The Three Laws of Algebra

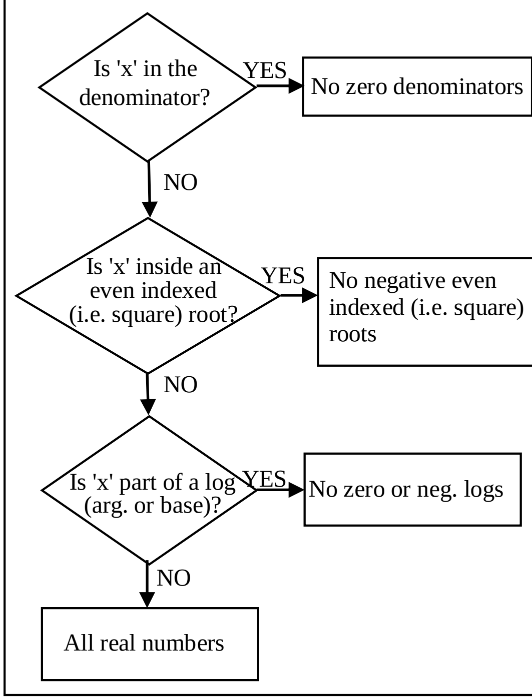
1. No zero denominators.
2. No negative even indexed radicals (i.e. no negative square roots)
3. No zero or negative logarithms

Line Information

1. To find the equation of a line:
    a) Find the slope, m
    b) Find a point on the line: (x_1,y_1)
    c) Write the equation of the line: $y - y_1 = m(x - x_1)$
2. To find the slope:
    a) $m = \frac{y_2 - y_1}{x_2 - x_1}$
3. $y = mx + b$ $\Longrightarrow$ slope - intercept form of a line
4. $y - y_1 = m(x - x_1)$ $\Longrightarrow$ point-slope form of a line
5. $m_{parallel} = m$ Parallel slopes are the same. **CHANGE NOTHING**
6. $m_{perpendicular} = -\frac{1}{m}$ **Flip old slope over AND CHANGE THE SIGN**
7. X-intercept: (x,0) **plug in 'zero' for y, and solve for x.**
8. Y-intercept: (0,y) **plug in 'zero' for x, and solve for y.**

DOMAIN: ALL "LEGAL" X-VALUES
RANGE: ALL "LEGAL" Y-VALUES

How to Find the Domain of a Function



Functions

Function: No repeated x-values (all x-values unique) → Passes **VERTICAL** line test

One-to-one function: No repeated y-values (all y-values unique) → Passes **HORIZONTAL** line test

Examples: {(–2,4), (–1,1), (0,0), (0,1),(2,4)} ← Not a function (x-values repeated)

{(–2,4), (–1,1), (0,0), (1,1),(2,4)} ← Function, but not one-to-one (y-values repeated)

{(–2,4), (–1,3), (0,2), (1,1),(2,0)} ← One-to-one function (x- & y-values unique)

Average Rate of Change of a Function

$$A.R.C. = \frac{f(b) - f(a)}{b - a}$$

Example:

$f(x) = x^2 + 2x$ on the interval [3,5]

$f(3) = 3^2 + 2(3) = 15$ → means the point: (3, 15)

$f(5) = 5^2 + 2(5) = 35$ → means the point (5,35)

Now, just find the slope between (3,15) & (5,35):

$$A.R.C. = slope = m = \frac{35-15}{5-3} = \frac{20}{2} = 10 \leftarrow \text{ANSWER}$$

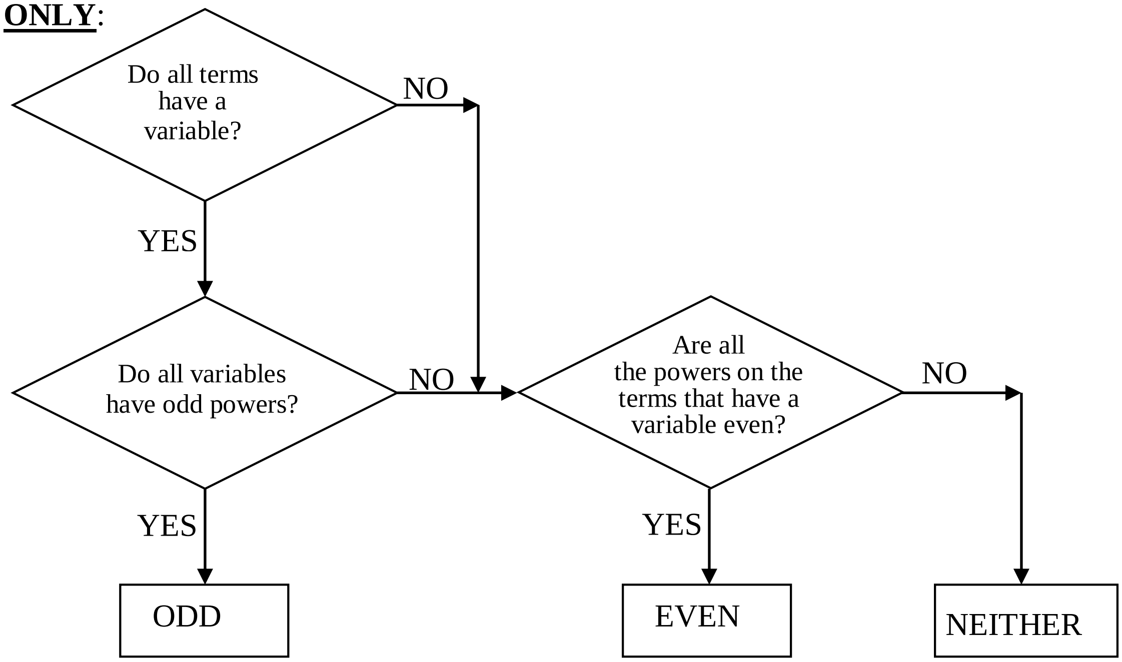
Odd, Even, Neither

For **ALL** functions, both polynomial and rational:

