EXAMPLES

1. The jet aircraft has a total mass of 22 *Mg* and a center of mass at *G*. Initially at take-off the engines provide a thrust of $2T = 4\ kN$ and $T' = 1.5\ kN$. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at *B*. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.

Data:

$m = 22\times10^6\ gr = 22\times10^3\ kg$ $\qquad a_G = ?$

$2T = 4000\ N$ $\qquad N_A = ?$

$T' = 1500\ N$ $\qquad N_B = ?$

1

1) $\sum F_x = m a_{Gx} \quad \Rightarrow \quad T' + 2T = m a_{Gx}$

$1{,}500 + 4{,}000 = 22{,}000\, a_{Gx} \quad \Rightarrow \quad a_{Gx} = 0.25\, m/s^2$

2) $\sum F_y = m a_{Gy} = 0 \quad \Rightarrow \quad N_A + 2N_B - 22{,}000\times 9.81 = 0$

$N_A + 2N_B = 215{,}820$

3) $\sum M_G = 0 \Rightarrow$

$-T'(2.5-1.2) - 2T(2.3-1.2) - 2N_B\times3 + N_A\times6 = 0$

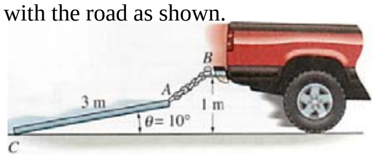
$N_A - N_B = 1{,}058.33$

$N_A = 72{,}646\, N = 72.6\, kN$

$N_B = 71{,}587\, N = 71.6\, kN$

2

2. The pipe has a length of 3 *m* and a mass of 500 *kg*. It is attached to the back of the truck using a 0.6 *m* long chain *AB*. If the coefficient of kinetic fiction at *C* is $\mu = 0.4$, determine the acceleration of the truck if the angle $\theta = 10^\circ$ with the road as shown.



Data:

$L_{CA} = 3m$

$L_{AB} = 0.6m$

$m = 500kg$

$W = 4{,}905N$

$\mu_k = 0.4$

$\theta = 10^\circ$

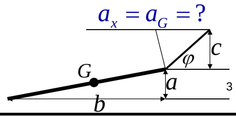
$a_x = a_G = ?$

SOLUTION

1) Geometry:

$\sin10^\circ = \dfrac{a}{3} \quad \Rightarrow \quad a = 0.52094\, m$

$\cos10^\circ = \dfrac{b}{3} \quad \Rightarrow \quad b = 2.9544\, m$



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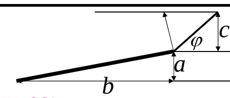
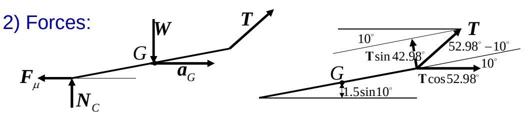
$c = 1.0 - 0.52094 = 0.47906\, m$



$\sin\varphi = \dfrac{0.47906}{0.6} = 0.79843 \Rightarrow \quad \varphi = 52.98^\circ$

2) Forces:



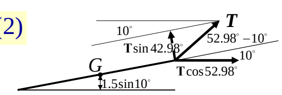
$\sum F_x = m a_G \quad \Rightarrow \quad -\mu N_C + T\cos52.98^\circ = m a_G$

$-0.4N_C + 0.60209T = 500\, a_G \qquad (1)$

$\sum F_y = 0 \quad \Rightarrow \quad N_C - W + T\sin52.98 = 0$

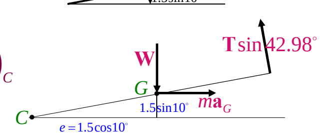
4

$N_C + 0.79843T = 4{,}905.0 \quad (2)$



3) Moments:

