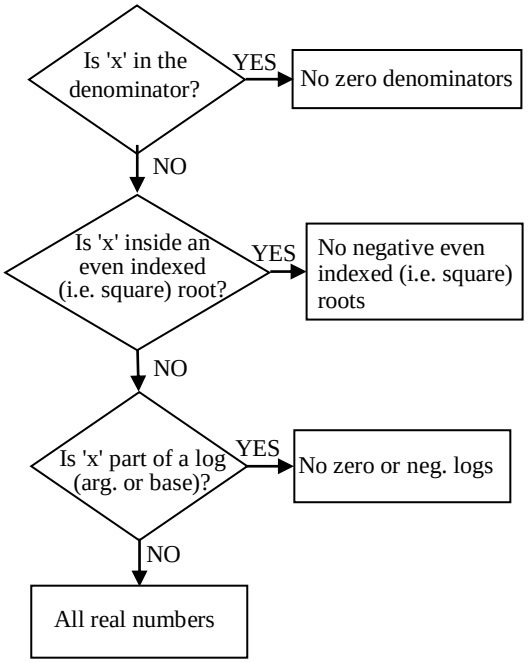
The Three Laws of Algebra

1. No zero denominators.
2. No negative even indexed radicals (i.e. no negative square roots)
3. No zero or negative logarithms

Line Information

1. To find the equation of a line:
   a) Find the slope, m
   b) Find a point on the line: (x_1,y_1)
   c) Write the equation of the line: $y - y_1 = m(x - x_1)$
2. To find the slope:
   a) $m = \frac{y_2 - y_1}{x_2 - x_1}$
3. $y = mx + b \implies$ slope - intercept form of a line
4. $y - y_1 = m(x - x_1) \implies$ point-slope form of a line
5. $m_{parallel} = m$ Parallel lines have the same slope. **DON'T CHANGE THE SLOPE!**
6. $m_{perpendicular} = -\frac{1}{m}$ Flip old slope over **AND CHANGE THE SIGN**
7. X-intercept: (x,0) "plug in 'zero' for y, and solve for x.
8. Y-intercept: (0,y) "plug in 'zero' for x, and solve for y.

How to Find the Domain of a Function



DOMAIN: ALL "LEGAL" X-VALUES
RANGE: ALL "LEGAL" Y-VALUES

Functions

Function: No repeated x-values (all x-values unique) → Passes **VERTICAL** line test

One-to-one function: No repeated y-values (all y-values unique) → Passes **HORIZONTAL** line test

Examples: {(–2,4), (–1,1), (0,0), (0,1),(2,4)} ← Not a function (x-values repeated)

{(–2,4), (–1,1), (0,0), (1,1),(2,4)} ← Function, but not one-to-one (y-values repeated)

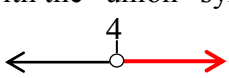
{(–2,4), (–1,3), (0,2), (1,1),(2,0)} ← One-to-one function (x- & y-values unique)

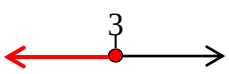
Inequality Rule

When multiplying or dividing both sides of an inequality by a **negative** number, you MUST switch the direction of the inequality. Example: $-2x > 4 \rightarrow x < -2$

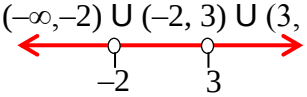
Interval Notation

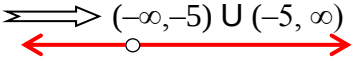
1. Interval Notation: left to right (or down to up). {smallest x, largest x} OR {smallest y, largest y}
   a) $>$ or $<$ or ∞ or $-\infty \implies$ round parenthesis: ( )
   b) $\geq$ or $\leq \implies$ square brackets: [ ]
   c) Join two or more sets with the "union" symbol: ∪
   d) Examples: 1) $x > -4 \implies (-4, \infty)$ 2) $x \leq 3 \implies (-\infty, 3]$



