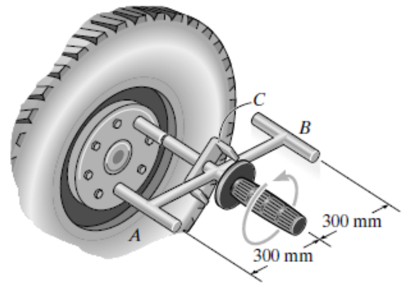
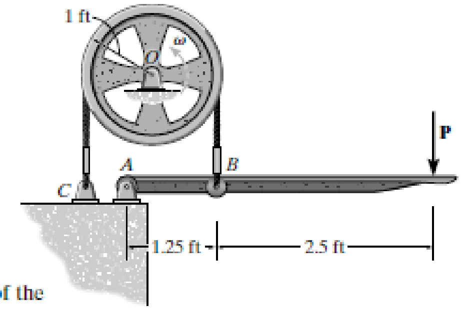
19–6. The impact wrench consists of a slender 1-kg rod AB which is 580 mm long, and cylindrical end weights at A and B that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to rotate about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod AB is given an angular velocity of 4 rad/s and it strikes the bracket C on the handle without rebounding, determine the angular impulse imparted to the lug nut.



$$I_{axle} = \frac{1}{12}(1)(0.6 - 0.02)^2 + 2\left[\frac{1}{2}(1)(0.01)^2 + 1(0.3)^2\right] = 0.2081\ \text{kg}\cdot\text{m}^2$$

$$\int M dt = I_{axle}\,\omega = 0.2081(4) = 0.833\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

•**19–13.** The 200-lb flywheel has a radius of gyration about its center of gravity O of $k_O = 0.75$ ft. If it rotates counterclockwise with a constant angular velocity of 1200 rev/min before the brake is applied, determine the required force **P** that must be applied to the handle to stop the wheel in 2 s. The coefficient of kinetic friction between the belt and the wheel rim is $\mu_k = 0.3$. (*Hint:* Recall from the statics text that the relation of the tension in the belt is given by $T_B = T_C e^{\mu\beta}$, where β is the angle of contact in radians.)



Principle of Angular Impulse and Momentum: The mass moment of inertia of the wheel about its mass center is $I_O = mk_O^2 = \left(\frac{200}{32.2}\right)(0.75^2) = 3.494 \text{ slug}\cdot\text{ft}^2$, and the initial angular velocity of the wheel is $\omega_1 = \left(1200\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 40\pi$ rad/s. Applying the angular impulse and momentum equation about point O using the free-body diagram shown in Fig. *a*,

$$\circlearrowleft + \quad I_O\omega_1 + \Sigma\int_{t_1}^{t_2} M_O\,dt = I_O\omega_2$$

$$3.494(40\pi) + T_C(2)(1) - T_B(2)(1) = 0$$

$$T_B - T_C = 219.52 \qquad \textbf{(1)}$$

Using the belt friction formula,

$$T_B = T_C e^{\mu\beta}$$

$$T_B = T_C e^{0.3(\pi)} \qquad \textbf{(2)}$$

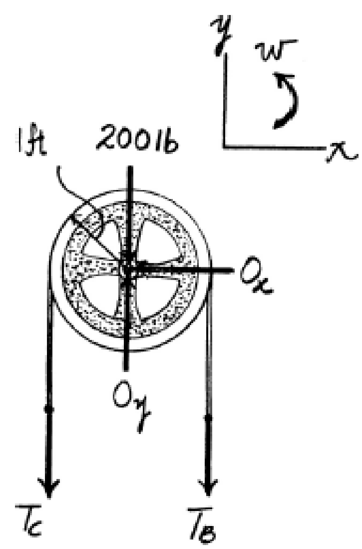
Solving Eqs. (1) and (2),

$$T_C = 140.15 \text{ lb} \qquad T_B = 359.67 \text{ lb}$$

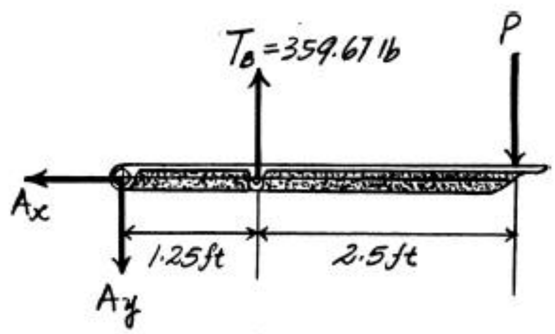
Equilibrium: Using this result and writing the moment equation of equilibrium about point A using the free-body diagram of the brake arm shown in Fig. *b*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 359.67(1.25) - P(3.75) = 0$$

