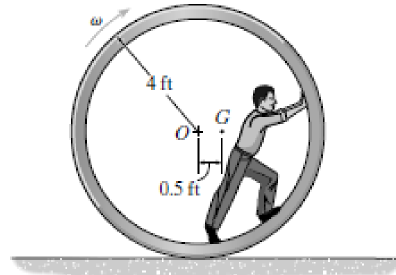
***17–100.** The circular concrete culvert rolls with an angular velocity of $\omega = 0.5$ rad/s when the man is at the position shown. At this instant the center of gravity of the culvert and the man is located at point G, and the radius of gyration about G is $k_G = 3.5$ ft. Determine the angular acceleration of the culvert. The combined weight of the culvert and the man is 500 lb. Assume that the culvert rolls without slipping, and the man does not move within the culvert.



Equations of Motion: The mass moment of inertia of the system about its mass center is $I_G = mk_G{}^2 = \frac{500}{32.2}(3.5^2) = 190.22\ \text{slug}\cdot\text{ft}^2$. Writing the moment equation of motion about point A, Fig. a,

$$+\Sigma M_A = \Sigma(M_k)_A; \quad -500(0.5) = -\frac{500}{32.2}(a_G)_x(4) - \frac{500}{32.2}(a_G)_y(0.5) - 190.22\alpha \quad \textbf{(1)}$$

Kinematics: Since the culvert rolls without slipping,

$$a_0 = \alpha r = \alpha(4) \rightarrow$$

Applying the relative acceleration equation and referring to Fig. b,

$$a_G = a_O + \alpha \times r_{G/O} - \omega^2\, \mathbf{r}_{G/A}$$

$$(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = 4\alpha\mathbf{i} + (-\alpha\mathbf{k}) \times (0.5\mathbf{i}) - 0.5^2(0.5\mathbf{i})$$

$$(a_G)_x\,\mathbf{i} - (a_G)_y\,\mathbf{j} = (4\alpha - 0.125)\mathbf{i} - 0.5\alpha\mathbf{j}$$

Equating the **i** and **j** components,

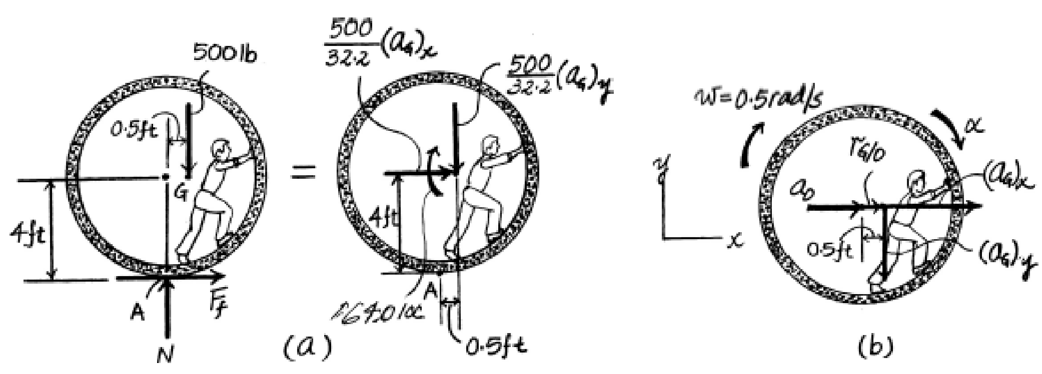
$$(a_G)_x = 4\alpha - 0.125 \quad \textbf{(2)}$$

$$(a_G)_y = 0.5\alpha \quad \textbf{(3)}$$

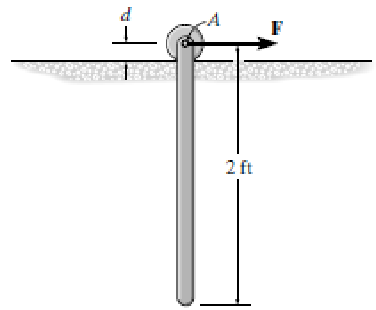
Substituting Eqs. (2) and (3) into Eq. (1),

$$-500(0.5) = -\frac{500}{32.2}(4\alpha - 0.125)(4) - \frac{500}{32.2}(0.5\alpha)(0.5) - 190.22\alpha$$

