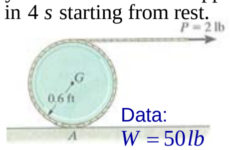
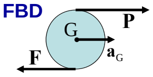
3. A cord of negligible mass is wrapped around the outer surface of the 50 lb cylinder and its end is subjected to a constant horizontal force of $P = 2\ lb$. If the cylinder rolls without slipping at A, determine its angular velocity in 4 s starting from rest. Neglect the thicknes of the cord.



SOLUTION

FBD



No slip
$\Downarrow$
$v_G = r\omega$

Data:
$W = 50\,lb$
$m = 1.5528\,slug$
$P = 2\,lb$
$v(0) = 0$
$\omega(4s) = ?$

$$I_G = \frac{1}{2} m r^2 = \frac{1}{2} \times 1.5528 \times 0.6^2 = 0.2795\,slug \cdot ft^2$$

$$L_x: \quad m(v_{Gx})_1 + \int_0^4 (P - F)\,dt = m(v_{Gx})_2$$

$$0 + (2 - F) \times 4 = 1.5528 \times (0.6\omega_2)$$

$$8 - 4F = 0.93168\omega_2 \quad (1)$$

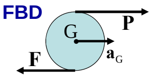
1

$$H_G: \; I_G\,\omega_1 + \int_0^4 (P + F)\,r\,dt = I_G\,\omega_2$$

$$(2 + F) \times 0.6 \times 4 = 0.2795\omega_2$$

$$8 + 4F = 0.46583\,\omega_2 \quad (2)$$

FBD



Adding (1) and (2) $16 = 1.3975\omega_2$

$$\omega_2 = 11.45\,rad/s$$

Alternatively

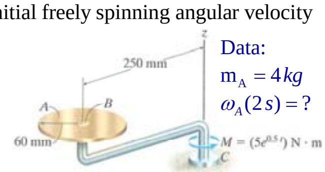
$$\left.\begin{aligned}\sum F_x &= m a_{Gx} \\ \sum M_G &= I_G\,\alpha\end{aligned}\right\} \Rightarrow \left.\begin{aligned} P - F &= m r \alpha \\ (P + F) r &= I_G \alpha\end{aligned}\right\} \Rightarrow \left\{\begin{aligned} 2 - F &= 0.93168\alpha \\ 2 + F &= 0.46583\alpha\end{aligned}\right.$$

Solving $F = 0.66667\,lb$ and $\alpha = 2.8622\,rad/s^2$

$$\omega_2 = \int_0^4 \alpha\,dt = \int_0^4 2.8622\,dt = 11.45$$

2

4. A 4 kg disk A is mounted on arm BC, which has a negligible mass. If a torque of $M = (5e^{0.5t})\,N \cdot m$, where t is in seconds, is applied to the arm at C, determine the angular velocity of BC in 2 s starting from rest. Solve the problem assuming that:
(a) The disk is set on a smooth bearing at B so that it rotates with curvilinear translation.
(b) The disk is fixed to the shaft BC.
(c) The disk is given an initial freely spinning angular velocity of $\omega_D = (-80k)\,rad/s$



Data:
$m_A = 4\,kg$
$\omega_A(2s) = ?$

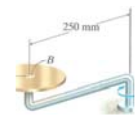
SOLUTION

$$I_G = \frac{1}{2} m r^2 = \frac{1}{2} \times 60 \times 0.6^2 = 0.0072\,kg \cdot m^2$$

3

(a) Smooth bearing, disk in pure translation:

$$(H_C)_1 + \int_0^2 M_C\,dt = (H_C)_2$$



$$0 + \int_0^2 5e^{0.5t}\,dt = m r v_B = m r^2 \omega_{BC}$$

$$\frac{5}{0.5}(e^1 - e^0) = 4 \times 0.25^2\,\omega \Rightarrow \quad \omega = 68.7\,rad/s$$

(b) Fixed disk, rigid body rotation:

$$\int_0^2 5e^{0.5t}\,dt = m r^2 \omega + I_G \omega$$

$$17.183 = 4 \times 0.25^2\,\omega + 0.0072\,\omega \Rightarrow \quad \omega = 66.8\,rad/s$$

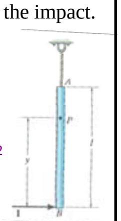
(c) Spinning freely: (also smooth bearing)

$$I_G\omega_0 + 17.183 = 0.25\omega + I_G\omega_0 \Rightarrow \quad \omega = 68.7\,rad/s$$

4. The slender rod has a mass m and is suspended at its end A by a cord. If the rod receives a horizontal blow giving it an impulse $\mathbf{I}$ at its bottom B, determine the location y of the point P about which the rod appears to rotate during the impact.

