EXAMPLES

1. The jet aircraft has a total mass of $22 \text{ Mg}$ and a center of mass at $G$. Initially at take-off the engines provide a thrust of $2T = 4 \text{ kN}$ and $T' = 1.5 \text{ kN}$. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the two wing wheels located at $B$. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.

Data:

$\begin{align*}
\text{m} &= 22 \times 10^3 \text{ kg} \\
2T &= 4000 \text{ N} \\
T' &= 1500 \text{ N}
\end{align*}$

$\text{Solution}$

1) Geometry:

$\sin 10^\circ = \frac{a}{3} \Rightarrow a = 0.52094 \text{ m}$

$\cos 10^\circ = \frac{b}{3} \Rightarrow b = 2.9544 \text{ m}$

$\text{2) Forces:}$

$\sum F_x = ma_{Gx} \Rightarrow \quad T' + 2T = ma_{Gx}$

$1500 + 4000 = 22000 a_{Gx} \Rightarrow \quad a_{Gx} = 0.025 \text{ m/s}^2$

$\sum F_y = ma_{Gy} = 0 \Rightarrow \quad N_A + 2N_B = 22000 \times 9.81 = 0$

$N_A + 2N_B = 215,820$

$3) \sum M_G = 0 \Rightarrow \quad -T(2.5 - 1.2) \quad -T(2.3 - 1.2) \quad -2N_B \times 3 + N_A \times 6 = 0$

$N_A - N_B = 1,058.33$

$N_A = 72,646 \text{ N} \quad N_B = 71,587 \text{ N}$

$\text{2. The pipe has a length of 3 m and a mass of 500 kg. It is attached to the back of the truck using a 0.6 m long chain AB. If the coefficient of kinetic friction at C is } \mu = 0.4,$

$determine the acceleration of the truck if the angle $\theta = 10^\circ$ with the road as shown.

Data:

$\begin{align*}
L_{CA} &= 3 \text{ m} \\
L_{AB} &= 0.6 \text{ m} \\
\mu &= 0.4 \\
W &= 4905 \text{ N} \\
m &= 500 \text{ kg}
\end{align*}$

$\text{Solution}$

1) Geometry:

$\sin 10^\circ = \frac{a}{3} \Rightarrow a = 0.52094 \text{ m}$

$\cos 10^\circ = \frac{b}{3} \Rightarrow b = 2.9544 \text{ m}$

2) Forces:

$\sum F_x = ma_{Gx} \Rightarrow \quad -\mu N_C + T \cos 52.98^\circ = ma_{Gx}$

$-0.4N_C + 0.60209T = 500a_{Gx}$

(1)

$\sum F_y = 0 \Rightarrow \quad N_C - W + T \sin 52.98^\circ = 0$

3) Moments:

$\sum M_C = \sum (M_k)_C$

$-W(1.5 \cos 10^\circ) + 3(T \sin 42.98^\circ) = -ma_{Gx}(1.5 \sin 10^\circ)$

$2.0452T - 72.457 = -130.23a_{Gx}$

(3)

Solving the equations $T = 3.39 \text{ kN}$, $N_C = 2.2 \text{ kN}$

$a_{Gx} = 2.33 \text{ m/s}^2$

3. The pipe has a mass of 460 kg and is held in place on the truck bed using the two boards $A$ and $B$. Determine the greatest acceleration of the truck so that the pipe begins to lose contact at $A$ and the bed of the truck and starts to pivot about $B$. Assume board $B$ will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board $B$ exert on the pipe during the acceleration?

Data:

$\begin{align*}
m &= 460 \text{ kg} \\
W &= 4512.6 \text{ N}
\end{align*}$
At the moment of interest $\alpha = 0$ and $N = 0$.

1) Equilibrium in the x - direction
\[
\sum F_x = ma_x \Rightarrow P_x = ma_x
\]

2) Equilibrium in the y - direction
\[
\sum F_y = ma_y \Rightarrow P_y - W = 0 \Rightarrow P_y = 4513 \text{ N}
\]

3) Equilibrium of moments about $G$
\[
\sum M_G = 0 \Rightarrow P_x \times 0.4 - P_y \times 0.3 = 0 \Rightarrow P_x = 3384 \text{ N}
\]
\[a_x = 7.36 \text{ m/s}^2\]

Curvilinear Translation:
In a body subjected to a Curvilinear Translation all the points in the body travel along parallel curvilinear paths.