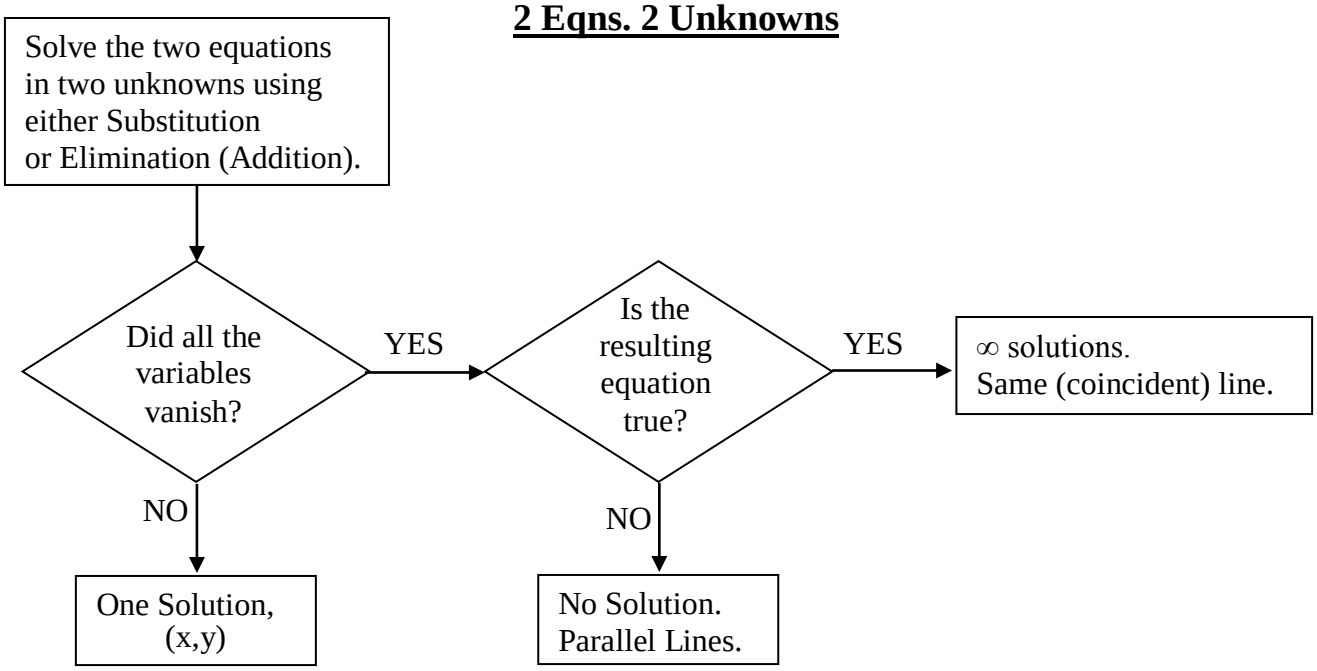


3) $x \neq -2$ & $x \neq 3 \implies (-\infty,-2) \cup (-2, 3) \cup (3, \infty)$ 4) $x \neq -5 \implies (-\infty,-5) \cup (-5, \infty)$



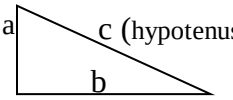


2 Eqns. 2 Unknowns



Other Handy Formulas

Pythagorean Theorem: $a^2 + b^2 = c^2$



Oompa, Loompa, Doompa Dee Dormula, To Solve This Equation, Use the Quadratic formula!

Quadratic Formula: Given $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Work Equation

$\dfrac{T}{A} + \dfrac{T}{B} = 1$ Where: **T** = together time, **A** = A's alone time, **B** = B's alone time

Properties of Exponents

1. $a^0 = 1$
2. $a^m \cdot a^n = a^{m+n}$
3. $\left(a^m\right)^n = a^{m \cdot n}$
4. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
5. $\dfrac{a^m}{a^n} = a^{m-n}$
6. $a^{-m} = \dfrac{1}{a^m}$
7. $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$
8. $\sqrt[n]{a^m} = \sqrt[n]{a}^m = a^{m/n}$
9. $(\text{negative})^{\text{Odd}} = \text{negative}$
   $(\text{negative})^{\text{Even}} = \text{positive}$

Logarithms

$\log_b x = y \rightarrow b^y = x, \quad b > 0, \; b \neq 1$ Example: $\log_2 16 = ? \rightarrow ? = 4$ since $2^4 = 16$

Scientific Notation

$1 \le x < 10 \times 10^{power}$ Note: power on exponent may be + or −. x may be decimal. × means "times."

Move decimal point to the right → subtract from exponent

Move decimal point to the left → add to exponent

Examples:
$0.00459 = 4.59 \times 10^{-3}$ Move decimal 3 places to the right
$459. = 4.59 \times 10^{2}$ Move decimal 2 places to the left