SOLUTION

$$I_G = \frac{1}{12} m \ell^2$$



Linear momentum: $\quad m(v_{Gx})_1 + \mathrm{I} = m(v_{Gx})_2$

Angular momentum: $\quad I_G\,\omega_1 + (\ell/2)\mathrm{I} = I_G\,\omega_2$

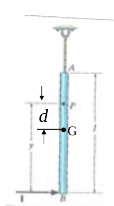
$$(v_{Gx})_1 = \omega_1 = 0 \Rightarrow \left\{\begin{aligned} \mathrm{I} &= m(v_{Gx})_2 \\ \frac{\ell}{2}\mathrm{I} &= \frac{1}{12} m \ell^2 \omega_2\end{aligned}\right. \Rightarrow \left\{\begin{aligned} (v_{Gx})_2 &= \frac{\mathrm{I}}{m} \\ \omega_2 &= \frac{6\mathrm{I}}{m\ell}\end{aligned}\right.$$

5

$$\mathbf{v}_P = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{P/G} = v_{Gx}\mathbf{i} + (\omega\mathbf{k}) \times (d\,\mathbf{j})$$

$$0 = (v_{Gx})_2 - \omega_2 d$$

$$d = \frac{(v_{Gx})_2}{\omega_2} = \frac{\mathrm{I}/m}{6\mathrm{I}/m\ell} = \frac{\ell}{6}$$



$$y = \frac{\ell}{2} + \frac{\ell}{6} = \frac{2\ell}{3}$$

6

5. The platform swing consists of a 200 lb flat plate suspended by four rods of negligible weight. When the swing is at rest, the 150 lb man jumps off the platform when his center of gravity G is 10 ft from the pin at A. This is done with a horizontal velocity of 5 ft/s, measured relative to the swing at the level of G. Determine the angular velocity he imparts to the swing just after jumping off.

SOLUTION

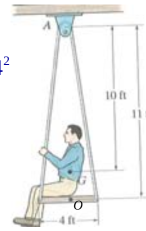
$$I_{PO} = \frac{1}{12} m d^2 = \frac{1}{12} \times 6.21 \times 4^2$$

$$I_{PO} = 8.28\, slug \cdot ft^2$$

$$I_{PA} = I_{PO} + m_P d_{OA}^2$$

$$I_{PA} = 8.28 + 6.21 \times 11^2$$

$$I_{PA} = 759.7\, slug \cdot ft^2$$



Data:
$W_P = 200\, lb$
$W_M = 150\, lb$
$m_P = 6.21\, slug$
$m_M = 4.658\, slug$
$v_{GM} = 5\, ft/s$
$\omega_P = ?$

7

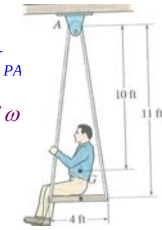
$$(H_A)_1 = (H_A)_2$$

$$0 = -r_{G/A}\, m_M\, v_M + r_{O/A}\, m_P\, v_P + I_{PA}$$

$$0 = -10 \times 4.658 \times 5 + 11 \times 6.21 \times 11 \omega + 759.7\, \omega$$

$$232.9 = 1511.1 \omega$$

$$\omega = 0.154\, rad/s$$



8

6. The pendulum consists of a 5 lb slender rod AB and a 10 lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile imbeds itself into the block, Determine the angular velocity of the pendulum just after impact.

SOLUTION

$$I_{RA} = \frac{1}{3} m \ell^2 = \frac{1}{3} \times 0.15528 \times 2^2$$

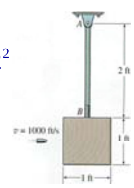
$$I_{RA} = 0.207\, slug \cdot ft^2$$

$$I_{BA} = \frac{1}{6} m d^2 + m r^2$$

$$= (0.31677 + 0.00621) \times \left(\frac{1}{6} \times 1^2 + 2.5^2 \right)$$

$$I_{BA} = 2.0326\, slug \cdot ft^2$$

(in I_{BA} the mass includes both the block and the projectile)



Data:
$W_R = 5\, lb$
$W_B = 10\, lb$
$W_p = 0.2\, lb$
$m_R = 0.15528\, slug$
$m_B = 0.31056\, slug$
$m_p = 0.00621\, slug$
$(v_p)_1 = 1000\, ft/s$
$\omega_1 = 0$
$\omega_2 = ?$

