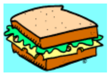
Solving an absolute value inequality: advanced

$|f(x)| < a$ -OR- $|f(x)| \le a$

$-a < f(x) < a$  $-a \le f(x) \le a$

When function is **less**, sandwich the function between ± value & solve.

$|f(x)| > a$ -OR- $|f(x)| \ge a$

$f(x) < -a$

$f(x) > a$

$f(x) \le -a$

$f(x) \ge a$

When function is **greater**, split the function like blackjack, & solve.

Mnemonic:
Sandwiches have **LESS** calories **than** banana splits.
Banana **SPLITS** have **GREATER** calories **than** sandwiches

As the above graphic summarizes:

If you have a **less than** problem, you **SANDWICH** your solution.

If you have a **greater than** problem, you **SPLIT** your hand like double aces in blackjack.

BEFORE you can use this technique, you must get the absolute value all by itself on one side of the inequality.

Example:

9|y+1|+4 > 31

Get the absolute value all by itself by subtracting 4 from both sides, then dividing both sides by 9:

|y+1| > 3 ← Since this is a "greater than," you split your hand like blackjack changing the sign on the right hand side for the "<" case:

y+1 < –3 -OR- y+ 1 > +3

Now, solve by subtracting one from each inequality:

y < –4 -OR- y > +2

Example:

5|v–1| – 3 ≤ 22

Get the absolute value all by itself by adding 3 to both sides, then dividing both sides by 5:

|v–1| ≤ 5 ← Since this is a "less than," you sandwich your solution between the + and – of the right hand side of the inequality:

–5 ≤ v–1 ≤ +5

Now solve by adding one to each side of the inequalities:

–4 ≤ v ≤ +6

EXCEPTION: Absolute value inequalities and negative numbers.

An absolute value MUST be greater than or equal to zero to have a solution.

Example:

|w + 1| ≤ –5 Has **NO SOLUTION** since it is impossible for an absolute value to be less than zero.

|w + 8| > –2 Has **ALL REAL NUMBERS** as its solution since an absolute value is always positive, hence greater than any negative number