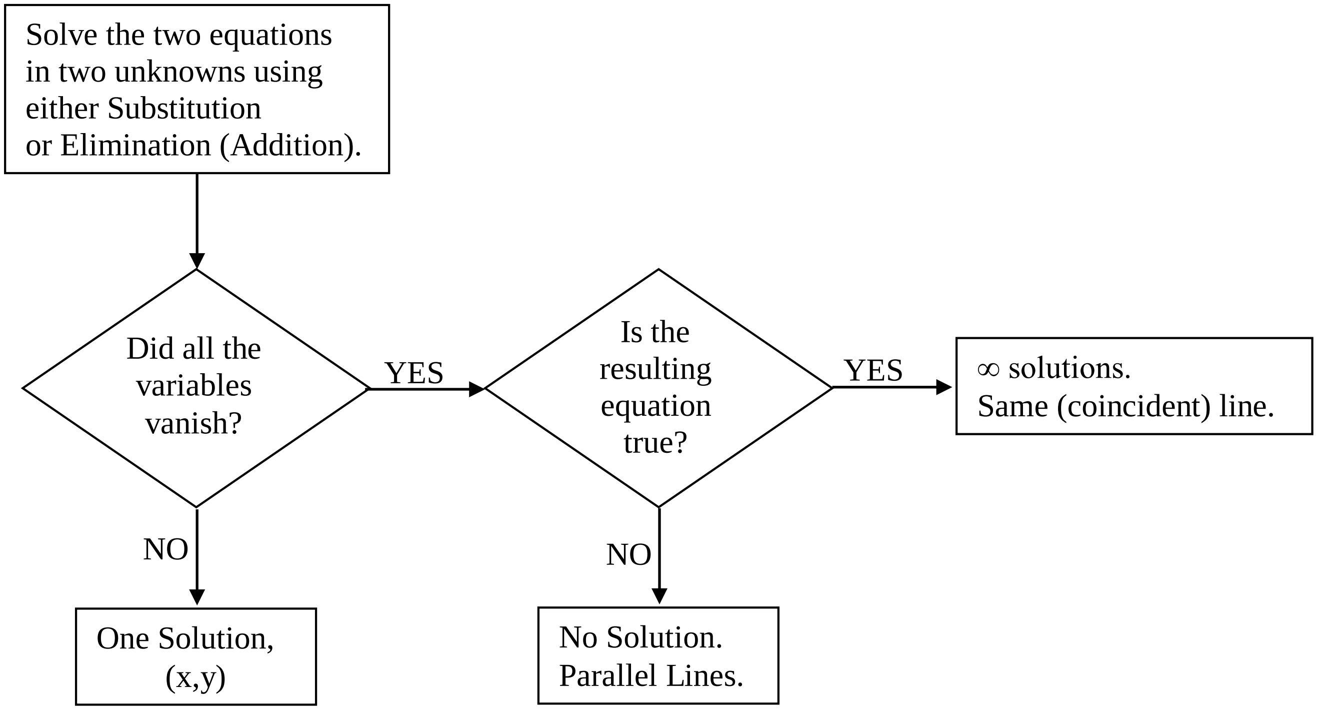
EVEN: $f(-x) = f(x)$ {symmetric with respect to the y-axis}

ODD: $f(-x) = -f(x)$ {Symmetric with respect to the origin}

For **polynomial** functions **ONLY**:

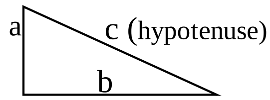


2 Eqns. 2 Unknowns



Other Handy Formulas

Pythagorean Theorem: $a^2 + b^2 = c^2$



Mid-Point Formula: $M = (x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Quadratic Formula: Given $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Work Equation

$\frac{T}{A} + \frac{T}{B} = 1$ Where: **T** = together time, **A** = A's alone time, **B** = B's alone time

Solving Polynomial and Rational Inequalities:

1. Get everything on the left side of the inequality, zero on the right.
   a) If it's a polynomial fraction, get everything over a common denominator and simplify.
2. Find the **zeros** and where the denominator (if applicable) is zero.
   a) Graph those points on a number line, using solid dots for ≤ & ≥ and open circles for < & > & zero denominator values.
3. **Choose** a point **in each interval** on your number line.
4. **Test** that chosen point. Only evaluate whether the test point is + or –, not the actual value of the point.
4. Write your **answer** in interval notation using the number line and the test values.
   a) Only write the + intervals if the original problem was > or ≥ 0
   b) Only write the – intervals if the original problem was < or ≤ 0

Example: $\frac{x+2}{2x-4} \geq 1$ ← first get zero on the left, then a single common fraction.

$$\frac{\boldsymbol{x+2}}{\boldsymbol{2x-4}} \geq \mathbf{1} \rightarrow \frac{x+2}{2x-4} - 1 \geq 0 \rightarrow \frac{x+2}{2x-4} - 1\left(\frac{2x-4}{2x-4}\right) \geq 0$$

$$\frac{x+2-(2x-4)}{2x-4} \geq 0 \rightarrow \frac{\boldsymbol{-x+6}}{\boldsymbol{2x-4}} \geq \mathbf{0}$$

So, now we have: $\frac{-x+6}{2x-4} \geq 0$

Zeros: 2 6

Choose: 0 +3 +7

Test: – + –

Test chosen points:

$\frac{-0+6}{2(0)-4} = -$

$\frac{-3+6}{2(3)-4} = +$

$\frac{-7+6}{2(7)-4} = -$

Answer: (2,6] ← since the original problem was ≥, we look for the +'s on our test.

Example: $x^3 + 8x^2 < 0$

$x^2(x + 8) < 0$

Zeros: –8 0

Choose: –9 –1 +1

Test: – + +

Test chosen points:

$(-9)^2(-9+8) = +81(-1) = -$

$(-1)^2(-1+8) = +1(+7) = +$

$(+1)^2(+1+8) = +1(+9) = +$

Answer: (–∞,–8) ← since the original problem was <, look for the –'s on the test.

Increasing/Decreasing Hints

ALWAYS LOOKING AT ONLY X-VALUES FROM LEFT TO RIGHT:

1. A function is **increasing** wherever the ladybug goes uphill.
2. A function is **decreasing** wherever the ladybug goes downhill.
3. A relative or local **maximum** is the Y-VALUE ONLY where the ladybug changes from going **uphill** to **downhill**.
   (a) The local maximum occurs **AT** the **x-value** of the maximum point.
4. A relative or local **minimum** is the Y-VALUE ONLY where the ladybug changes from going **downhill** to **uphill**.
   (a) The local minimum occurs **AT** the **x-value** of the minimum point.
5. Absolute maximum is the Y-VALUE ONLY of the highest point on the graph (if the graph has arrows on its ends, the abs. max. is $+\infty$, or the horizontal asymptote).
6. Absolute minimum is the Y-VALUE ONLY of the lowest point on the graph (if the graph has arrows on its ends, the abs. min. is $-\infty$, or the horizontal asymptote).