FACTORING TECHNIQUES

1. Factor out common factors.

   Example: $3x^2 - 15x + 3 = 3x(x^2 - 5x + 1)$

2. Difference of squares: $a^2 - b^2 = (a + b)(a - b)$

   Example: $36x^2 - 121 = (6x)^2 - 11^2 = (6x + 11)(6x - 11)$

3. Square of a sum: $(a + b)^2 = a^2 + 2ab + b^2$.

   Example: $25x^2 + 20x + 4 = (5x + 2)^2$

4. Square of a difference: $(a - b)^2 = a^2 - 2ab + b^2$.

   Example: $100x^2 - 60x + 9 = (10x - 3)^2$

5. Factoring trinomials in the form: $x^2 + bx + c$.

   Example: $x^2 - 14x + 48 = (x - 8)(x - 6)$

6. Factoring trinomials in the form: $ax^2 + bx + c$.

   Example: $28x^2 - 17x - 3 = (7x + 1)(4x - 3)$

7. Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

   Example: $8x^3 - 1 = (2x)^3 - 1^3 = (2x - 1)(4x^2 + 2x + 1)$

8. Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

   Example: $27x^6 + 64y^3 = (3x^2)^3 + (4y)^3 = (3x^2 + 4y)(9x^4 - 12x^2y + 16y^2)$

9. Factor by grouping.

   Example: $3x^2 + x - 6x - 2 = x(3x + 1) - 2(3x + 1) = (3x + 1)(x - 2)$

Percentage Formulae

1. Selling price = Cost of item (in $) + Markup (in $, **not** %)
2. Markup (in $) = Markup rate (convert from % to decimal) • Cost of item (in $)

-OR- Markup (in $) = Sold Price (in $) – Original Cost (in $)

3. Markup rate = Markup (in $) ÷ Original Cost of item (in $), then (multiply by 100 to get the percentage)
4. Selling price = List price of item (in $) – Discount (in $, **not** %)
5. Discount (in $) = Discount rate (convert from % to decimal) • List price (in $)

-OR- Original cost (in $) – Sale Price (in $)

6. Discount rate = Discount (in $) ÷ List price (in $), then (multiply by 100 to get the percentage)

Word Problem Examples

Percentage Calculations:

You receive a 6% raise and are now making $12500. What was your salary before the raise?

Old salary plus 6% of old salary is new salary.

1x + .06x = $12,500 ⟹ 1.06x = $12500 ⟹ x = $11,792.45

You buy a box of day-old cookies for 15% off list(street or retail) price. If the cookies sell for $5.35 normally, how much do you pay with the discount (i.e. what is the sale price)?

List price less 15% of list price is sale price.

5.35 – .15(5.35) = x ⟹ $5.35 – 0.80 = $4.55

-OR-

If the item is 15% OFF, then it is 85% ON: $5.35(.85) = $4.55

Money/Interest 2 eqns. 2 unknowns:

You invest $5000 in two accounts. One pays 3% interest, the other 7%. You earn $225 interest. How much did you invest in each account?

		3%		7%		Total
Line 1	Invest	x	+	y	=	5000
Line 2	Interest	.03x	+	.07y	=	225
Line 3		–.03x	+	–.03y	=	–150
				.04y	=	75 ⟹ y = $1875 @ 7%
						x = $5000 – y = $3125 @ 3%

Multiply **everything** on Line 1 by the **negative** of the smaller of the two numbers on Line 2 (–.03) & write that result on Line 3.

Add Line 2 & Line 3 then solve for the variable

Mixture Problems:

How much 15% anti-freeze should you mix with 45% anti-freeze to get 19 gallons of 26% anti-freeze?

		15%		45%		Total (26%)
Line 1	Gallons	x	+	y	=	19
Line 2	Chemical	.15x	+	.45y	=	4.94
Line 3		–.15x	+	–.15y	=	–2.85
				.3y	=	2.09 ⟹ y = 7 gallons of 45%
						x = 19 – y = 12 gallons of 15%

UBER-IMPORTANT!
Calculate this value by multiplying the total % (.26) by the total amount (19 gallons)

Multiply **everything** on Line 1 by the **negative** of the smaller of the two numbers on Line 2 (–.15) & write that result on Line 3.
Add Line 2 & Line 3 then solve for the variable

Distance – Rate – Time Problems:

Julio and Alejandro entered the New York marathon last year. After two hours, Alejandro was 3 miles ahead of Julio. If Julio's speed was three-fourths Alejandro's speed, how fast were the runners going?

	Distance	=	Rate	*	Time
Julio	D		$\frac{3}{4}R$		2
Alejandro	D +3		R		2

$D = 2\left(\frac{3}{4}R\right) \Rightarrow D = \frac{3}{2}R$

D + 3 = 2R $\Rightarrow \frac{3}{2}R + 3 = 2R \Rightarrow$ 3 = ½ R $\Rightarrow$ **6** = R

Alejandro: **6** mph Julio: $\frac{3}{4}R = 4\frac{1}{2}$ mph

Airplane/Boat Problems:

An airplane travel 330 miles in 3 hrs. against the wind and makes the return trip in 2 hrs. Find the speed of the plane with no wind (still air) and the speed of the wind.