Here it is convenient to write the equations of motion in normal and tangential coordinates.
\[
\sum F = ma, \quad \sum F = ma, \quad \sum M_G = 0
\]

If the summation of moments about $G$ is replaced by the summation about another point $B$ then we must account for the kinetic moments
\[
\sum M_b = \sum (M_b)_{a} = e \cdot (ma_{a}) - h \cdot (ma_{a})
\]

EXAMPLES

1. The arm $BDE$ of the industrial robot manufactured by Cincinnati Milacron is activated by applying the torque of $50 \text{ Nm}$ to link $CD$. Determine the reactions at the pins $B$ and $D$ when the links are in the position shown and have an angular velocity of $2 \text{ rad/s}$.

Data:
- $m_{a1} = 10 \text{ kg}$
- $m_{a2} = 12 \text{ kg}$
- $M = 50 \text{ Nm}$
- $\omega = 2 \text{ rad/s}$

This is a curvilinear translation

1) Element CD: Rotation about fixed point $C$
\[a_C = \omega^2 r_{D/C} = 2^2 \times 0.6 \text{ m/s}^2\]
\[a_D = 2.4 \text{ m/s}^2\]
\[\sum M_C = 0 \Rightarrow D_x \times 0.6 - 50 = 0 \Rightarrow D_x = 83.3 \text{ N}\]

2) In member $BDE$ take $\sum M_D = \sum (M_D)_{a}$, this eliminates $D_x$.
\[W_1 = 10 \times 9.81 = 98.1 \text{ N}\]
\[W_2 = 12 \times 9.81 = 117.72 \text{ N}\]
\[-B_x \times 0.22 - 98.1 \times 0.365 - 117.72 \times 1.1 = (-10 \times 2.4) \times 0.365 - (12 \times 2.4) \times 1.1\]
\[B_x = -568 \text{ N}\]

3) Find $D_y$ using equilibrium in the y-direction
\[B_y + D_y - W_1 - W_2 = -m_1a_{o1} - m_2a_{o2}\]
\[-567.54 + D_y - 98.1 - 117.72 = -24 - 28.8\]
\[D_y = 731 \text{ N}\]

4) Find $B_y$ using equilibrium in the x-direction. Because at this instant $a_x = a_y = 0$ we have
\[B_x + D_x = 0 \Rightarrow B_y = -83.3 \text{ N}\]

2. The two 3-lb rods $EF$ and $HI$ are fixed (welded) to the link $AC$ at $E$. Determine the normal force $N_E$, the shear force $V_E$, and moment $M_E$, which the bar $AC$ exerts on $FE$ at $E$ if at the instant $\theta = 30^\circ$ link $AB$ has an angular velocity $\omega = 5 \text{ rad/s}$ and an angular acceleration $\alpha = 8 \text{ rad/s}^2$ as shown.

Data:
- $W_{HI} = W_{EF} = 3 \text{ lb}$
- $m_{EF} = 0.093168 \text{ slug}$
- $\omega = 5 \text{ rad/s}$
- $\alpha = 8 \text{ rad/s}^2$
- $N_E = ?$
- $V_E = ?$
- $M_E = ?$

SOLUTION
The bars EFHI are undergoing a curvilinear translation, therefore their angular velocity and acceleration must be zero.

