# Supplement #8

1. A small shop sells ties for \$2.00 each. The daily cost function is $C(x) = x^2 - 2x - 12$, where x is dozens of ties sold daily. Find the value of x that will maximize the store's daily profit & graph the profit function.
2. The demand equation for a product is $p = 2 - .001x$. Find the value of x and the corresponding price p that maximize the revenue.
3. Some years ago it was estimated that the demand for steel approximately satisfied the equation $p = 256 - 50x$, and the total cost of producing x units of steel was $C(x) = 182 + 56x$. (The quantity x was measured in millions of pounds and the price and total cost were measured in millions of dollars.) Determine the level of production and the corresponding price that maximize the profits.
4. Until recently hamburgers at the city sports arena cost \$2 each. The food concessionaire sold an average of 10,000 hamburgers on a game night. When the price was raised to \$2.40, hamburger sales dropped off to an average of 8000 per night.
   (a) Assuming a linear demand curve, find the price of a hamburger that will maximize the nightly hamburger **revenue**.
   (b) Suppose that the concessionaire has fixed costs of \$1000 per night and the variable cost is \$.60 per hamburger. Find the price of a hamburger that will maximize the nightly hamburger **profit**.
5. The average ticket price for a concert at the Opera House was \$50. The average attendance was 4000. When the ticket price was raised to \$52, attendance declined to an average of 3800 persons per performance. What should the ticket price be in order to maximize the revenue for the Opera House? (Assume a linear demand curve.)
6. The monthly demand equation for an electric utility company is estimated to be: $p = 60 - (10^{-5})x$, where p is measured in dollars and x is measured in thousands of kilowatt-hours. The utility has fixed costs of \$7,000,000 per month and variable cost of \$30 per 1000 kilowatt-hours of electricity generated, so that the cost function is: $C(x) = 7 \bullet 10^6 + 30x$.
   (a) Find the value of x and the corresponding price for 1000 kilowatt-hours that maximize the utility's profit.
   (b) Suppose that rising fuel costs increase the utility's variable cost from \$30 to \$40 so that its new cost function is: $C(x) = 7 \bullet 10^6 + 40x$.
   Should the utility pass all this increase of \$10 per thousand-kilowatt hours on to consumers? Explain your answer.
7. Suppose that the demand equation for a monopolist is $p = 150 - .02x$ and the cost function is $C(x) = 10x + 300$. Find the value of x that maximizes the profit.
8. A rectangular garden of area 75 $ft.^2$ is to be surrounded on three sides by a brick wall costing \$10 per foot and on one side by a fence costing \$5 per foot. Find the dimensions of the garden such that the cost of materials is minimized.
9. A closed rectangular box with square base and a volume of 12 $ft.^3$ is to be constructed using two different types of materials. The top is made of a metal costing \$2 per square foot and the remainder of wood costing \$1 per square foot. Find the dimensions of the box for which the cost of materials is minimized.
10. A farmer has \$1500 available to build an E-shaped fence along a straight river so as to create two identical rectangular pastures. Materials for the side parallel to the river cost \$6 per foot and the materials for the three sections perpendicular to the river cost \$5 per foot find the dimensions for which the total area is as large as possible.
11. A Norman window is a window which consists of a rectangle capped by a semicircular region. Find the value of x such that the perimeter of the window will be 14 feet and the area of the window will be as large as possible. (See figure next page)
12. Design in open rectangular box with square ends, having volume 36 $in.^3$, that minimizes the amount of material required for construction.
13. A certain airline requires that rectangular packages carried on an airplane by passengers be such that the sum of the three dimensions is at most 120 cm. Find the dimensions of the square-ended rectangular package of greatest volume that meets this requirement.

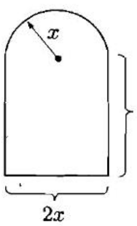
14. An athletic field consists of a rectangular region with a semicircular region at each end. The perimeter will be used for a 440-yard track. Find the value of x for which the area of the rectangular region is as large as possible. (See figure below)

15. A rectangular page is to contain 50 in.$^2$ of print. The page has to have a 1-inch margin on top and at the bottom and a 1/2-inch margin on each side. Find the dimensions of the page that minimizes the amount of paper used. . (See figure below)

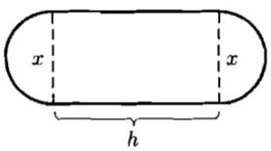
16. An open rectangular box is to be constructed by cutting square corners out of a 16- by 16-inch piece of cardboard and folding up the flaps. Find the value of x for which the volume of the box will be as large as possible. . (See figure below)

17. A closed rectangular box is to be constructed with a base that is twice as long as it is wide. Suppose that the total surface area must be 27 ft.$^2$. Find the dimensions of the box that will maximize the volume.

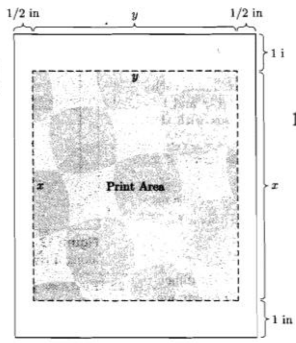
18. A botanical display is to be constructed as a rectangular region with a river as one side and a sidewalk 2 meters wide along the inside edges of the other three sides. The area for the plants must be 800 square meters. Find the outside dimensions of the region that minimizes the area of the sidewalk and hence minimizes the amount of concrete needed for the sidewalk. . (See figure below)



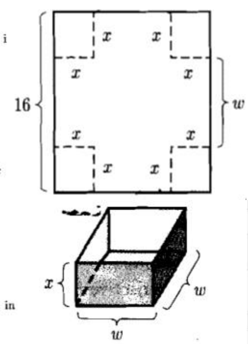
Prob. #11



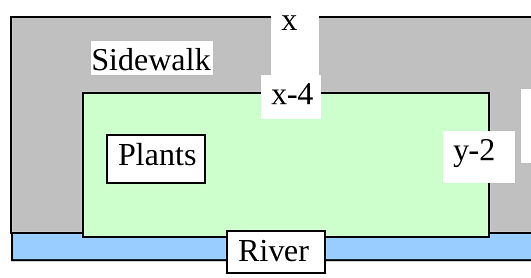
Prob. #14



Prob. #15



Prob. #16



Prob. #18

1. 2 dozen ties
2. x=1000 p = $1
3. x = 2 (million tons) p = $156 per ton
4. (a) (10,000, $2)
   (b) (8500, $2.30)
5. $45/ticket $202,500
6. (a) x = 1.5 million 1000 kw-hrs $45/1000 kw-hr
   (b) x = 1 million 1000 kw-hrs $50/1000 kw-hr only pass on $5 of cost
7. x = 3500
8. 10' X 7.5'
9. 2' X 3'
10. 125' X 50'

11. $x = \frac{14}{\pi + 4}$
12. 3" X 3" X 4"
13. 40 cm X 40 cm X 40 cm
14. $x = \frac{220}{\pi}$
15. 6" X 12"
16. $x = \frac{8}{3}$"
17. $\frac{3}{2} ft. \ X \ 3 ft. \ X \ 2 ft.$
18. 44m X 22m