$$P = 120 \text{ lb} \qquad \textbf{Ans.}$$

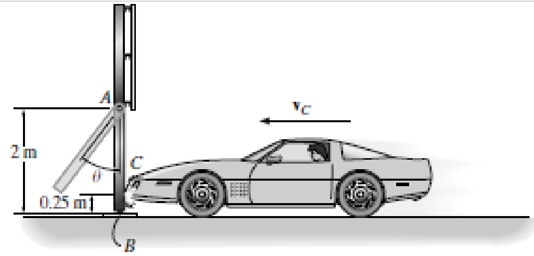


(a)



(b)

•**19–21.** For safety reasons, the 20-kg supporting leg of a sign is designed to break away with negligible resistance at B when the leg is subjected to the impact of a car. Assuming that the leg is pinned at A and approximates a thin rod, determine the impulse the car bumper exerts on it, if after the impact the leg appears to rotate clockwise to a maximum angle of $\theta_{max} = 150°$.



(+↻) $$I_A \omega_1 + \Sigma \int_{t_1}^{t_2} M_A\, dt = I_A \omega_2$$

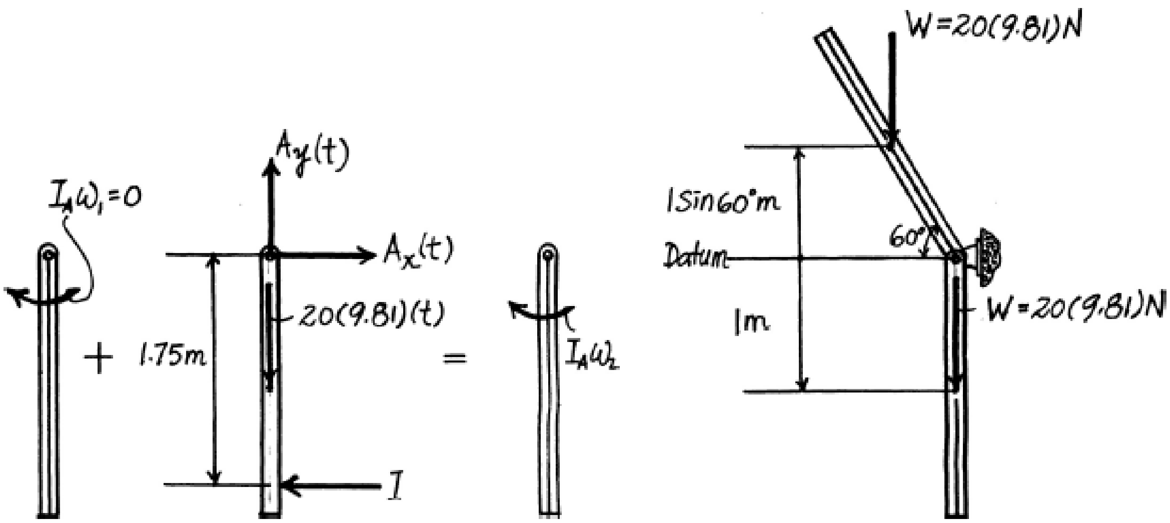
$$0 + I(1.75) = \left[\frac{1}{3}(20)(2)^2\right]\omega_2$$

$$\omega_2 = 0.065625I$$

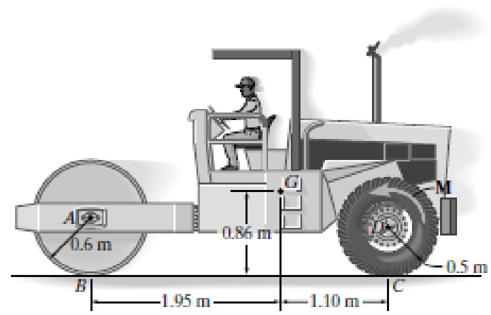
$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}\left[\frac{1}{3}(20)(2)^2\right](0.065625I)^2 + 20(9.81)(-1) = 0 + 20(9.81)(1 \sin 60°)$$