9

$$(H_A)_1 = (H_A)_2$$

$$(H_A)_1 = r_{B/A}\, m_p\, (v_p)_1$$

$$= 2.5 \times 0.00621 \times 1000$$

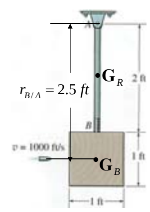
$$(H_A)_1 = 15.525\, slug \cdot m^2/s$$

$$(H_A)_2 = I_{RA}\, \omega + I_{BA}\, \omega$$

$$= 0.207\, \omega + 2.0326\, \omega$$

$$(H_A)_2 = 2.2396\, \omega$$

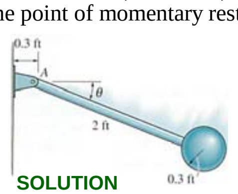
$$\omega = 6.93\, rad/s$$



Note: If we neglect the mass of the projectile in block B we get $I_{BA} = 1.9927$ and $\omega = 7.06$ (a 1.7% difference)

10

7. The pendulum consists of a 10 lb solid ball and 4 lb rod. If it is released from rest when $\theta_1 = 0$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take $e = 0.6$.



Data:
$W_B = 10\, lb$
$W_R = 4\, lb$
$m_B = 0.31056\, slug$
$m_R = 0.12422\, slug$
$\omega_1 = 0$
$e = 0.6$

SOLUTION

$$I_{G(B)} = \frac{2}{5} m_B r_B^2 = \frac{2}{5} \times 0.31056 \times 0.3^2 = 0.01118\, slug \cdot ft^2$$

$$I_{A(R)} = \frac{1}{3} m_R \ell^2 = \frac{1}{3} \times 0.12422 \times 2^2 = 0.16563\, slug \cdot ft^2$$

$$I_{A(B)} = I_{G(B)} + m_B r_{B/A}^2 = 0.01118 + 0.31056 \times 2.3^2 = 1.654\, slug \cdot ft^2$$

$$I_A = I_{A(R)} + I_{A(B)} = 1.8197\, slug \cdot ft^2$$

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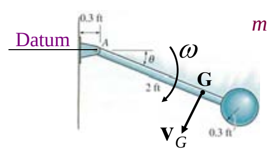

$$m = m_R + m_B = 0.31056 + 0.12422$$

$$m = 0.43478\, slug$$

$$r_{G/A} = \frac{0.12422 \times 1 + 0.31056 \times 2.3}{0.43478}$$

$$r_{G/A} = 1.92858$$

1) Velocity just before striking the wall:

$$W = W_R + W_B = 14\, lb$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 = \frac{1}{2} I_A \omega_1^2 + \frac{1}{2} m v_G^2 + W\, r_{G/A}$$

$$0 = \frac{1}{2} \times 1.8197\, \omega_1^2 + \frac{1}{2} \times 0.43478 \times (1.92858\, \omega_1)^2 - 14 \times 1.92858$$

$$\omega_1 = -3.9639\, rad/s$$

$$(v_{Bx})_1 = -9.1169\, ft/s \quad (v_{Bx} = 2.3\omega)$$

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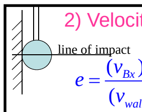
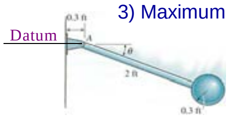
2) Velocity just after impact with the wall, central impact:



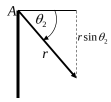
$$e=\frac{(v_{Bx})_2-(v_{wall})_2}{(v_{wall})_1-(v_{Bx})_1}=-\frac{(v_{Bx})_2}{(v_{Bx})_1}\Rightarrow(v_{Bx})_2=-0.6\times(-9.1169)$$

$$(v_{Bx})_2=5.4701\ ft/s\ ,\ \omega_2=2.3783\,rads$$

3) Maximum return angle:



$$T_2+V_2=T_3+V_3$$



$$\frac{1}{2}I_A\omega_2^2+\frac{1}{2}mv_G{}^2-W\,r_{G/A}=0-W\,r_{G/A}\sin\theta_2$$

$$\frac{1}{2}\times1.8197\times2.3783^2+\frac{1}{2}\times0.43478\times(1.92858\times2.3783)^2$$

$$-14\times1.92858=-14\times1.92858\times\sin\theta_2$$

13

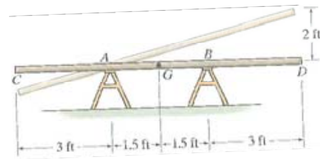
$$-17.2801=-27\sin\theta_2$$

$$\sin\theta_2=0.64$$

$$\theta_2=39.8^\circ$$

8. The plank has a weight of 30 *lb*, center of gravity at *G*, and it rests on the two sawhorses *A* and *B*. If the end *D* is raised 2 *ft* above the top of the sawhorses and is released from rest, determine how high end *C* will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about *A*, stikes and pivots on the saw horses at *B*, and rotates clockwise off the sawhorses at *A*.

