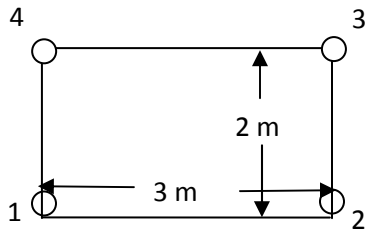
Sample Exam 2 Problems

Problems involving two dimensional elements

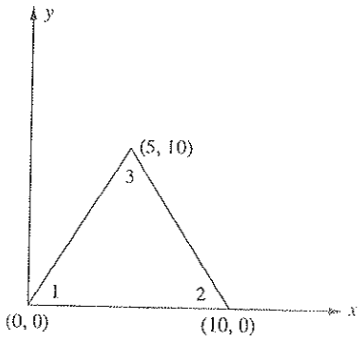
1. For the bilinear membrane finite element in PLANE STRESS shown in the figure, $E = 30 \times 10^6 \, N/m^2$ and $\nu = 0.3$. Forces are applied at the nodes that produce displacements $v_1 = 0.01m$ and $u_3 = -0.01m$ while keeping $u_1 = u_2 = u_4 = v_2 = v_3 = v_4 = 0$.

a) Write the bilinear shape functions for the element (15 points)
b) Find the matrix $\mathbf{B}$ for the element (15 points)
c) Find the strains in the element at the point $(x, y) = (1, 1)$ (10 points)
d) Find the stresses in the element at the $(x, y) = (1, 1)$ (10 points)



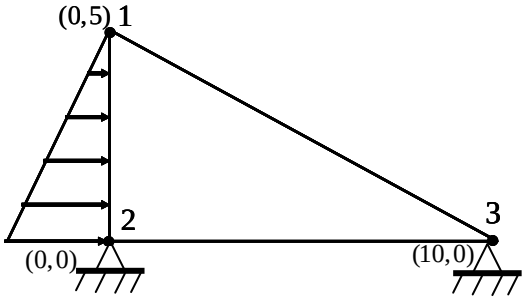
2. The triangular element in the figure has $E = 70GPa$, $\nu = 0.3$, and is such that the shape functions $N_1(x, y)$ and $N_2(x, y)$ are 0.15 and 0.25, respectively, when evaluated at the point P. The nodal coordinates are given in m.

a) Determine the x- and y-coordinates of point P (20 pts)
b) If the element is in a state of plane stress, and the displacements are zero except for $u_3 = 0.001m$ and $v_3 = 0.001m$. Find the strains in the element (20 pts)



1. The PLANE STRESS triangle in the figure is subjected to the distributed load $p_x(y) = 2{,}000(5-x)\,N/m$ on its left side. If $E = 2MPa$ and $\nu = 0.3$:

a) Find the shape functions (5 points)
b) Find the load equivalent nodal forces (15 points)
c) Find the matrix **B** (10 points)
d) Find the displacements at node 1 (15 points). (HINT: you only need the upper left 2 x 2 part of the element stiffness matrix)
e) Find he stresses in the element (5 points)



2. For the plane stress triangle in the figure, determine The forces at the nodes necessary to produce the Displacements:

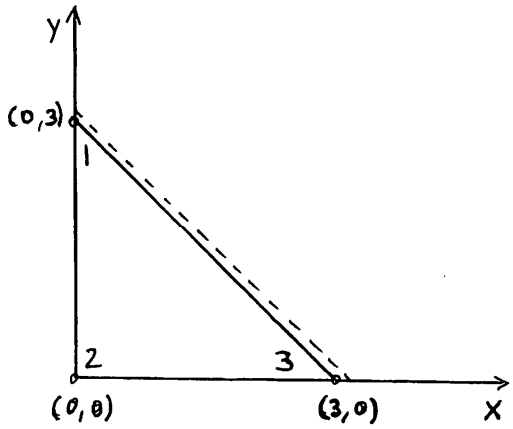
$u_1 = 0.0 \quad v_1 = 0.01$

$u_2 = 0.0 \quad v_2 = 0.0$

$u_3 = 0.01 \quad v_3 = 0.0$

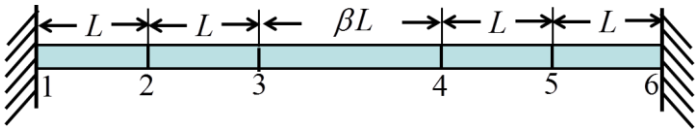
The material properties are (60 points)

$E = 10^7 lb/in^2,\ \nu = 0.3\ and\ t = 0.1in.$

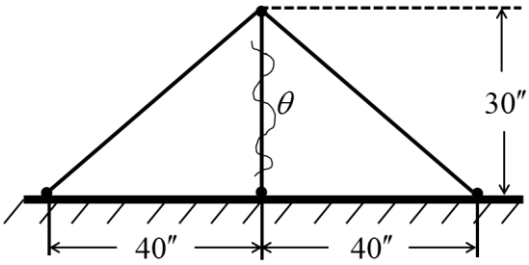


Sample exam problems involving thermal stresses

1. In the one dimensional system shown the central rod element 3-4 is heated to a temperature T. If the Young's modulus of the bars is E and the thermal expansion coefficient is α, calculate: a) The displacements at the nodes (15 pts.)
   b) The stresses in the bars (15 pts.)



2.Three rod elements are connected to a foundation as shown. All elements have a cross-sectional area $A = 2in^2$ and Young's modulus $E = 30\times10^6\ psi$. If the central element is heated to a temperature $T = 50^\circ F$ above the other elements and the coefficient of thermal expansion of the central element is $\alpha = 6.0\times10^{-6}(in/in)/^\circ F$ find the deflections and stresses in the structure. (40 pts.)



3.Three elastic rods are placed between a rigid support and a rigid flange. The block is at a distance δ from another rigid part. All three rods have a cross sectional area *A*, Young's modulus *E* and thermal expansion coefficient α. The middle rod is fully insulated from the rest of the system and is heated by a coil. Find the value of the temperature *T* at which the gap δ becomes closed (25 pts.)