$I = 79.8$ N · s **Ans.**



19–30. The frame of the roller has a mass of 5.5 Mg and a center of mass at G. The roller has a mass of 2 Mg and a radius of gyration about its mass center of $k_A = 0.45$ m. If a torque of $M = 600$ N·m is applied to the rear wheels, determine the speed of the compactor in $t = 4$ s, starting from rest. No slipping occurs. Neglect the mass of the driving wheels.



Driving Wheels: (mass is neglected)

$$\circlearrowleft + \Sigma M_D = 0; \qquad 600 - F_C(0.5) = 0$$

$$F_C = 1200 \text{ N}$$

Frame and driving wheels:

$$(\overset{+}{\leftarrow}) \qquad m(v_{Gx})_1 + \Sigma \int F_x\, dt = m(v_{Gx})_2$$

$$0 + 1200(4) - A_x(4) = 5500 v_G$$

$$A_x = 1200 - 1375 v_G \qquad \textbf{(1)}$$

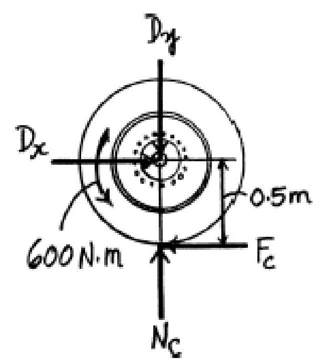
Roller:

$$v_G = v_A = 0.6\omega$$

$$(\circlearrowleft +) \qquad (H_B)_1 + \Sigma \int M_B\, dt = (H_B)_2$$

$$0 + A_x(4)(0.6) = \left[2000(0.45)^2\right]\left(\frac{v_G}{0.6}\right) + \left[2000(v_G)\right](0.6)$$

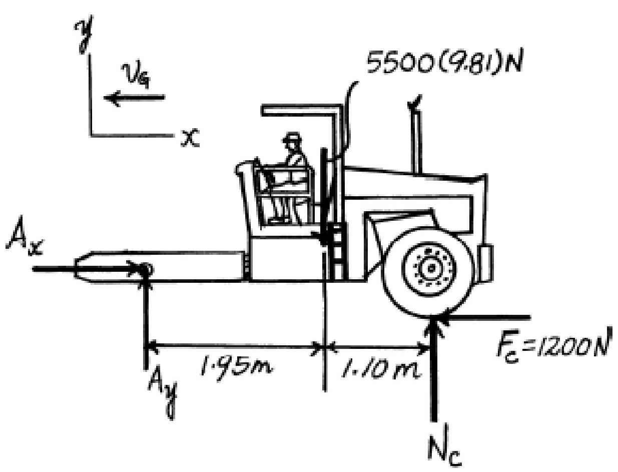
$$A_x = 781.25 v_G \qquad \textbf{(2)}$$

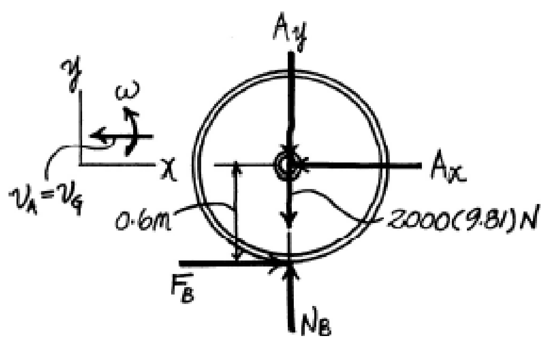


Solving Eqs. (1) and (2):