Graphing Polynomial Functions:

1. Determine the power function, $f(x) = \pm x^n$, by adding up all the x's.
2. Fill in the chart:

Zeros $\rightarrow$ the x-intercepts of the function	
Multiplicity $\rightarrow$ the power on the $(x - a)$ term	
Odd/Even $\rightarrow$ Is the **multiplicity** an **odd** number or an **even** number?	
Cross/Touch $\rightarrow$ if **odd** then **cross**, $\rightarrow$ if **even**, then **touch**	

3. Determine the end behavior from the power function:

$+x^{even}$	↑ ↑	As $x \rightarrow \infty$	$f(x) \rightarrow \infty$
		As $x \rightarrow -\infty$	$f(x) \rightarrow \infty$
$-x^{even}$	↓ ↓	As $x \rightarrow \infty$	$f(x) \rightarrow -\infty$
		As $x \rightarrow -\infty$	$f(x) \rightarrow -\infty$
$+x^{odd}$	↓ ↑	As $x \rightarrow \infty$	$f(x) \rightarrow \infty$
		As $x \rightarrow -\infty$	$f(x) \rightarrow -\infty$
$-x^{odd}$	↑ ↓	As $x \rightarrow \infty$	$f(x) \rightarrow -\infty$
		As $x \rightarrow -\infty$	$f(x) \rightarrow \infty$

4. Doman: All real numbers

Example: $r(x) = -3x^2(x+3)(x-2)^3$

Power function: $p(x) = -3x^6$

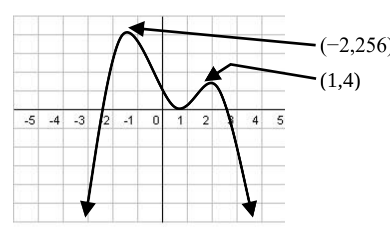
Sign and leading coefficient on function are sign and leading coefficient on power function

Z	−3	0	2
M	1	2	3
O/E	O	E	O
C/T	C	T	C

Add up numbers in this row to get degree

E.B. (determined from the power function, $-x^6$)

$-x^{even}$	↓ ↓	As $x \rightarrow \infty$	$f(x) \rightarrow -\infty$
		As $x \rightarrow -\infty$	$f(x) \rightarrow -\infty$



Note: A polynomial function will AT MOST have one fewer bumps than the degree of the power function.

Inverse Functions

$f^{-1}(x)$: Just switch x & y then solve equation for y.

Example: $f(x) = \frac{4x-1}{2x+3} \Longrightarrow y = \frac{4x-1}{2x+3}$

1. Switch x & y variables: $x = \frac{4y-1}{2y+3}$
2. Solve for y:
   a) Multiply both sides by 2y+3 to clear the fraction: $x(2y + 3) = 4y - 1$
   b) Distribute the x: $2xy + 3x = 4y - 1$
   c) Get all of your y's on one side, all of your "not y's" on the other side: $2xy - 4y = -3x - 1$
   d) Factor out the common y: $y(2x - 4) = -3x - 1$
   e) Divide by 2x – 4: $y = \frac{-3x-1}{2x-4} \Longrightarrow f^{-1}(x) = \frac{-3x-1}{2x-4}$

Composition of Functions

1. $(f \circ g)(x) = f(g(x))$ Example: {note: dog(3) ≠ god(3), that is compositions are not necessarily commutative}

Given the functions: d(x) = x^2; o(x) = 3x; g(x) = x − 2

$(d \circ o \circ g)(x) = (d(o(g(x)))) = 9x^2 - 36x + 36$

d(x) = x^2 = $(3x - 6)^2$ = $9x^2 - 36x + 36$

o(x) = 3x = 3(x − 2) = 3x − 6

g(x) = x − 2

$(d \circ o \circ g)(3) = (d(o(g(3)))) = 9$

d(x) = x^2 = $(3)^2$ = 9

o(x) = 3x = 3(1) = 3

g(x) = x − 2 = 3 − 2 = 1

3

$(g \circ o \circ d)(3) = (g(o(d(3)))) = 25$

g(x) = x − 2 = 27 − 2 = 25

o(x) = 3x = 3(9) = 27

d(x) = x^2 = $(3)^2$ = 9

3

2. If $(f \circ g)(x) = x$ AND $(g \circ f)(x) = x$, then f(x) & g(x) are inverse functions. That is $f^{-1}(x) = g(x)$

The Transformed Function

- Functions can be affected by four different transformations. **Not all functions will have all of these transformations.**
- **The order of the transformations matters sometimes**.

You will always transform a function correctly if you do it in this order:

1) A horizontal shift. (left or right): **shift RIGHT if c is NEGATIVE, shift LEFT if c is POSITIVE**
2) A **horizontal** reflection. (about the **y axis**)
3) A horizontal stretch or compression.
4) A **vertical** reflection. (about the **x axis**)
5) A vertical stretch or compression.
6) A vertical shift. (up or down): **shift DOWN if d is NEGATIVE, shift UP if d is POSITIVE**

$$F(x) = \underset{4}{-}\,\underset{5}{a}\,f(\underset{2}{-}\,\underset{3}{b}x + \underset{1}{c}) + \underset{6}{d}$$

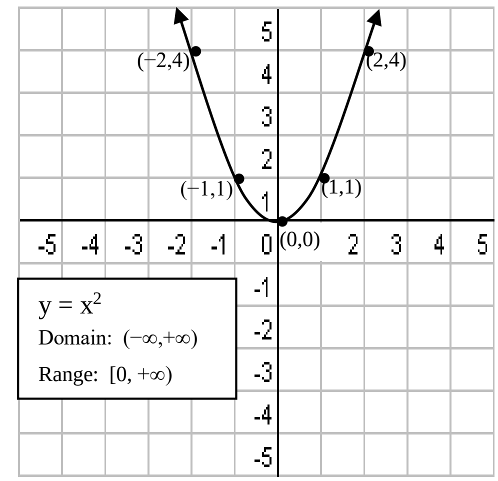
A minus sign here causes a reflection about the **x axis**.

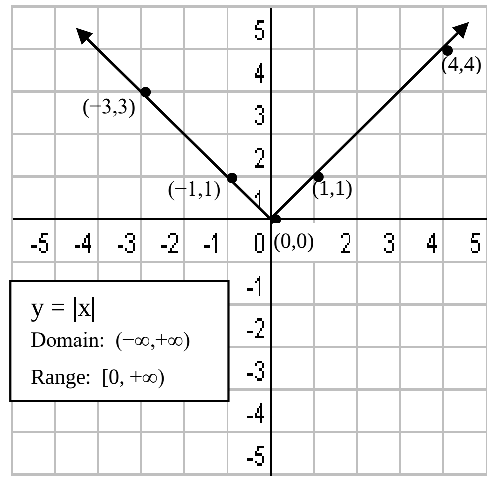
A minus sign here causes a reflection about the **y axis**.