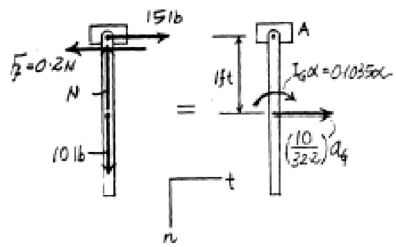
$$\alpha = 0.582\ \text{rad/s}^2 \quad \textbf{Ans.}$$



•**17–109.** Solve Prob. 17–108 assuming that the roller at A is replaced by a slider block having a negligible mass. The coefficient of kinetic friction between the block and the track is $\mu_k = 0.2$. Neglect the dimension d and the size of the block in the computations.



Equations of Motion: The mass moment of inertia of the rod about its mass center is given by $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{10}{32.2}\right)(2^2) = 0.1035 \text{ slug}\cdot\text{ft}^2$. At the instant force **F** is applied, the angular velocity of the rod $\omega = 0$. Thus, the normal component of acceleration of the mass center for the rod $(a_G)_n = 0$. Applying Eq. 17–16, we have



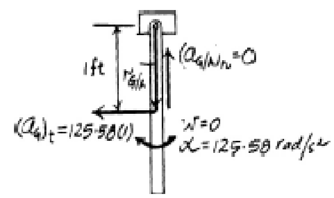
$$\Sigma F_n = m(a_G)_n; \qquad 10 - N = 0 \qquad N = 10.0 \text{ lb}$$

$$\Sigma F_t = m(a_G)_t; \qquad 15 - 0.2(10.0) = \left(\frac{10}{32.2}\right)a_G \qquad a_G = 41.86 \text{ ft/s}^2$$

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 0 = \left(\frac{10}{32.2}\right)(41.86)(1) - 0.1035\alpha$$

$$\alpha = 125.58 \text{ rad/s}^2$$

Kinematics: Since $\omega = 0$, $(a_{G/A})_n = 0$. The acceleration of block A can be obtain by analyzing the motion of points A and G. Applying Eq. 16–17, we have



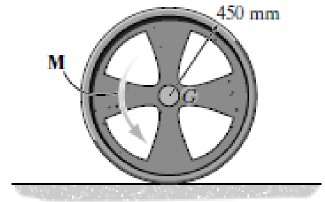
$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$\begin{bmatrix} 41.86 \\ \rightarrow \end{bmatrix} = \begin{bmatrix} a_A \\ \rightarrow \end{bmatrix} + \begin{bmatrix} 125.58(1) \\ \leftarrow \end{bmatrix} + [0]$$

$(\overset{+}{\rightarrow})$ $$41.86 = a_A - 125.58$$

$$a_A = 167 \text{ ft/s}^2$$ **Ans.**

17–122. The 75-kg wheel has a radius of gyration about its mass center of $k_G = 375$ mm. If it is subjected to a torque of $M = 150\ \text{N}\cdot\text{m}$, determine its angular acceleration. The coefficients of static and kinetic friction between the wheel and the ground are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively.



Equations of Motion: The mass moment of inertia of the wheel about its mass center is $I_G = mk_G{}^2 = 75\left(0.375^2\right) = 10.55\ \text{kg}\cdot\text{m}^2$. Writing the moment equation of motion about point A, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(\mu_k)_A;\quad 150 = 75a_G(0.45) + 10.55\alpha \qquad \textbf{(1)}$$

Assuming that the wheel rolls without slipping,

$$a_G = \alpha r_G = \alpha(0.45) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$a_G = 2.623\ \text{m/s}^2$$

$$\alpha = 5.829\ \text{rad/s}^2$$

Writing the force equations of motion along the x and y axes,

$$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N - 75(9.81) = 0 \qquad N = 735.75\ \text{N}$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad F_f = 75(2.623) = 196.72\ \text{N}$$