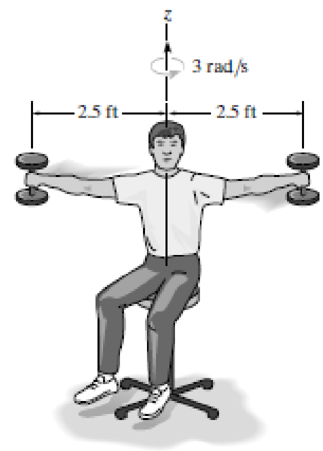
$$A_x = 435 \text{ N}$$

$$v_G = 0.557 \text{ m/s} \qquad \textbf{Ans.}$$





•**19–37.** The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is rotating at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration $k_z = 0.55$ ft about the z axis. Neglect the mass of his arms and the size of the weights for the calculation.



Mass Moment of Inertia: The mass moment inertia of the man and the weights about z axis when the man arms are fully stretched is

$$(I_z)_1 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(2.5^2)\right] = 3.444 \text{ slug}\cdot\text{ft}^2$$

The mass moment inertia of the man and the weights about z axis when the weights are drawn in to a distance 0.3 ft from z axis

$$(I_z)_2 = \left(\frac{160}{32.2}\right)(0.55^2) + 2\left[\frac{5}{32.2}(0.3^2)\right] = 1.531 \text{ slug}\cdot\text{ft}^2$$

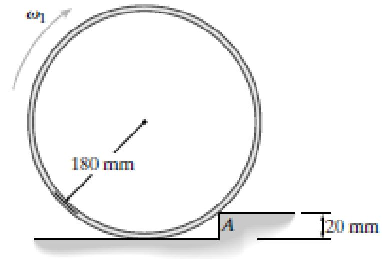
Conservation of Angular Momentum: Applying Eq. 19–17, we have

$$(H_z)_1 = (H_z)_2$$

$$3.444(3) = 1.531(\omega_z)_2$$

$$(\omega_z)_2 = 6.75 \text{ rad/s} \qquad \textbf{Ans.}$$

***19–44.** The 15-kg thin ring strikes the 20-mm-high step. Determine the smallest angular velocity ω_1 the ring can have so that it will just roll over the step at A without slipping



The weight is non-impulsive.

$$(H_A)_1 = (H_A)_2$$

$$15(\omega_1)(0.18)(0.18 - 0.02) + \left[15(0.18)^2\right](\omega_1) = \left[15(0.18)^2 + 15(0.18)^2\right]\omega_2$$

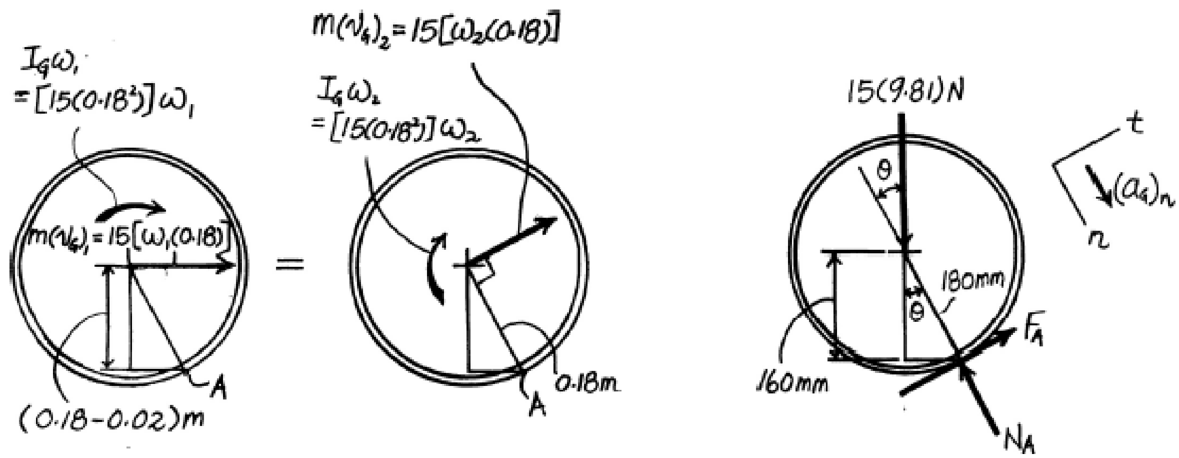
$$\omega_2 = 0.9444\omega_1$$

$$+\searrow \Sigma F_n = m(a_G)_n; \qquad (15)(9.81)\cos\theta - N_A = 15\omega_2^2(0.18)$$