$\sum M_C = \sum(\mathcal{M}_k)_C$



$-W(1.5\cos10^\circ) + 3(T\sin42.98^\circ) = -m a_G(1.5\sin10^\circ)$

$2.0452T - 72.457 = -130.23 a_G \qquad (3)$

Solving the equations $\quad T = 3.39\, kN \quad , \quad N_C = 2.2\, kN$

$a_G = 2.33\, m/s^2$

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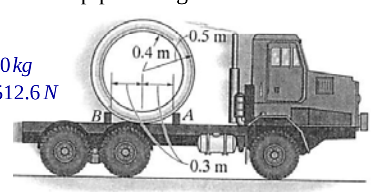
3. The pipe has a mass of 460 *kg* and is held in place on the truck bed using the two boards *A* and *B*. Determine the greatest acceleration of the truck so that the pipe begins to lose contact at *A* and the bed of the truck and starts to pivot about *B*. Assume board *B* will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board *B* exert on the pipe during the acceleration?

Data:

$m = 460\, kg$

$W = 4512.6\, N$



SOLUTION

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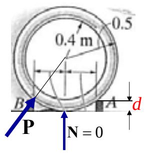
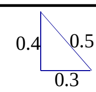
At the moment of interest
$\alpha = 0$ and $\mathbf{N} = 0$.





$d = 0.1$

1) Equilibrium in the x - direction

$$\sum F_x = m a_x \Rightarrow P_x = m a_x$$

2) Equilibrium in the y - direction

$$\sum F_y = m a_y \Rightarrow P_y - W = 0 \Rightarrow P_y = 4513\,N$$

3) Equilibrium of moments about G

$$\sum M_G = 0 \Rightarrow P_x \times 0.4 - P_y \times 0.3 = 0 \Rightarrow P_x = 3384\,N$$

$$a_x = 7.36\,m/s^2$$

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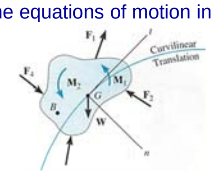
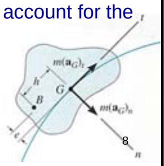
Curvilinear Translation:
In a body subjected to a Curvilinear Translation all the points in the body travel along parallel curvilinear paths.
Here it is convenient to write the equations of motion in normal and tangential coordinates.

$$\sum F_n = m a_{Gn}$$
$$\sum F_t = m a_{Gt}$$
$$\sum M_G = 0$$



If the summation of moments about *G* is replaced by the summation about another point B then we must account for the kinetic moments

$$\sum M_B = \sum (\mathcal{M}_k)_B = e \cdot (m a_{Gt}) - h \cdot (m a_{Gn})$$



EXAMPLES

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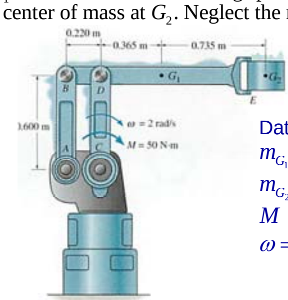
1. The arm *BDE* of the industrial robot manufactured by Cincinnati Milacron is activated by applying the torque of $M = 50\ N \cdot m$ to link *CD*. Determine the reactions at the pins *B* and *D* when the links are in the position shown and have an angular velocity of $2\ rad/s$. The uniform arm *BDE* has a mass of $10\ kg$ and a center of mass at G_1. The container held in its grip at *E* has a mass of $12\ kg$ and a center of mass at G_2. Neglect the mass of links *AB* and *CD*



Data:
$m_{G_1} = 10\,kg$
$m_{G_2} = 12\,kg$
$M = 50\,N \cdot m$
$\omega = 2\ rad/s$

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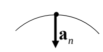
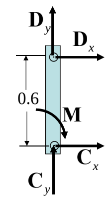
This is a curvilinear translation

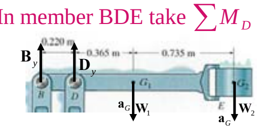
1) Element CD: Rotation about fixed point C

$$a_D = \omega^2 r_{D/C} = 2^2 \times 0.6$$
$$a_D = 2.4\ m/s^2 = a_G$$
$$\sum M_C = 0 \Rightarrow D_x \times 0.6 - 50 = 0$$
$$D_x = 83.3\,N$$