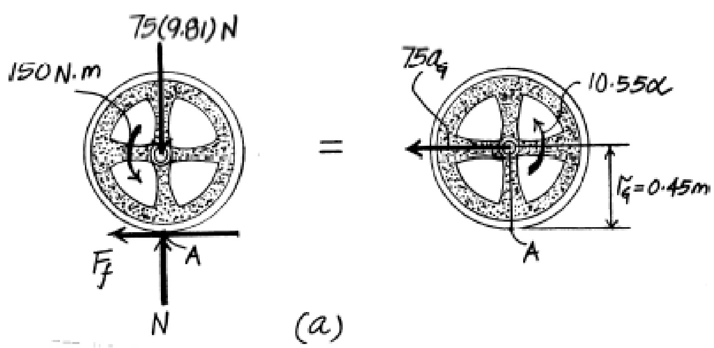
Since $F_f > \mu_k N = 0.2(735.75) = 147.15$ N, the wheel slips. The solution must be reworked using $F_f = \mu_k N = 0.15(735.75) = 110.36$ N. Thus,

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 110.36 = 75a_G \qquad a_G = 1.4715\ \text{m/s}^2$$

Substituting this result into Eq. (1), we obtain

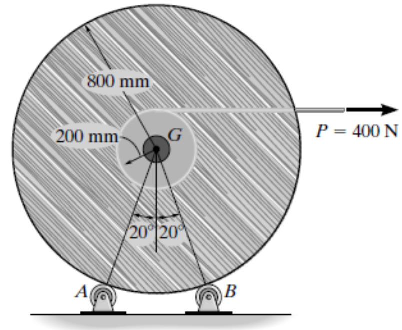
$$150 = 75(1.4715)(0.45) + 10.55\alpha$$

$$\alpha = 9.513\ \text{rad/s}^2 = 9.51\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



(a)

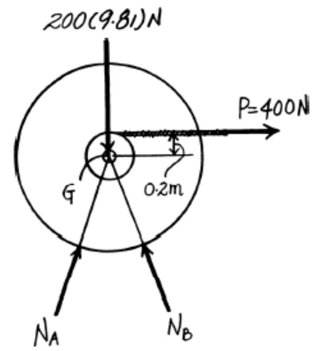
*18–4. The spool of cable, originally at rest, has a mass of 200 kg and a radius of gyration of $k_G = 325$ mm. If the spool rests on two small rollers A and B and a constant horizontal force of $P = 400$ N is applied to the end of the cable, determine the angular velocity of the spool when 8 m of cable has been unwound. Neglect friction and the mass of the rollers and unwound cable.



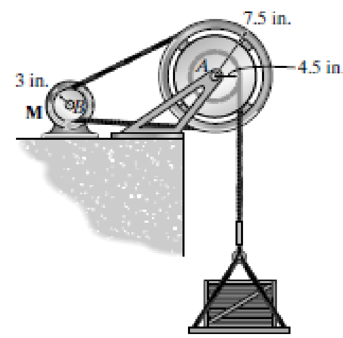
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + (400)(8) = \frac{1}{2}\left[200(0.325)^2\right]\omega_2^2$$

$$\omega_2 = 17.4 \text{ rad/s}$$ **Ans.**



18–19. The wheel and the attached reel have a combined weight of 50 lb and a radius of gyration about their center of $k_A = 6$ in. If pulley B that is attached to the motor is subjected to a torque of $M = 50$ lb · ft, determine the velocity of the 200-lb crate after the pulley has turned 5 revolutions. Neglect the mass of the pulley.