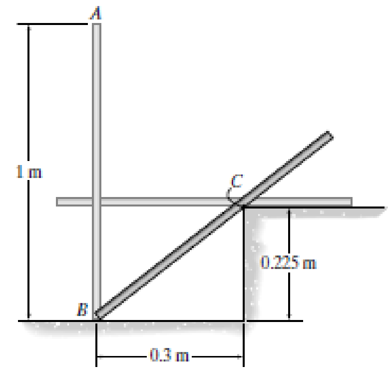
When hoop is about to rebound, $N_A - 0$. Also, $\cos\theta = \frac{160}{180}$, and so

$$\omega_2 = 6.9602 \text{ rad/s}$$

$$\omega_1 = \frac{6.9602}{0.9444} = 7.37 \text{ rad/s} \qquad \textbf{Ans.}$$



•**19–49.** The uniform 6-kg slender rod AB is given a slight horizontal disturbance when it is in the vertical position and rotates about B without slipping. Subsequently, it strikes the step at C. The impact is perfectly plastic and so the rod rotates about C without slipping after the impact. Determine the angular velocity of the rod when it is in the horizontal position shown.



Conservation of Energy: From the geometry of Fig. a, $\theta = \tan^{-1}\left(\frac{0.225}{0.3}\right) = 36.87°$ and $BC = \sqrt{0.3^2 + 0.225^2} = 0.375$ m. Thus, $r_{CG} = 0.5 - 0.375 = 0.125$ m. With reference to the datum, $V_1 = W(y_G)_1 = 6(9.81)(0.5) = 29.43$ J, $V_2 = V_3 = W(y_G)_3 = 6(9.81)(0.5 \sin 36.87°) = 17.658$ J, and $V_4 = W(y_G)_4 = 6(9.81)(0.225) = 13.2435$ J. Since the rod is initially at rest, $T_1 = 0$. The rod rotates about point B before impact. Thus, $(v_G)_2 = \omega_2 r_{BG} = \omega_2(0.5)$. The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(6)(1^2) = 0.5 \text{ kg}\cdot\text{m}^2$. Then, $T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(6)\left[\omega_2(0.5)\right]^2 + \frac{1}{2}(0.5)\omega_2^2 = 1\omega_2^2$. Therefore,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 29.43 = 1\omega_2^2 + 17.658$$

$$\omega_2 = 3.431 \text{ rad/s}$$

The rod rotates about point C after impact. Thus, $v_G = \omega r_{CG} = \omega(0.125)$. Then,

$$T = \frac{1}{2}m(v_G)^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(6)\left[\omega(0.125)\right]^2 + \frac{1}{2}(0.5)\omega^2 = 0.296875\omega^2$$

so that

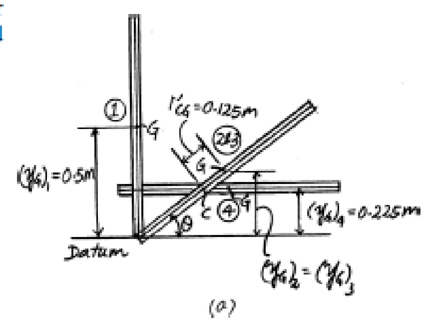
$$T_3 = 0.296875\omega_3^2 \text{ and } T_4 = 0.296875\omega_4^2$$

$$T_3 + V_3 = T_4 + V_4$$

$$0.296875\omega_3^2 + 17.658 = 0.296875\omega_4^2 + 13.2435$$

$$\omega_4^2 - \omega_3^2 = 14.87 \quad \textbf{(1)}$$

Conservation of Angular Momentum: Referring to Fig. b, the sum of the angular impulses about point C is zero. Thus, angular momentum of the rod is conserved about this point during the impact. Then,