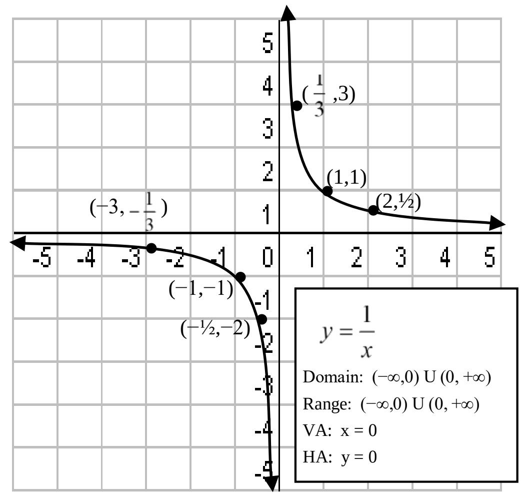
If the transformations are applied in the **order listed above**:

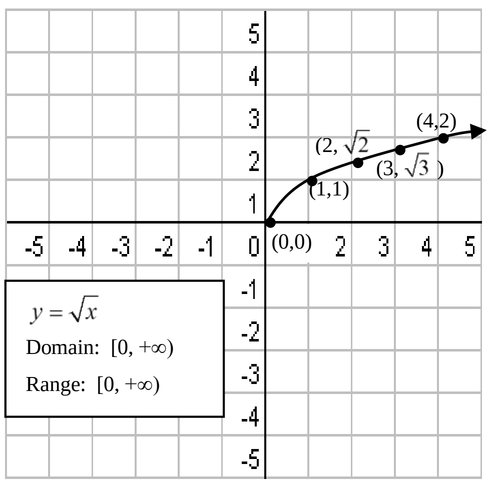
- The constant **c** causes a shift left or right. (subtract or add **c** from the x coordinate of each point on the graph)
  The shift is to the **right** if c<0 (**negative**), to the **left** if c>0 (**positive**).
- The constant **d** causes a shift up or down. (subtract or add **d** to the y coordinate of each point on the graph)
  The shift is **down** if d<0 (**negative**), **up** if d>0 (**positive**).
- The constant **b** causes a horizontal stretch or compression. It is a compression if b>1, a stretch if b<1.
  (divide the x coordinate of each point on the graph by b)
- The constant **a** causes a vertical stretch or compression. It is a stretch if a>1, a compression if a<1.
  (multiply the y coordinate of each point on the graph by a)