2) In member BDE take $\sum M_D = \sum (\mathcal{M}_k)_D$, this eliminates D_y



$W_1 = 10 \times 9.81 = 98.1\,N$
$W_2 = 12 \times 9.81 = 117.72$

$$-B_y \times 0.22 - 98.1 \times 0.365 - 117.72 \times 1.1 = (-10 \times 2.4) \times 0.365 - (12 \times 2.4) \times 1.1$$

$$B_y = -568\,N$$

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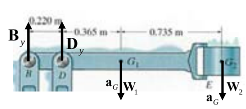
3) Find D_y using equilibrium in the y-direction

$$B_y + D_y - W_1 - W_2 = -m_1 a_{G_1} - m_2 a_{G_2}$$



$$-567.54 + D_y - 98.1 - 117.72 = -24 - 28.8$$

$$D_y = 731\,N$$

4) Find B_x using equilibrium in the x-direction. Because at this instant $a_x = a_t = 0$ we have

$B_x + D_x = 0 \Rightarrow B_x = -83.3\,N$

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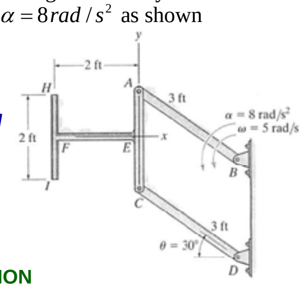
2. The two 3-*lb* rods *EF* and *HI* are fixed (welded) to the link *AC* at *E*. Determine the normal force N_E, the shear force V_E, and moment M_E, which the bar *AC* exerts on *FE* at *E* if at the instant $\theta = 30^\circ$ link *AB* has an angular velocity $\omega = 5\,rad/s$ and an angular acceleration $\alpha = 8\,rad/s^2$ as shown

Data:
$W_{HI} = W_{EF} = 3\,lb$
$m_{HI} = m_{EF} = 0.093168\,slug$
$\omega = 5\,rad/s$
$\alpha = 8\,rad/s^2$
$N_E = ?$
$V_E = ?$
$M_E = ?$



SOLUTION

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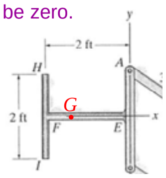
The bars EFHI are undergoing a curvilinear translation, therefore their angular velocity and acceleration must be zero.

1) Find the position of G:

$G_y = 0$

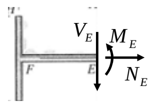
$G_x = \dfrac{3lb \times (-1\,ft) + 3lb \times (-2\,ft)}{6lb} = -1.5\,ft$



2) Equilibrium in the x-direction:

$$\sum F_x = N_E = 2\,m\,a_{Gx}$$

3) Equilibrium in the y-direction:



$$\sum F_y = -2W - V_E = 2\,m\,a_{Gy}$$

4) Equilibrium of moments about G:

$$\sum M_G = M_E - V_E \times 1.5 = 0$$

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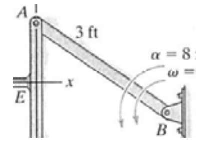
5) We must have $\mathbf{a}_G = \mathbf{a}_A$ and we can find $\mathbf{a}_A$:



$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_{A/B} - \omega^2 \mathbf{r}_{A/B} \quad \mathbf{r}_{A/B} = -3\cos 30^\circ\,\mathbf{i} + 3\sin 30^\circ$

$\mathbf{a}_A = (8\mathbf{k}) \times (-2.5981\mathbf{i} + 1.5\mathbf{j}) - 5^2 \times (-2.5981\mathbf{i} + 1.5\mathbf{j})$

$\mathbf{a}_A = 53\mathbf{i} - 58.3\,\mathbf{j}$

$N_E = 9.87\,lb$

$V_E = -4.86\,lb$

$M_E = -7.29\,lb \cdot ft$

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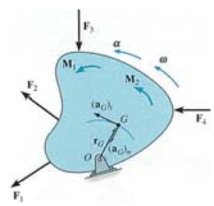
ROTATION ABOUT A FIXED AXIS