Data:
$W=30\,lb$
$m=0.93168\,slug$



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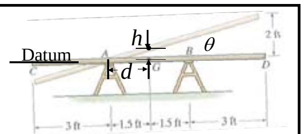
SOLUTION



$$\sin\theta=\frac{2}{6}\Rightarrow\theta=19.47^\circ$$

$$h=1.5\sin\theta=0.5\ ft$$

$$I_G=\frac{1}{12}m\ell^2=6.2888\,slug\cdot ft^2$$

$$I_A=I_G+md^2=6.2888+0.93168\times1.5^2=8.3851\,slug\cdot ft^2$$

1) Velocity right before striking sawhorse B:

$$T_0+V_0=T_1+V_1$$

$$0+Wh=\frac{1}{2}I_A\omega_1^2+0\ \Rightarrow\ 30\times0.5=0.5\times8.3851\omega_1^2$$

$$\omega_1=-1.8915\ rad/s$$

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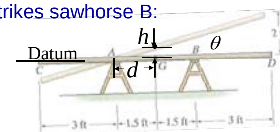
2) Velocity right after the plank strikes sawhorse B:



$$(H_A)_1=(H_A)_2$$

$$I_A\omega_1+d\,m(v_G)_1=I_A\omega_2+d\,m(v_G)_2$$

$$8.3851\times(-1.8915)-1.5\times0.93168\times[(-1.8915)\times1.5]=$$
$$8.3851\times\omega_2+1.5\times0.93168\times(1.5\times\omega_2)$$

$$\omega_2=-\omega_1=1.892\quad\Rightarrow\quad\theta_2=\theta_1$$

This happens because the losses at impact in *B* have not been taken into account. For this we need to add in the left hand side the term $\int_0^t 2d\,F\,dt$ which is the impulse due to deformation when the plank hits the sawhorse *B*. However we have no iformation about this force *F*.

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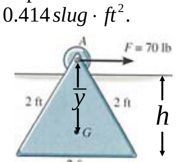
9. The uniform plate weights 40 *lb* and is supported by a roller at *A*.If a horizontal force *F* = 70 *lb* is suddenly applied to the roller, determine the acceleration of the center of mass of the roller at the instant the force is applied. The plate has a moment of inertia about its center of mass of $I_G=0.414\,slug\cdot ft^2$. Neglect the weight of the roller.

SOLUTION



$$\bar{y}=\frac{h}{3}=\frac{2\sqrt{3}}{3}=1.1547\ ft$$

1) $\sum F_x=F=m\,a_{Gx}\Rightarrow70=1.2422\,a_{Gx}$

$$a_{Gx}=56.352\ ft/s^2$$

Data:
$W=40\,lb$
$m=1.2422\,slug$
$I_G=0.414\,slug\cdot ft^2$

2) $\sum M_A=0=-m\,a_{Gx}\bar{y}+I_G\alpha$

$$-1.2422\times56.352\times1.1547+0.414\alpha=0$$

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$$\alpha=195.24\,rad/s^2$$

3) $\mathbf{a}_A=\mathbf{a}_G+\boldsymbol{\alpha}\times\mathbf{r}_{A/G}-\omega^2\mathbf{r}_{A/G}$

$$a_A\mathbf{i}=56.532\,\mathbf{i}+(195.24\,\mathbf{k})\times(1.1547\,\mathbf{j})+0$$

$$a_A=282\ ft/s^2$$

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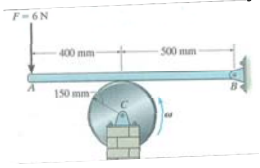
10. The 15 kg cylinder is rotating with an angular velocity of 40 rad/s. If a force $F = 6N$ is applied to link AB, as shown, determine the time needed to stop the rotation. The coefficient of kinetic friction etween AB and the cylinder is $\mu_k = 0.4$.



Data:
$m = 15\,kg$
$\omega = 40\,rad/s$
$F = 6\,N$
$\mu_k = 0.4$

SOLUTION

To find **N** take moments about B to avoid finding those reactions.

1) $\sum M_B = F \times 0.9 - N \times 0.5 = 0$

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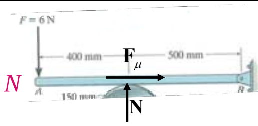
$N = 10.8(N)$

$F_\mu = 0.4\,N = 0.4 \times 10.8 = 4.32\,N$



2) $(\mathbf{H}_C)_1 + \int_0^t \mathbf{M}_C\,dt = 0$

$(H_C)_1 = I_C\omega$

$I_C = \frac{1}{2} m r^2 = \frac{1}{2} \times 15 \times 0.15^2 = 0.16875\,kg \cdot m^2$

$I_C\omega + \int_0^t (F_\mu r)\,dt = 0$

$0.1688 \times 40 - \int_0^t (4.32 \times 0.15)\,dt = 0$

$6.752 - 6.48t = 0 \Rightarrow$ $t = 10.42\,s$

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