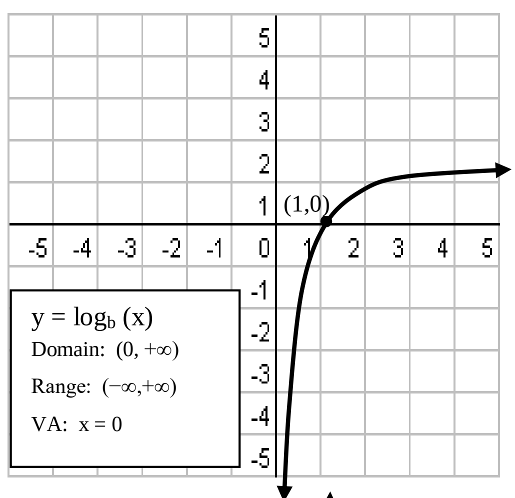
Reference Graphs

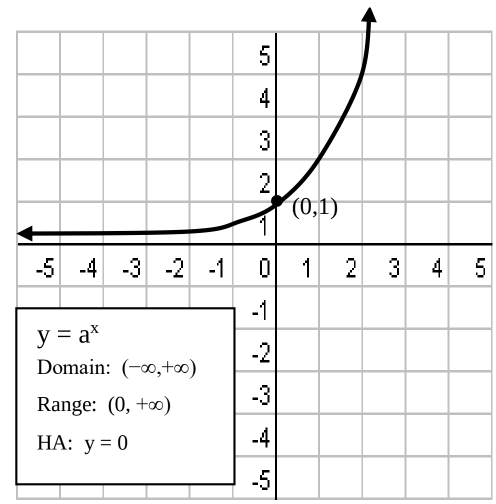


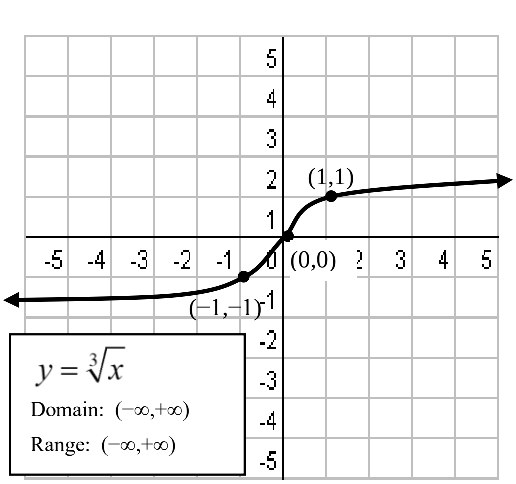


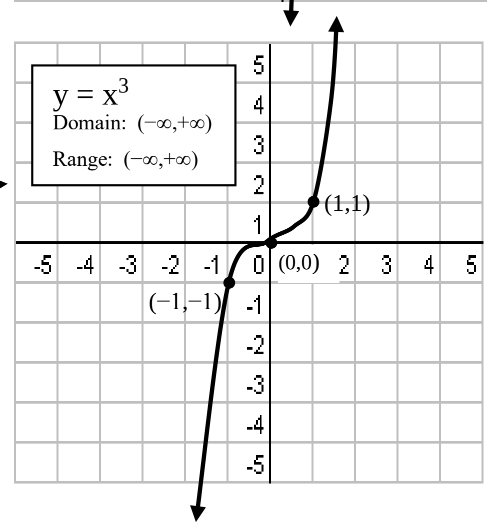


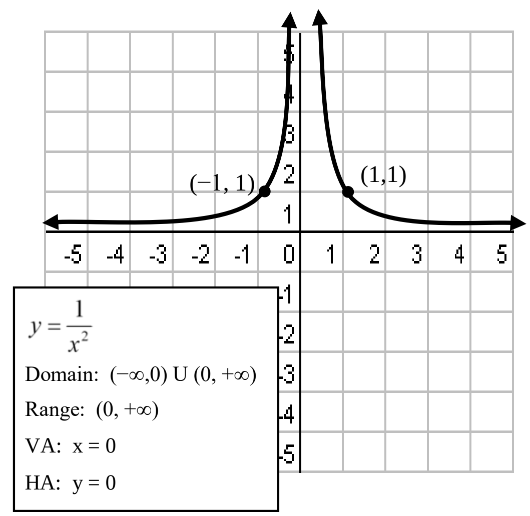












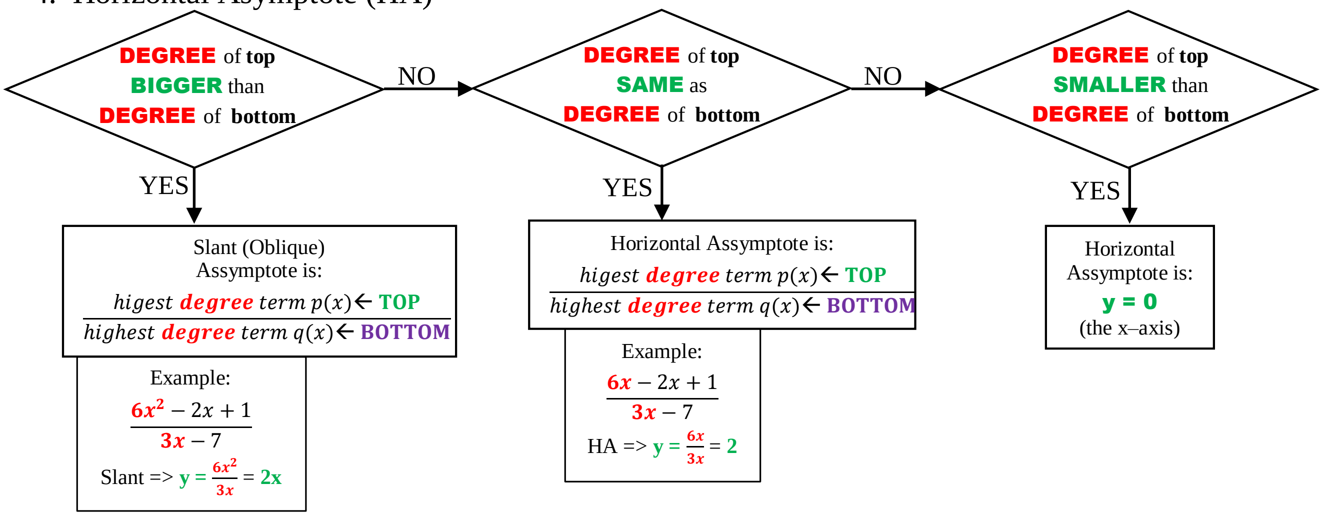
1) A horizontal shift. (left or right): shift **RIGHT** if c is **NEGATIVE**, shift **LEFT** if c is **POSITIVE**
2) A horizontal reflection. (about the y axis)
3) A vertical reflection. (about the x axis)
4) A vertical shift. (up or down): shift **DOWN** if d is **NEGATIVE**, shift **UP** if d is **POSITIVE**

$$F(X) = \overset{3}{-}F(\overset{2}{-}X\overset{1}{+}C)\overset{4}{+}D$$

For: $r(x) = \dfrac{p(x) \leftarrow \text{TOP}}{q(x) \leftarrow \text{BOTTOM}}$

Rational Functions

1. Vertical Asymptote (VA):
   (a) Solve q(x) = 0
   (b) i.e. Set bottom (denominator) of fraction = 0 & solve
2. x-Intercept(s): (?,0)
   (a) Solve p(x) = 0
   (b) i.e. Set top (numerator) of fraction = 0 & solve
3. y-Intercept(s): (0,?)
   (a) Solve r(0)
   (b) i.e. plug in zero for all the x's and solve
4. Horizontal Asymptote (HA)



5. End Behavior:
   As $x \to +\infty$ $\quad r(x) \to$ HA or slant (oblique)
   $x \to -\infty$ $\quad r(x) \to$ HA or slant (oblique)
6. Domain: $x \neq VA$ or $(-\infty, VA) \cup (VA, \infty)$

   Range: **For 1st degree polynomials ONLY:** $y \neq HA$ or $(-\infty, HA) \cup (HA, \infty)$ **i.e.** $\frac{1}{x}, \frac{-3x+2}{x-4}$

   For polynomials greater than degree 1, HA must be determined from the graph. i.e. $\frac{1}{x^2}, \frac{8x}{x^2-3}, \frac{x^2-2x+1}{(x-1)^2}$

Compound Interest & Exponential Growth/Decay

Simple Interest:

1. I = Prt $\Longrightarrow$ I = \$ interest earned; P = principal (starting amount); r = interest rate; t = time (in yrs.)
2. A = P(1+ rt) $\Longrightarrow$ A = \$ final amount; P = principal (starting amount); r = interest rate; t = time (in yrs.)

Compound 'n' times per year:

3. $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $\Longrightarrow$ A = \$ final amount; P = principal (starting amount); r = interest rate; n = # times compounded per year; t = time (in yrs.)

Compound Continuously, Population Growth, Radioactive Decay:

4. $A(t) = Pe^{rt}$ $\Longrightarrow$ A = final amount; P = starting amount or present value; r = growth/decay constant or interest rate; t = time
5. Doubling time: $r = \dfrac{\ln(2)}{doubling\ \ time}$ Half-life: $r = \dfrac{\ln\left(\frac{1}{2}\right)}{half\ -life}$

   Tripling time: $r = \dfrac{\ln(3)}{tripling\ \ time}$ Quadrupling time: $r = \dfrac{\ln(4)}{quadrupling\ \ time}$, etc.

Note: $half\ -life = \dfrac{\ln\left(\frac{1}{2}\right)}{r}$

$doubling\ \ time = \dfrac{\ln(2)}{r}$

$tripling\ \ time = \dfrac{\ln(3)}{r}$, etc.

Revenue Equation

1. Revenue: R(x) = items • price. Price abbreviated with the letter, p. Items represented by the letter, x.
   (The '•' above means "times.")

2. To find the **number of items** needed to get a maximum or minimum:

   (a) Find the x-value of the vertex of the revenue equation using $x = -\frac{b}{2a}$.

3. To find the actual maximum or minimum value for revenue or profit:
   (a) Take the x-value from 2.(a) above
   (b) Plug that x-value into the original revenue equation and solve for R(x) {that is, 'y.'}

4. To find the ideal price to charge:
   (a) Take the x-value from 2.(a) above
   (b) Plug that x-value into the original price equation & solve for the price, p.