$$(H_C)_1 = (H_C)_2$$

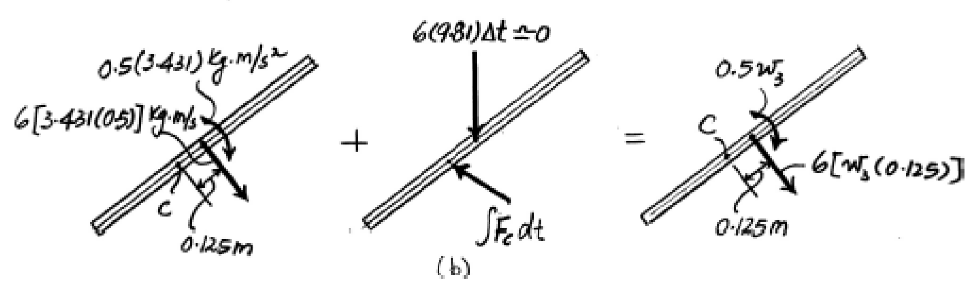
$$6\left[3.431(0.5)\right](0.125) + 0.5(3.431) = 6\left[\omega_3(0.125)\right](0.125) + 0.5\omega_3$$

$$\omega_3 = 5.056 \text{ rad/s}$$

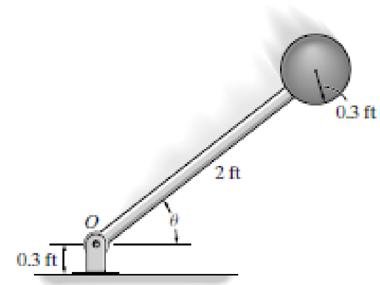
Substituting this result into Eq. (1), we obtain

$$\omega_4^2 - (5.056)^2 = 14.87$$

$$\omega_4 = 6.36 \text{ rad/s} \quad \textbf{Ans.}$$



19–55. The pendulum consists of a 10-lb sphere and 4-lb rod. If it is released from rest when $\theta = 90°$, determine the angle θ of rebound after the sphere strikes the floor. Take $e = 0.8$.



$$I_A = \frac{1}{3}\left(\frac{4}{32.2}\right)(2)^2 + \frac{2}{5}\left(\frac{10}{32.2}\right)(0.3)^2 + \left(\frac{10}{32.2}\right)(2.3)^2 = 1.8197 \text{ slug} \cdot \text{ft}^2$$

Just before impact:

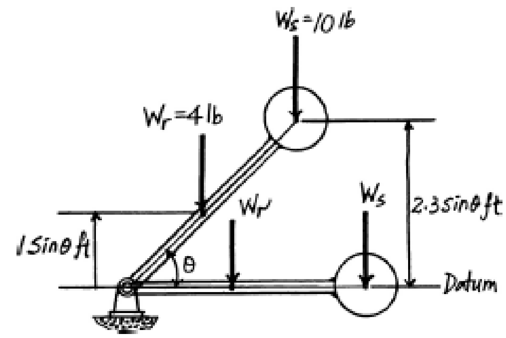
Datum through O.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 4(1) + 10(2.3) = \frac{1}{2}(1.8197)\omega^2 + 0$$

$$\omega_2 = 5.4475 \text{ rad/s}$$

$$v = 2.3(5.4475) = 12.529 \text{ ft/s}$$



Since the floor does not move,

$$(+\uparrow) \qquad e = 0.8 = \frac{(v_P) - 0}{0 - (-12.529)}$$

$$(v_P)_3 = 10.023 \text{ ft/s}$$

$$\omega_3 = \frac{10.023}{2.3} = 4.358 \text{ rad/s}$$

$$T_3 + V_3 = T_4 + V_4$$

$$\frac{1}{2}(1.8197)(4.358)^2 + 0 = 4(1 \sin\theta_1) + 10(2.3 \sin\theta_1)$$

$$\theta_1 = 39.8° \qquad \textbf{Ans.}$$