Kinetic Energy and Work: Since the wheel at A rotates about a fixed axis, $v_C = \omega r_C = \omega(0.375)$. The mass moment of inertia of wheel A about its mass center is $I_A = mk_A{}^2 = \left(\frac{50}{32.2}\right)(0.5^2) = 0.3882$ slug · ft². Thus, the kinetic energy of the system is

$$T = T_A + T_C$$

$$= \frac{1}{2} I_A\,\omega^2 + \frac{1}{2} m_C\,v_C{}^2$$

$$= \frac{1}{2}(0.3882)\omega^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left[\omega(0.375)\right]^2$$

$$= 0.6308\omega^2$$

Since the system is initially at rest, $T_1 = 0$. Referring to Fig. b, $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{W}_A$ do no work, $\mathbf{M}$ does positive work, and $\mathbf{W}_C$ does negative work. When pulley B rotates $\theta_B = (5 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 10\pi$ rad, the wheel rotates through an angle of $\theta_A = \frac{r_B}{r_A}\theta_B = \left(\frac{0.25}{0.625}\right)(10\pi) = 4\pi$. Thus, the crate displaces upwards through a distance of $s_C = r_C\,\theta_A = 0.375(4\pi) = 1.5\pi$ ft. Thus, the work done by $\mathbf{M}$ and $\mathbf{W}_C$ is

$U_M = M\theta_B = 50(10\pi) = 500\pi$ ft · lb

$U_{W_C} = -W_C s_C = -200(1.5\pi) = -300\pi$ ft · lb

Principle of Work and Energy:

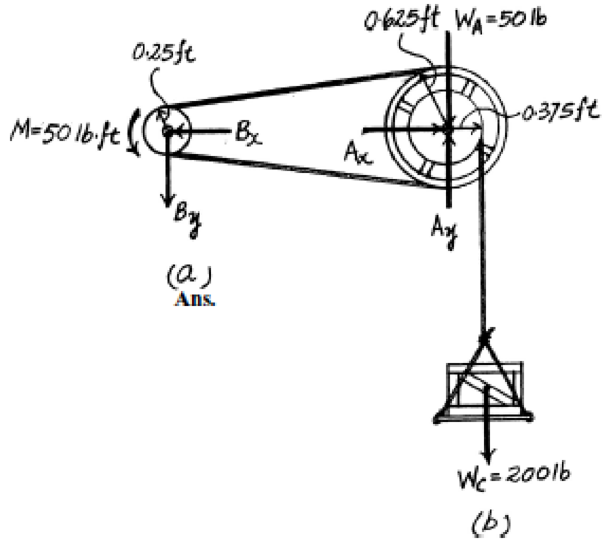
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [500\pi - 300\pi] = 0.6308\omega^2$$

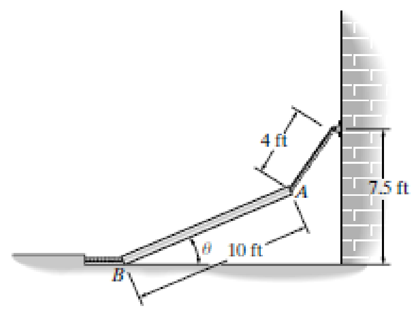
$$\omega = 31.56 \text{ rad/s}$$

Thus,

$$v_C = 31.56(0.375) = 11.8 \text{ ft/s}\uparrow$$ **Ans.**



18–31. The slender beam having a weight of 150 lb is supported by two cables. If the cable at end *B* is cut so that the beam is released from rest when $\theta = 30°$, determine the speed at which end *A* strikes the wall. Neglect friction at *B*.

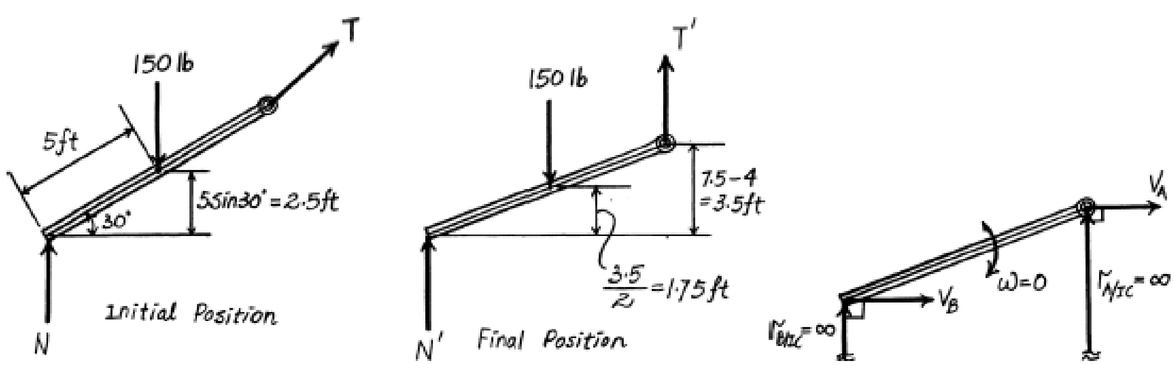


In the final position, the rod is in translation since the *IC* is at infinity.