5. Example:
   (a) We are told the price of a item, as a function of the number of items, x, is:
   $$p(x) = -3x + 12$$
   (b) Then the revenue equation, as a function of the number of items, x, will be:
   $R(x) = \text{items} \cdot \text{price} = x[-3x + 12] = -3x^2 + 12x$. (frowning parabola means MAX)
   (c) Then the number of items we need to maximize the revenue is:
   $$x = -\frac{b}{2a} = -\frac{12}{2(-3)} = 2 \ \textit{items}$$
   (d) The maximum revenue is: Notice: one number is ½ the other and opposite in sign!
   $$R(2) = -3(2)^2 + 12(2) = -12 + 24 = \$12 \text{ max. revenue}$$
   (d) The price is:
   $$p = -3(2) + 12 = \$6/\text{item}$$

Parabolas:

Standard Form: $f(x) = ax^2 + bx + c$
Transformation Form: $f(x) = a(x - h)^2 + k$

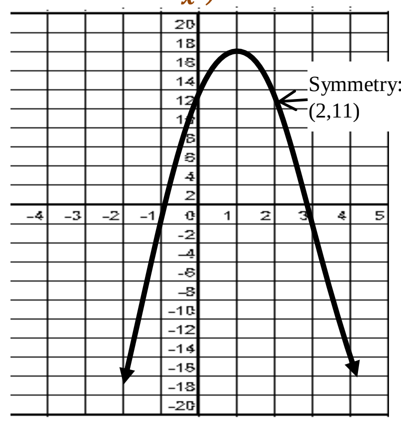
Change sign on h.
Leave k as it is.
Do **NOT** change the sign on k.

1. Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ -OR- (h,k)
2. a < 0, frowns $\Rightarrow$ Vertex is maximum
   a > 0, smiles $\Rightarrow$ Vertex is minimum
3. Axis of Symmetry is the line: $x = -\frac{b}{2a}$ -OR- x = h
4. Y-intercept: (0,c)
5. X-intercepts: $\left(\frac{-b+\sqrt{b^2-4ac}}{2a}, 0\right) \ AND \ \left(\frac{-b-\sqrt{b^2-4ac}}{2a}, 0\right)$ $\Rightarrow$ Factor if you can, if not, quadratic formula.
6. Domain: All real numbers
7. . Range: a < 0: $\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$ *or* $(-\infty, k)$ *or* $(-\infty, y - \textit{value of vertex})$
   a > 0: $\left[f\left(-\frac{b}{2a}\right), \infty\right)$ *or* (k, ∞) *or* $(y - \textit{value of vertex}, \infty)$

Parabola Example:

The price of an item, x, is given by: $\mathbf{p(x) = -5x + 10 + \frac{11}{x}}$ How many items must be sold to maximize the revenue? Graph the revenue function labeling the vertex, points of symmetry and all intercepts.

Solution: The revenue (items • price) brought in by selling x items is: $R(x) = x\left(-5x + 10 + \frac{11}{x}\right)$



R(x) = –5x² + 10x + 11

Vertex: $-\frac{B}{2A} = -\frac{10}{2(-5)} = -\frac{10}{-10} = 1$

$R(1) = -5(1)^2 + 10(1) + 11 = 16 \rightarrow$ **Vertex: (1, 16)**

Y-intercept: $R(0) = -5(0)^2 + 10(0) + 11 = 11 \rightarrow$ **Y.I.: (0, 11)**

X-Intercepts: $0 = -5x^2 + 10x + 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{10^2 - 4(-5)(11)}}{2(-5)} == \frac{-10 \pm \sqrt{100 + 220}}{-10} == \frac{-10 \pm \sqrt{320}}{-10} = \frac{-10 \pm \sqrt{64 * 5}}{-10}$$

$$\frac{-10 \pm 8\sqrt{5}}{-10} = \frac{-2\left(5 \pm 4\sqrt{5}\right)}{-10} = \frac{\left(5 \pm 4\sqrt{5}\right)}{5} = \frac{5}{5} \pm \frac{4\sqrt{5}}{5} = 1 \pm \frac{4\sqrt{5}}{5} \rightarrow X.I.: \left(1 + \frac{4\sqrt{5}}{5}, 0\right) \quad \left(1 - \frac{4\sqrt{5}}{5}, 0\right)$$

(2.8 , 0) **(–0.8 , 0)**

Word Problem Examples

Percentage Calculations:

You receive a 6% raise and are now making $12500. What was your salary before the raise?

Old salary plus 6% of old salary is new salary.

1x + .06x = $12,500 ⟹ 1.06x = $12500 ⟹ x = $11,792.45

You buy a box of day-old cookies for 15% off list (street or retail) price. If the cookies sell for $5.35 normally, how much do you pay with the discount (i.e. what is the sale price)?

List price less 15% of list price is sale price.

5.35 – .15(5.35) = x ⟹ $5.35 – 0.80 = $4.55

-OR-

If the item is 15% OFF, then it is 85% ON: $5.35(.85) = $4.55

Money/Interest 2 eqns. 2 unknowns:

You invest $5000 in two accounts. One pays 3% interest, the other 7%. You earn $225 interest. How much did you invest in each account?

		3%		7%		Total
Line 1	Invest	x	+	y	=	5000
Line 2	Interest	.03x	+	.07y	=	225
Line 3		–.03x	+	–.03y	=	–150
				.04y	=	75

Multiply **everything** on **Line 1** by the **negative** of the smaller of the two numbers on **Line 2** (–.03) & write that result on **Line 3**.

Add **Line 2** & **Line 3** then solve for the variable

.04y = 75 ⟹ y = $1875 @ 7%

x = $5000 – y = $3125 @ 3%

Mixture Problems:

How much 15% anti-freeze should you mix with 45% anti-freeze to get 19 gallons of 26% anti-freeze?

		15%		45%		Total (26%)
Line 1	Gallons	x	+	y	=	19
Line 2	Chemical	.15x	+	.45y	=	4.94
Line 3		–.15x	+	–.15y	=	–2.85
				.3y	=	2.09

UBER-IMPORTANT!
Calculate this value by multiplying the total % (.26) by the total amount (19 gallons)

Multiply **everything** on **Line 1** by the **negative** of the smaller of the two numbers on **Line 2** (–.15) & write that result on **Line 3**.
Add **Line 2** & **Line 3** then solve for the variable

$.3y = 2.09 \Longrightarrow y = 7$ gallons of 45%

$x = 19 - y = 12$ gallons of 15%

Distance – Rate – Time Problems:

Julio and Alejandro entered the New York marathon last year. After two hours, Alejandro was 3 miles ahead of Julio. If Julio's speed was three-fourths Alejandro's speed, how fast were the runners going?