	Distance	=	Rate	*	Time
Against	330		P – w		3
With	330		P + w		2

330 = 3(P – w) $\Rightarrow$ 110 = P – w

330 = 2(P + w) $\Rightarrow$ 165 = P + w

275 = 2P $\Rightarrow$ P = **137.5** mph

165 = P + w $\Rightarrow$ w = **27.5** mph

Parabolas:

Standard Form: $f(x) = ax^2 + bx + c$

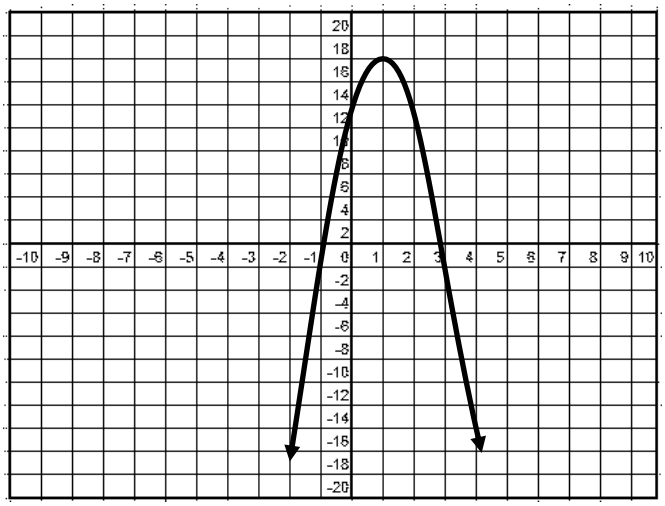
1. Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$
2. a < 0, frowns $\Longrightarrow$ Vertex is maximum
   a > 0, smiles $\Longrightarrow$ Vertex is minimum
3. Axis of Symmetry is the line: $x = -\frac{b}{2a}$
4. Y-intercept: (0,c)
5. X-intercepts: $\left(\frac{-b+\sqrt{b^2-4ac}}{2a}, 0\right) AND \left(\frac{-b-\sqrt{b^2-4ac}}{2a}, 0\right)$ $\Longrightarrow$ Factor if you can, if not, quadratic formula.
6. Domain: All real numbers
7. Range: a < 0: $\left(-\infty, f\left(-\frac{b}{2a}\right)\right] \; or \; (-\infty, y - value\ of\ vertex]$

   a > 0: $\left[f\left(-\frac{b}{2a}\right), \infty\right) or \; [y - value\ of\ vertex, \infty)$

Example:

The total cost (in dollars) for a company to produce and sell x items per week is **$C(x) = x^2 + 20x - 11$**. The revenue brought in by selling x items is given by, **$R(x) = -4x^2 + 30x$**. How many items must be sold to maximize the profit? Graph the profit function labeling the vertex, points of symmetry and all intercepts.

$P(x) = R(x) - C(x) = [-4x^2 + 30x] - [x^2 + 20x - 11]$ → **$P(x) = -5x^2 + 10x + 11$**



Vertex: $-\frac{B}{2A} = -\frac{10}{2(-5)} = -\frac{10}{-10} = 1$

$P(1) = -5(1)^2 + 10(1) + 11 = 16$ → **Vertex: (1, 16)**

Y-intercept: $P(0) = -5(0)^2 + 10(0) + 11 = 16$ → **Y.I.: (0, 11)**

X-Intercepts: $0 = -5x^2 + 10x + 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{10^2 - 4(-5)(11)}}{2(-5)} = = \frac{-10 \pm \sqrt{100 + 220}}{-10} = = \frac{-10 \pm \sqrt{320}}{-10} = \frac{-10 \pm \sqrt{64 * 5}}{-10}$$

$$\frac{-10 \pm 8\sqrt{5}}{-10} = \frac{-2(5 \pm 4\sqrt{5})}{-10} = \frac{(5 \pm 4\sqrt{5})}{5} = \frac{5}{5} \pm \frac{4\sqrt{5}}{5} = 1 \pm \frac{4\sqrt{5}}{5} \rightarrow X.I.: \left(1 + \frac{4\sqrt{5}}{5}, 0\right) \left(1 - \frac{4\sqrt{5}}{5}, 0\right)$$

(2.8 , 0) **(–0.8 , 0)**

Domain: $(-\infty, +\infty)$

Range: $(-\infty, 16]$

Axis of Symmetry: x = 1