$\mathbf{a}_G = \alpha\, r_G\, \mathbf{a}_{Gt} + \omega^2 r_G\, \mathbf{a}_{Gn}$

$\sum F_n = m\,a_{Gn} = m\,\omega^2 r_G$

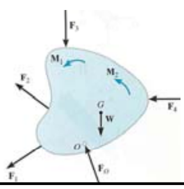
$\sum F_t = m\,a_{Gt} = m\,\alpha\, r_G$

$\sum M_G = I_G \alpha$



Free Body Diagram

Kinetic Diagram



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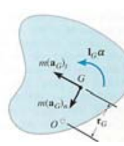

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The moment equation can be replaced by a summation about any point P lying inside or outside the body. In that case we must take into account the moments $\sum(\mathcal{M}_k)_P$ due to $I_g\alpha$, $m\,a_{Gn}$ and $m\,a_{Gt}$.

In many problems it is convenient to choose moments about O. This eliminates the unknown rection $\mathbf{F}_O$, and the kinetic moments become

$$\sum M_O = \sum(\mathcal{M}_k)_O = m\,r_G\,a_{Gt} + I_G\alpha$$

Note that the component $m\,a_{Gn}$ does not appear, because its line of action goes through the point O.

Furthermore, using $a_{Gn} = r_G\,\alpha$ we have $\sum M_O = r_G^2 m\alpha + I_G\alpha$ or $\sum M_O = \left(r_G^2 m + I_G\right)\alpha$ and from the parallel axis theorem $I_O = I_G + m r_G^2$. Therefore we also have $\sum M_O = I_O\alpha$

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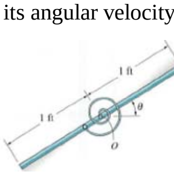
EXAMPLES

1. The 10 lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5\ lb \cdot ft/rad$, so that the torque developed is $M = (5\theta)\ lb \cdot ft$, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90^\circ$, determine its angular velocity when $\theta = 45^\circ$.

SOLUTION

There are no forces in so only the moments equation applies:

$$\sum M_O = I_O\,\alpha$$



Data:

$W = 10\,lb$

$m = 0.31056\,slug$

$k = 5\,lb \cdot ft/rad$

$M = 5\theta\,lb \cdot ft$

$\theta_0 = \pi/2$

$\theta_1 = \pi/4$

$\omega(\pi/4) = ?$

1) Moment of inertia:

$$I_O = \frac{1}{12} m\,\ell^2 = \frac{1}{12} \times 0.31056 \times 2^2 = 0.10352\,slug \cdot ft^2$$

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2) Equilibrium of moments:

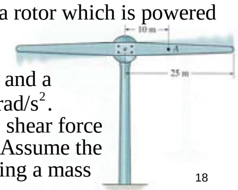
$$\sum M_O = -5\theta = 0.10356\,\alpha \;\Rightarrow\; \alpha = -48.281\theta$$

3) Kinematics: $\alpha\,d\theta = \omega\,d\omega$

$$-\int_{\pi/2}^{\pi/4} 48.281\theta\,d\theta = \int_0^{\omega} \omega\,d\omega \;\Rightarrow\; \frac{1}{2}\omega^2 = -24.141\theta^2\Big|_{\pi/2}^{\pi/4}$$

$$\omega = 9.45\,rad/s$$

2. The lightweight turbine consists of a rotor which is powered from a torque applied at its center. at the instant the rotor is horizontal it has an angular velocity 0f 15 rad/s and a clockwise angular acceleration of 8 rad/s^2. Determine the internal normal force, shear force and moment at a section through A. Assume the rotor is a 50 m long slender rod, having a mass of 3 kg/m.



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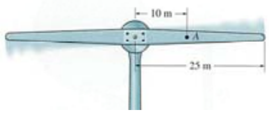
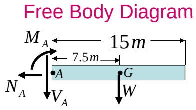


Data:
$\omega = 15 rad/s$
$\alpha = 8 rad/s^2$
$\gamma = 3 kg/m$
$N_A = ?$
$V_A = ?$
$M_A = ?$

SOLUTION

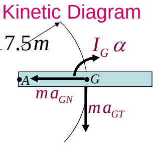
Consider the segment of the blade to the right of *A*.