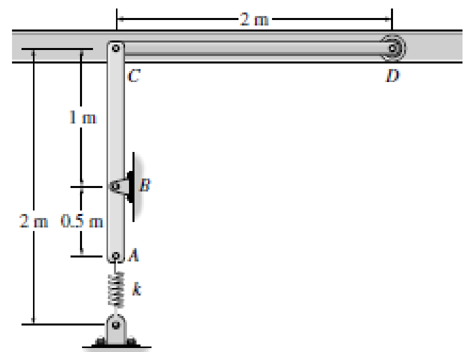
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150(2.5 - 1.75) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_G^2$$

$$v_G = v_A = 6.95 \text{ ft/s}$$ **Ans.**



•**18–49.** The garage door *CD* has a mass of 50 kg and can be treated as a thin plate. Determine the required unstretched length of each of the two side springs when the door is in the open position, so that when the door falls freely from the open position it comes to rest when it reaches the fully closed position, i.e., when *AC* rotates 180°. Each of the two side springs has a stiffness of $k = 350$ N/m. Neglect the mass of the side bars *AC*.



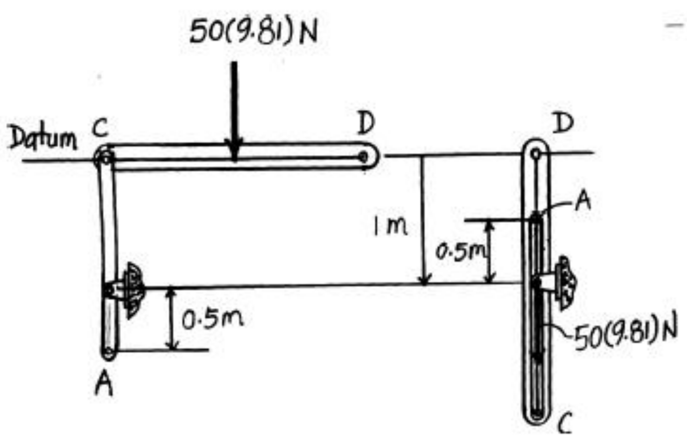
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}(350)(x_1)^2\right] = 0 + 2\left[\frac{1}{2}(350)(x_1 + 1)^2\right] - 50(9.81)(1)$$

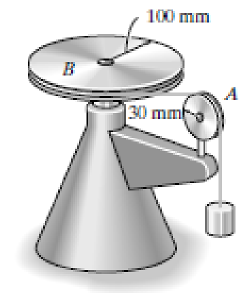
$$x_1 = 0.201 \text{ m}$$

Thus,

$$l_0 = 0.5 \text{ m} - 0.201 \text{ m} = 299 \text{ mm}$$ **Ans.**



18–60. The assembly consists of a 3-kg pulley A and 10-kg pulley B. If a 2-kg block is suspended from the cord, determine the block's speed after it descends 0.5 m starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.



$$T_1 + V_1 = T_2 + V_2$$

$$[0 + 0 + 0] + [0] = \frac{1}{2}\left[\frac{1}{2}(3)(0.03)^2\right]\omega_A^2 + \frac{1}{2}\left[\frac{1}{2}(10)(0.1)^2\right]\omega_B^2 + \frac{1}{2}(2)(v_C)^2 - 2(9.81)(0.5)$$

$$v_C = \omega_B(0.1) = 0.03\,\omega_A$$

Thus,

$$\omega_B = 10\,v_C$$

$$\omega = 33.33 v_C$$

Substituting and solving yields,

$$v_C = 1.52 \text{ m/s}$$ **Ans.**