	Distance	=	Rate	*	Time
Julio	D		$\frac{3}{4}R$		2
Alejandro	D +3		R		2

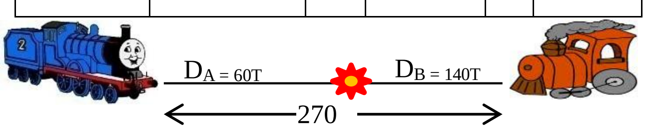
$D = 2\left(\frac{3}{4}R\right) \Longrightarrow D = \frac{3}{2}R$

$D + 3 = 2R \Longrightarrow \frac{3}{2}R + 3 = 2R \Longrightarrow 3 = \frac{1}{2}R \Longrightarrow 6 = R$

Alejandro: **6** mph Julio: $\frac{3}{4}R = 4\frac{1}{2}$ mph

Train A travels 60 kph south. On the same track train B travels 104 kph north. If the trains depart at the same time and their points of departure are 270 km apart, when do the trains collide?

	Distance	=	Rate	*	Time
Train A	D_A		60		T
Train B	D_B		104		T



$D_A = 60T$
$D_B = 104T$
$D_A + D_B = 270$, so $60T + 104T = 270$
$164T = 270$
$T =$ **1.65** hrs. after departure

Airplane/Boat Problems:

An airplane travel 330 miles in 3 hrs. against the wind and makes the return trip in 2 hrs. Find the speed of the plane with no wind (still air) and the speed of the wind.

	Distance	=	Rate	*	Time
Against	330		P – w		3
With	330		P + w		2

$330 = 3(P - w) \Longrightarrow 110 = P - w$
$330 = 2(P + w) \Longrightarrow 165 = P + w$
$275 = 2P \Longrightarrow P =$ **137.5** mph
$165 = P + w \Longrightarrow w =$ **27.5** mph

Properties of Exponents

1. $a^0 = 1$
2. $a^m \cdot a^n = a^{m+n}$
3. $(a^m)^n = a^{m \cdot n}$
4. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
5. $\frac{a^m}{a^n} = a^{m-n}$
6. $a^{-m} = \frac{1}{a^m}$
7. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
8. $\sqrt[n]{a^m} = \sqrt[n]{a}^{\,m} = a^{m/n}$
9. $(\text{negative})^{\text{Odd}} = \text{negative}$

   $(\text{negative})^{\text{Even}} = \text{positive}$

Examples: *Property* 2: $5^3 \cdot 5^{x+1} = 5^{x+4}$ *Property* 8: $\sqrt{x} = x^{\frac{1}{2}}$; $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

Property 5 & 6: $\frac{x^2}{x^7} = x^{2-7} = x^{-5} = \frac{1}{x^5}$ *Property* 3: $(4^{x-2})^{x^2} = 4^{x^3-2x^2}$

Properties of Logarithms

$\log_b x = y \rightarrow b^y = x, \quad b > 0,\ b \neq 1$

Example: $\log_2 16 = ? \rightarrow ? = 4$ since $2^4 = 16$

For all real numbers a, b, N, M>0, a≠1:

1. $\log_b(1) = 0$
2. $\log_b(b) = 1$
3. $\log_b(b^x) = x$
4. $b^{\log_b(x)} = x$, x>0
5. $\log_b(M \cdot N) = \log_b(M) + \log_b(N)$
6. $\log_b(N^m) = m\log_b(N)$
7. $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$
8. $\log_b(a) = x \implies b^x = a$
9. $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$ for a, b, x >0
10. ln(x) is the same thing as $\log_e(x)$

1. $\ln(1) = 0$
2. $\ln(e) = 1$
3. $\ln(e^x) = x$
4. $e^{\ln(x)} = x$, x>0
5. $\ln(M \cdot N) = \ln(M) + \ln(N)$
6. $\ln(N^m) = m\ln(N)$
7. $\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N)$
8. $\ln(a) = x \implies e^x = a$

Examples:

5. $\ln(x^2+1)(x^3-3) = \ln(x^2+1) + \ln(x^3-3)$
6. $\ln(x^2+1)^3(x^3-3)^2$

   $= 3\ln(x^2+1) + 2\ln(x^3-3)$
7. $\ln\frac{(x^2+1)^2}{\left(\sqrt[3]{x^3-3}\right)} = 2\ln(x^2+1) - \frac{1}{3}\ln(x^3-3)$

Solving Logarithmic Equations

1. Get all of the log terms on one side of the equal sign and all of the not log terms on the other side of the equation.
2. Combine all of the log terms into a single log using the properties above.
3. Get rid of the log using property #4 above, remembering to apply that property to BOTH side of the equation.
4. Solve the remaining equation for the variable.
5. CHECK your solution to make sure one or more of the answers doesn't generate a zero or negative log argument.
6. Example:

   $\log_2(x) - 2 = 1 - \log_2(x-7)$

   Step 1: $\log_2(x) + \log_2(x-7) = 3$

   Step 2: $\log_2[x(x-7)] = 3$ ← property #5 above

   Step 3: $2^{\log_2[x(x-7)]} = 3 = 2^3$ ← property #4 above

   Step 4: $x(x-7) = 8 \rightarrow x^2-7x-8 = 0 \rightarrow (x-8)(x+1) = 0 \rightarrow$ Solutions are: $x = +8$ and $x = -1$

   Step 5: CHECK answer: **x = –1 generates a neg. log so discard answer.**

   x = +8 checks OK so keep answer.

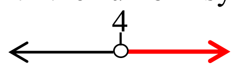
Interval Notation

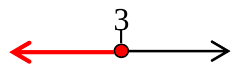
1. Interval Notation: left to right (or down to up). {smallest x, largest x} OR {smallest y, largest y}

   a) > or < or ∞ or –∞ ⟹ round parenthesis: ( )

   b) ≥ or ≤ ⟹ square brackets: [ ]

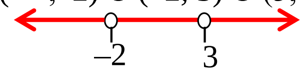
   c) Join two or more sets with the “union” symbol: U

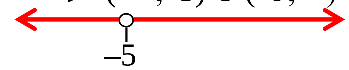




   d) Examples: 1) x > –4 ⟹ (–4, ∞) 2) x ≤ 3 ⟹ (–∞, 3]

   3) x ≠ –2 & x ≠ 3 ⟹ (–∞,–2) U (–2, 3) U (3, ∞) 4) x ≠ –5 ⟹ (–∞,–5) U (–5, ∞)





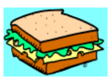
Inequality Rules

When multiplying or dividing an inequality by a **negative** number, **switch the direction** of the inequality.