Free Body Diagram



$m = 45 kg$
$W = 441.45 N$

Kinetic Diagram



1) Equilibrium in the Normal direction:

$$\sum F_N = m a_{GN} \Rightarrow N_A = m\omega^2 r_{G/O} = 45 \times 15^2 \times 17.5$$

$$N_A = 177188 N = 177.2 kN$$

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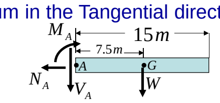
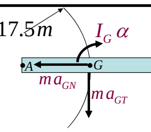
2) Equilibrium in the Tangential direction:





$$\sum F_T = m a_{GT} \Rightarrow V_A + W = m\alpha r_{G/O} \Rightarrow V_A + 441.45 = 45 \times 8 \times 17.5$$

$$V_A = 5858.6 N = 5.86 kN$$

3) Find the Moment of Inertia:

$$I_G = \frac{1}{12} m \ell^2 = \frac{1}{12} \times 45 \times 15^2 = 843.75 kg \cdot m^2$$

4) Equilibrium of Moments about *A*: Recall that $a_{GT} = \alpha r_{G/O}$

$$\sum M_A = \sum (\mathcal{M}_k)_A \Rightarrow M_A + W r_{G/A} = m(\alpha r_{G/O}) r_{G/A} + I_G \alpha$$

$$M_A + 441.45 \times 7.5 = 45 \times (8 \times 17.5) \times 7.5 + 843.75 \times 8$$

$$M_A = 50689 N \cdot m = 50.7 kN \cdot m$$

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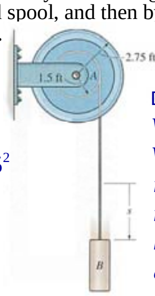
3. The cord is wrapped around the inner core of the spool. If a 5 *lb* block *B* is suspended from the cord and released from rest, determine the spool's angular velocity when $t = 3 s$. Neglect the mass of the cord. The spool has a weight of 180 *lb* and the radius of gyration about the axle *A* is $k_A = 1.25 ft$. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.



SOLUTION

First calculate the Moment of Inertia about *A*:

$$I_A = m_A k_A^2 = 5.5901 \times 1.25^2$$

$$I_A = 8.7345 slug \cdot ft^2$$

Data:
$W_A = 180 lb$
$W_B = 5 lb$
$m_A = 5.5901 slug$
$m_B = 0.15528 slug$
$k_A = 1.25 ft$
$\omega(3s) = ?$

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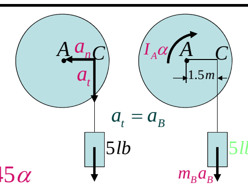
1) Consider the whole system:

$$\sum M_A = \sum (\mathcal{M}_k)_A$$
$$\Downarrow$$
$$W_B r_{B/A} = m_B r_{C/A}^2 \alpha + I_A \alpha$$



$$5 \times 1.5 = 0.15528 \times 1.5^2 \alpha + 8.7345 \alpha$$

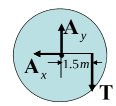
$$\alpha = 0.82564 \ rad/s^2$$

$$\omega = \omega_0 + \alpha t = 0.82564 \times 3 \Rightarrow \omega = 2.48 rad/s$$

2) Consider each particle separate.

a) Spool $\quad \sum M_A = I_A \alpha$

$$T \times 1.5 = 8.7345\alpha \Rightarrow T = 5.823\alpha$$



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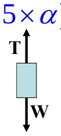
b) Weight *B*:

$$\sum F_y = m_B a_{By} \Rightarrow 5 - T = 0.15528 \times (1.5 \times \alpha)$$

$$T = 5 - 0.23292 \times \alpha$$



Solving the system:

$$\alpha = 0.826 rad/s^2$$

Same as before

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