1) Find the position of G:

\[ G_x = 0 \]
\[ G_y = \frac{3lb \times (-1 ft) + 3lb \times (-2 ft)}{6lb} = -1.5 ft \]

2) Equilibrium in the x-direction:

\[ \sum F_x = N_E = 2ma_{Gx} \]

3) Equilibrium in the y-direction:

\[ \sum F_y = -2W - V_E = 2ma_{Gy} \]

4) Equilibrium of moments about G:

\[ \sum M_G = M_E - V_E \times 1.5 = 0 \]

5) We must have \( \mathbf{a}_G = \mathbf{a}_x \) and we can find \( \mathbf{a}_x \):

\[ \mathbf{a}_x = \mathbf{a} \times \mathbf{r}_{4/B} - \mathbf{a} \mathbf{r}_{4/B} = -3\cos30^\circ \mathbf{i} + 3\sin30^\circ \mathbf{j} \]
\[ \mathbf{a}_x = (8k) \times (-2.5981\mathbf{i} + 1.5\mathbf{j}) - 5^2 \times (-2.5981\mathbf{i} + 1.5\mathbf{j}) \]
\[ \mathbf{a}_x = 531 - 58.3 \mathbf{j} \]

The moment equation can be replaced by a summation about any point P lying inside or outside the body. In that case we must take into account the moments due to \( \sum I_m \) due to \( I_\alpha \), \( ma_{Gx} \) and \( ma_{Gy} \).

In many problems it is convenient to choose moments about \( O \). This eliminates the unknown rection \( F_O \), and the kinetic moments become

\[ \sum M_O = \sum (M_k) = m\omega_a + I_\alpha \]

Note that the component \( ma_{Gx} \) does not appear, because its line of action goes through the point \( O \).

Furthermore, using \( a_{Gx} = \mathbf{r}_a \mathbf{\alpha} \) we have \( \sum M_O = \mathbf{r}_a \mathbf{\alpha} + I_\alpha \) or \( \sum M_O = (\mathbf{r}_a^2 + I) \mathbf{\alpha} \) and from the parallel axis theorem \( I_\alpha = I + m\mathbf{\alpha}^2 \). Therefore we also have \( \sum M_O = I_\alpha \)

2) Equilibrium of moments:

\[ \sum M_O = -5\theta = 0.10356\alpha \Rightarrow \alpha = -48.281\theta \]

3) Kinematics:

\[ \int_{t/2}^{t/4} 48.281\theta d\theta = \frac{\theta^2}{2} = -24.141\theta^2 \]
\[ \theta = 9.45 \text{ rad/s} \]

2. The lightweight turbine consists of a rotor which is powered from a torque applied at its center. At the instant the rotor is horizontal it has an angular velocity of 15 rad/s and a clockwise angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force and moment at a section through A. Assume the rotor is a 50 m long slender rod, having a mass of 3 kg/m.
3. The cord is wrapped around the inner core of the spool. If a 5 lb block is suspended from the cord and released from rest, determine the spool's angular velocity when t = 3 s. Neglect the mass of the cord. The spool has a weight of 180 lb and the radius of gyration about the axle A is $r_A = 1.25$ ft. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

**SOLUTION**

First calculate the Moment of Inertia about A:

$I_A = m_A k_A^2 = 5.590 \times 1.25^2$

$I_A = 8.7345 \text{slug} \cdot \text{ft}^2$

Data:

- $W_A = 180 \text{lb}$
- $W_B = 5 \text{lb}$
- $m_B = 5.5901 \text{slug}$
- $m_B = 0.15528 \text{slug}$
- $k_A = 1.25 \text{ft}$
- $\omega(3s) = ?$

1) Consider the whole system:

$\sum M_A = \sum (M_i)_A$

$W_A r_A = m_B \omega^2 + I_A \omega$

$5 \times 1.5 = 0.15528 \times 1.5^2 \omega + 8.7345 \omega$

$\omega = 0.82564 \text{ rad} / \text{s}$

$\omega = \omega_B + \omega_t = 0.82564 \times 3 \Rightarrow \omega = 2.48 \text{ rad} / \text{s}$

2) Consider each particle separately.

a) Spool

$\sum M_A = I_A \omega$

$T \times 1.5 = 8.7345 \omega \Rightarrow T = 5.823 \omega$

b) Weight B:

$\sum F_y = m_B a_B \Rightarrow 5 - T = 0.15528 \times 1.5 \omega$

$T = 5 - 0.23292 \times \omega$

Solving the system:

$\omega = 0.826 \text{ rad} / \text{s}$

Same as before