4x^2 = 6xy$; $y = \frac{27-4x^2}{6x}$

$V = 2x^2y = 2x^2\left(\frac{27-4x^2}{6x}\right) = x\left(\frac{27-4x^2}{3}\right) = x\left(\frac{27}{3} - \frac{4x^2}{3}\right)$

$V = 9x - \frac{4}{3}x^3$; $V' = 9 - 4x^2 = 0$; $9 = 4x^2$; $\frac{9}{4} = x^2$; $\boxed{\frac{3}{2} = x}$

$y = \frac{27-4x^2}{6x} = \frac{27-4(3/2)^2}{6(3/2)} = \frac{27}{9} - \frac{9}{9} = 3 - 1 = \boxed{2}$

Box is 3/2" x 2" x 3" ← (2x)

18. 

$800 = xy$

$\frac{800}{x} = y$

SIDEWALK = $\underbrace{2}_{\text{2 vertical sidewalk segments}}\underbrace{(2(y+2))}_{\text{1 horizontal sidewalk segment}} + 2x$

$= 4y + 8 + 2x$

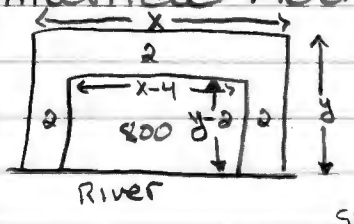
SIDEWALK $= 4\left(\frac{800}{x}\right) + 8 + 2x = \frac{3200}{x} + 8 + 2x = 3200x^{-1} + 8 + 2x$

SIDEWALK$' = -3200x^{-2} + 2 = \emptyset \Rightarrow 2 = 3200x^{-2} = \frac{3200}{x^2}$

$2x^2 = 3200$; $x^2 = 1600$; $x = 40$; $y = \frac{800}{x} = \frac{800}{40} = 20$

OUTSIDE DIMENSIONS: $x + 4 = 40 + 4 = \boxed{44}$ $y + 2 = 20 + 2 = \boxed{22}$

Alternate Method



Plants $= 800 = (x-4)(y-2)$; $\frac{800}{x-4} + 2 = y$

SIDEWALK $= xy - 800$ {overall rectangle minus inner (plants) rectangle

SIDEWALK $= x\left(\frac{800}{x-4} + 2\right) - 800 = \frac{800x}{x-4} + 2x - 800$

Quotient Rule: $u = 800x$ $v = x-4$

$u' = 800$ $v' = 1$

SIDEWALK$' = \frac{800(x-4) - (800x)(1)}{(x-4)^2} + 2 = \frac{800x - 3200 - 800x}{(x-4)^2} + 2 = \emptyset$

$\emptyset = -\frac{3200}{(x-4)^2} + 2$; $\frac{3200}{(x-4)^2} = 2$; $3200 = 2(x-4)^2$; $1600 = (x-4)^2$

$40 = x - 4$; $\boxed{x = 44}$ $y = \frac{800}{x-4} + 2 = \frac{800}{44-4} + 2 = \frac{800}{40} + 2$

$\boxed{y = 22}$