Example: –2x > 4 → x < –2

Absolute Value Inequalities

$|f(x)| < a$ -OR- $|f(x)| \leq a$

$-a < f(x) < a$  $-a \leq f(x) \leq a$

When function is **less**, sandwich the function between ± value & solve.

$|f(x)| > a$ -OR- $|f(x)| \geq a$

$f(x) < -a$   $f(x) \leq -a$

$f(x) > a$ $f(x) \geq a$

When function is **greater**, split the function like blackjack, & solve.

If you have a **less than** problem, you **SANDWICH** your solution.
If you have a **greater than** problem, you **SPLIT** your hand like double aces in blackjack.

BEFORE you can use this technique, you must get the absolute value all by itself on one side of the inequality.

Example:

9|y+1|+4 > 31

Get the absolute value all by itself by subtracting 4 from both sides, then dividing both sides by 9:

|y+1| > 3 ← Since this is a “greater than,” you split your hand like blackjack changing the sign on the right hand side for the “<” case:

y+1 < –3 and y+ 1 > +3

Now, solve by subtracting one from each inequality:

y < –4 or y > +2

Example:

5|v–1| – 3 ≤ 22

Get the absolute value all by itself by adding 3 to both sides, then dividing both sides by 5:

|v–1| ≤ 5 ← Since this is a “less than,” you sandwich your solution between the + and – of the right hand side of the inequality:

–5 ≤ v–1 ≤ +5

Now solve by adding one to each side of the inequalities:

4 ≤ v ≤ +6

Circles by Completing the Square

Circle in standard form: $(x-h)^2+(y-k)^2=r^2$ $\Longrightarrow$ Found by Completing the Square

change sign on both h & k   sq. root

Center: (h , k)   Radius: r

Example:

$x^2 + y^2 + 4x - 2y - 20 = 0$

Rearrange the equation with the x's next to each other, the y's next to each other and the constant on the right hand side of the equal sign.

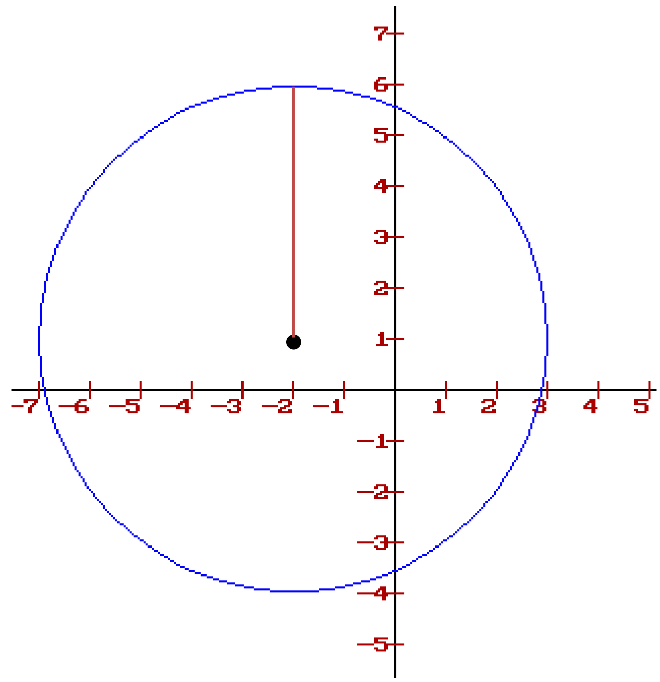
$x^2 + 4x + y^2 - 2y = 20$

$x^2 + 4x + \underline{\ 4\ } + y^2 - 2y + \underline{\ 1\ } = \underbrace{20 + \underline{\ 4\ } + \underline{\ 1\ }}$

`÷ 2   Square   ÷ 2   Square

$(x + \underline{\ 2\ })^2 + (y - \underline{\ 1\ })^2 = 25$

Change Sign   Square Root



Center: (–2, 1)   Radius: 5

Y-Intercept(s):

Set "x" equal to zero and solve equation for "y."

$(0 + 2)^2 + (y - 1)^2 = 25$

$4 + (y - 1)^2 = 25$

$(y - 1)^2 = 21$ ← Take ± square root

$y - 1 = \pm\sqrt{21}$

+ case: $y = 1 + \sqrt{21}$ – case: $y = 1 - \sqrt{21}$

Y– Intercepts: $(0 , 1 + \sqrt{21})$ & $(0 , 1 - \sqrt{21})$

X-Intercept(s):

Set "y" equal to zero and solve equation for "x."

$(x + 2)^2 + (0 - 1)^2 = 25$

$(x + 2)^2 + 1 = 25$

$(x + 2)^2 = 24$ ← Take ± square root

$x + 2 = \pm\sqrt{24} = \pm\sqrt{4}\sqrt{6} = \pm 2\sqrt{6}$

+ case: $x = -2 + 2\sqrt{6}$ – case: $x = -2 - 2\sqrt{6}$

X– Intercepts: $(-2 + 2\sqrt{6} , 0)$ & $(-2 - 2\sqrt{6} , 0)$

FACTORING TECHNIQUES

1. Factor out common factors.

   Example: $3x^3 - 15x^2 + 3x = 3x(x^2 - 5x + 1)$

2. Difference of squares: $a^2 - b^2 = (a + b)(a - b)$

   Example: $36x^2 - 121 = (6x)^2 - 11^2 = (6x + 11)(6x - 11)$

3. Square of a sum: $(a + b)^2 = a^2 + 2ab + b^2$.

   Example: $25x^2 + 20x + 4 = (5x + 2)^2$

4. Square of a difference: $(a - b)^2 = a^2 - 2ab + b^2$.

   Example: $100x^2 - 60x + 9 = (10x - 3)^2$

5. Factoring trinomials in the form: $x^2 + bx + c$.

   Example: $x^2 - 14x + 48 = (x - 8)(x - 6)$

6. Factoring trinomials in the form: $ax^2 + bx + c$.

   Example: $28x^2 - 17x - 3 = (7x + 1)(4x - 3)$

7. Difference of cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

   Example: $8x^3 - 1 = (2x)^3 - 1^3 = (2x - 1)(4x^2 + 2x + 1)$

8. Sum of cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

   Example: $27x^6 + 64y^3 = (3x^2)^3 + (4y)^3 = (3x^2 + 4y)(9x^4 - 12x^2y + 16y^2)$

9. Factor by grouping.

   Example: $3x^2 + x - 6x - 2 = x(3x + 1) - 2(3x + 1) = (3x + 1)(x - 2)$